Spring 2020

控制系統
Control Systems

Unit 6B
Bode Plot Techniques

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Bode Plot Techniques

- The hand plotting was developed by H.W. Bode
  at Bell Laboratories between 1932-1942.
- Now, most control system designers use computer programs
  to illustrate the Bode plot.
- However, it is still important to develop good intuition
  so that you can quickly identify erroneous computer result
  and perform sanity check
  and determine approximate result by hand

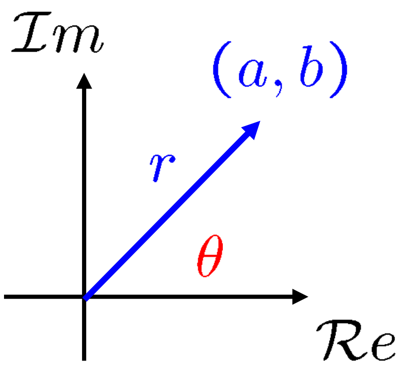
Bode Plot Techniques

- The idea in Bode's method is to plot

  magnitude curves using a logarithmic scale and

  phase curves using a linear scale

Magnitude & Phase Representation:



$$a + jb \quad \Rightarrow \quad \begin{cases} r = \sqrt{a^2 + b^2} \\ \tan(\theta) = \frac{b}{a} \end{cases}$$

$$\Rightarrow \quad a + jb \quad = \quad r e^{j\theta}$$

$$|a + jb| \quad = \quad \sqrt{a^2 + b^2} \qquad \left|\frac{1}{a+jb}\right| \quad = \quad \frac{1}{\sqrt{a^2+b^2}}$$

$$\angle a + jb \quad = \quad \tan^{-1}(\tfrac{b}{a}) \qquad \angle \frac{1}{a+jb} \quad = \quad -\tan^{-1}(\tfrac{b}{a})$$

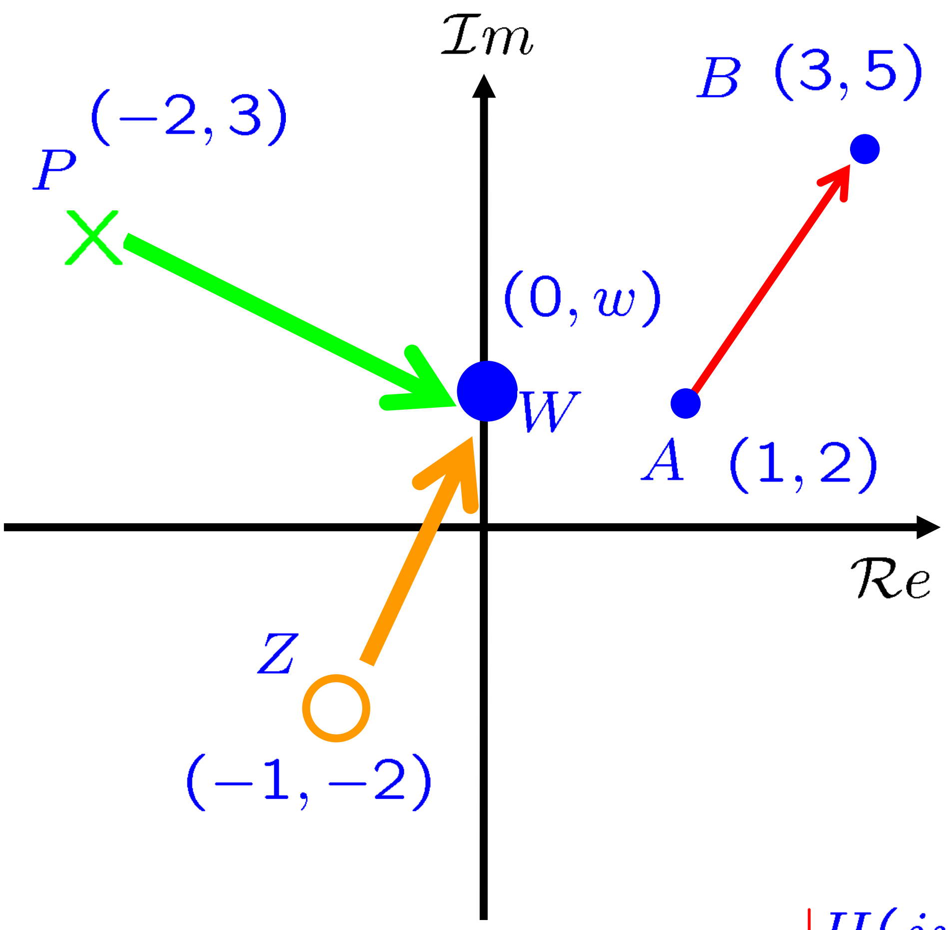
$$X(jw) = \mathcal{R}e\,\{X(jw)\} + j\ \mathcal{I}m\,\{X(jw)\} \qquad = \left|X(jw)\right| e^{j\angle X(jw)}$$

$$X(e^{jw}) = \mathcal{R}e\left\{X(e^{jw})\right\} + j\ \mathcal{I}m\left\{X(e^{jw})\right\} \qquad = \left|X(e^{jw})\right| e^{j\angle X(e^{jw})}$$

$\left|X(jw)\right|$ or $\left|X(e^{jw})\right|$: magnitude

$\angle X(jw)$ or $\angle X(e^{jw})$: phase angle

- In s-plane:



$$\overrightarrow{AB} = (3+5j) - (1+2j)$$
$$= 2+3j$$

$$\overrightarrow{AB} = (3,5) - (1,2) = (2,3)$$

$$|\overrightarrow{AB}| = \sqrt{2^2+3^2}$$

$$\angle\overrightarrow{AB} = \tan^{-1}\frac{(5-2)}{(3-1)}$$

$$H(s) = \frac{s-(-1-2j)}{s-(-2+3j)}$$

$$|H(jw)| = \frac{|jw-(-1-2j)|}{|jw-(-2+3j)|} = \frac{|\overrightarrow{ZW}|}{|\overrightarrow{PW}|}$$

$$\angle H(jw) = \angle\overrightarrow{ZW} - \angle\overrightarrow{PW}$$

- For example,

$$G(jw) = \frac{\vec{s_1}\ \vec{s_2}}{\vec{s_3}\ \vec{s_4}\ \vec{s_5}} = \frac{(\ r_1\ e^{j\theta_1}\ )\ )\ r_2\ e^{j\theta_2}\ )}{(\ r_3\ e^{j\theta_3}\ )\ (\ r_4\ e^{j\theta_4}\ )\ (\ r_5\ e^{j\theta_5}\ )}$$

$$= \frac{r_1\ r_2}{r_3\ r_4\ r_5}\ e^{j(\theta_1+\theta_2-\theta_3-\theta_4-\theta_5)}$$

$$|G(jw)| = \frac{r_1\ r_2}{r_3\ r_4\ r_5} \qquad \angle G(jw) = (\theta_1 + \theta_2 - \theta_3 - \theta_4 - \theta_5)$$

$$\log_{10} |G(jw)| = \log_{10} r_1 + \log_{10} r_2 - \log_{10} r_3 - \log_{10} r_4 - \log_{10} r_5$$

- Power db:

$$|G(jw)|_{\text{db}} = 10 \log_{10} \frac{P_2}{P_1}$$

- Voltage db:

$$|G(jw)|_{\text{db}} = 20 \log_{10} \frac{V_2}{V_1}$$

Advantages of Using Bode Plot Techniques

- Advantages of working with Frequency Response in terms of Bode Plots

1. Dynamic compensator design can be based entirely on Bode plots.
2. Bode plots can be determined experimentally.
3. Bode plots of systems in series (or tandem) simply add, which is quite convenient.
4. The use of a log scale permits a much wider range of frequencies to be displayed on a single plot than is possible with linear scales.

Bode Form of the Transfer Function

- The open-loop transfer function:

$$K\ G(s) = K \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

$$K\ G(jw) = K_0\ (jw)^n \frac{(jw\tau_1+1)\ (jw\tau_2+1)\cdots}{(jw\tau_a+1)\ (jw\tau_b+1)\cdots}$$

- For example,

$$K\ G(jw) = K_0 \frac{(jw\tau_1+1)}{(jw)^2\ (jw\tau_a+1)}$$

$$\angle\ K\ G(jw) = \angle\ K_0 + \angle\ (jw\tau_1+1) - \angle(jw)^2 - \angle(jw\tau_a+1)$$

$$\log|K\ G(jw)| = \log|K_0| + \log|(jw\tau_1+1)| - \log|(jw)^2| - \log|(jw\tau_a+1)|$$

$$|K\ G(jw)|_{\text{db}} = 20\log|K_0| + 20\log|(jw\tau_1+1)| - 20\log|(jw)^2| - 20\log|(jw\tau_a+1)|$$

Classes of Terms of Transfer Function

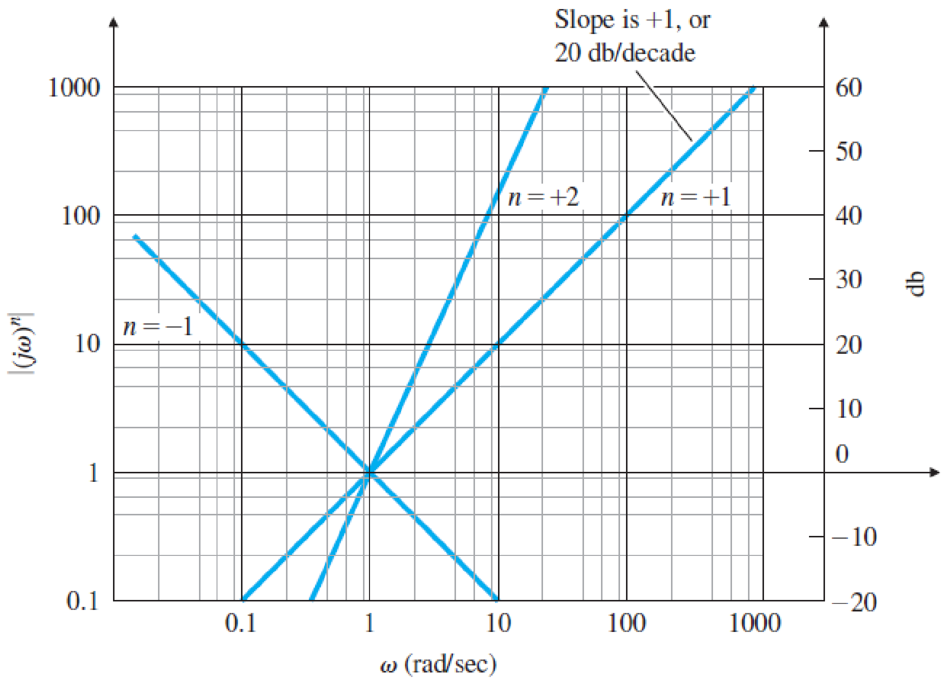
- Class 1: Singularities at the origin $K_0 \, (jw)^n$
- Class 2: Frist-order term $(jw\tau + 1)^{\pm 1}$
- Class 3: Second-order term $\left[\left(\frac{jw}{w_n} \right)^2 + 2\,\zeta \, \frac{jw}{w_n} + 1 \right]^{\pm 1}$

- Class 1: $K_0 \, (jw)^n$

$$\log K_0 |(jw)^n|$$

$$= \log K_0 + n \log |jw|$$

$$\angle K_0 \, (jw)^n = n \times 90^0$$



- Class 2: Magnitude

$$(j w \tau + 1)^{\pm 1}$$

a) For $\omega\tau << 1$, $j\omega\tau + 1 \cong 1$

b) For $\omega\tau >> 1$, $j\omega\tau + 1 \cong j\omega\tau$

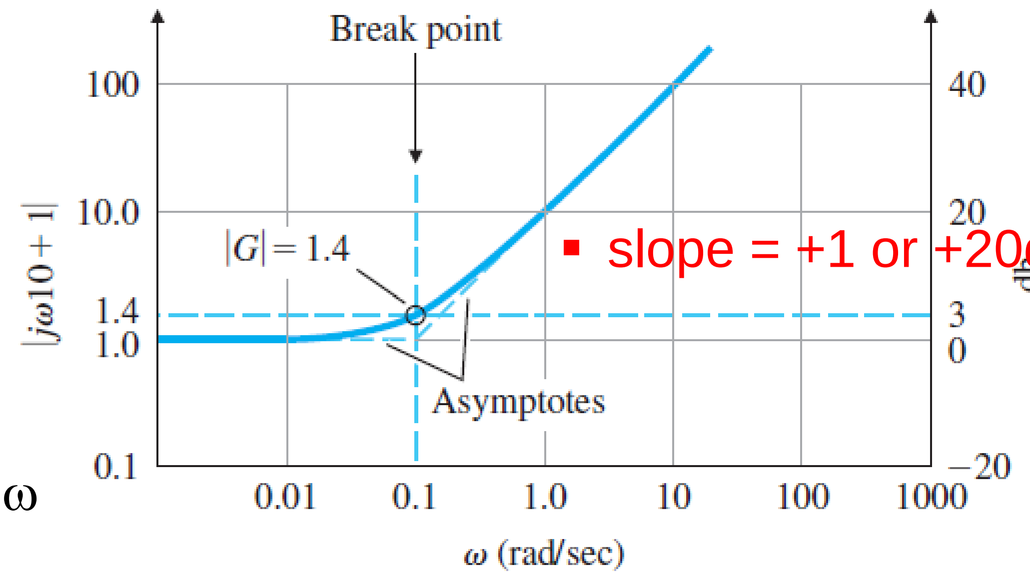
$$? \; G(s) = \frac{1}{10\, s + 1}$$

- $\omega = 1/\tau$: Break Point

- For example,

$$G(s) = 10\, s + 1$$

$$G(jw) = j(10w) + 1$$



- slope = +1 or +20db

a) For $10\omega << 1$, $j10\omega + 1 \cong 1$

b) For $10\omega >> 1$, $j10\omega + 1 \cong j10\omega$

- $\omega = 1/10$: Break Point

$$|G(j0.1)| = |j(1) + 1| = 1.414 = +3 \text{ db}$$

Classes of Terms of Transfer Function

- Class 2: Phase

$$(j w \tau + 1)^{\pm 1}$$

a) For $\omega\tau << 1$, $\angle 1 = 0^o$

b) For $\omega\tau >> 1$, $\angle j\omega\tau = 90^o$

c) For $\omega\tau \cong 1$, $\angle (j\omega\tau+1) \cong 45^o$

$$? \; G(s) = \frac{1}{10\,s + 1}$$

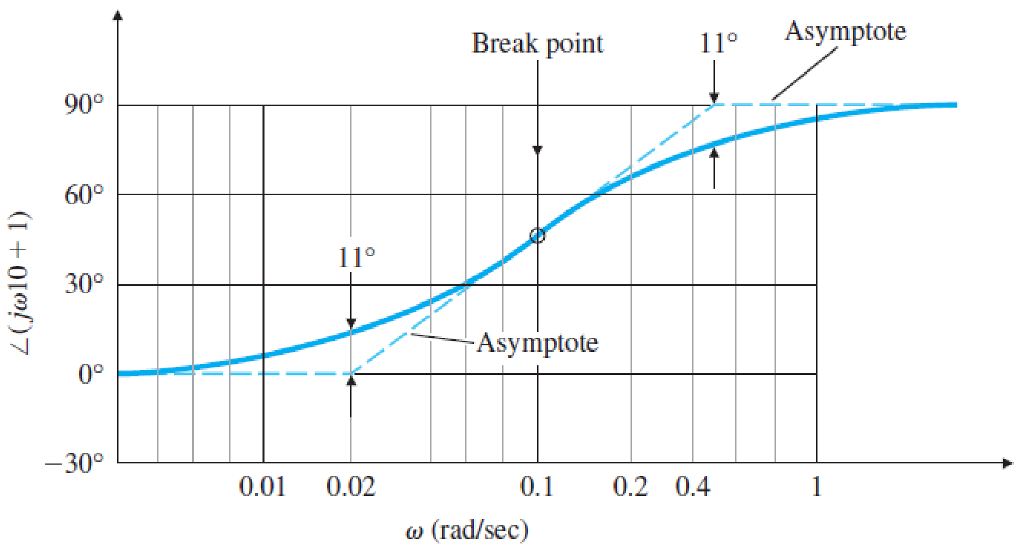
- $\omega = 1/\tau$: Break Point

- For example,

$$G(s) = 10\,s + 1$$

$$G(jw) = j(10w) + 1$$

- $\omega = 1/10$: Break Point

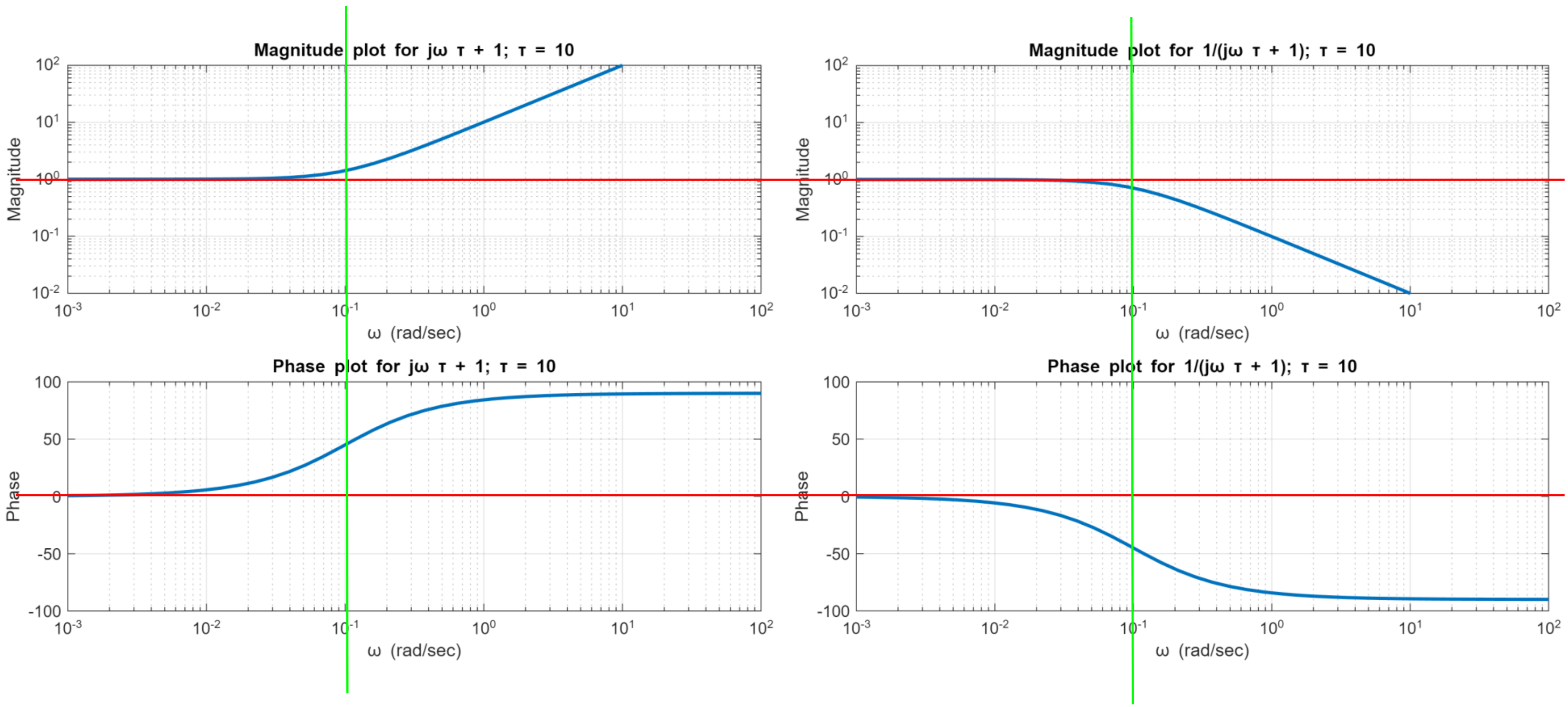


Classes of Terms of Transfer Function

- Class 2:

$$(j w \tau + 1)^{\pm 1}$$

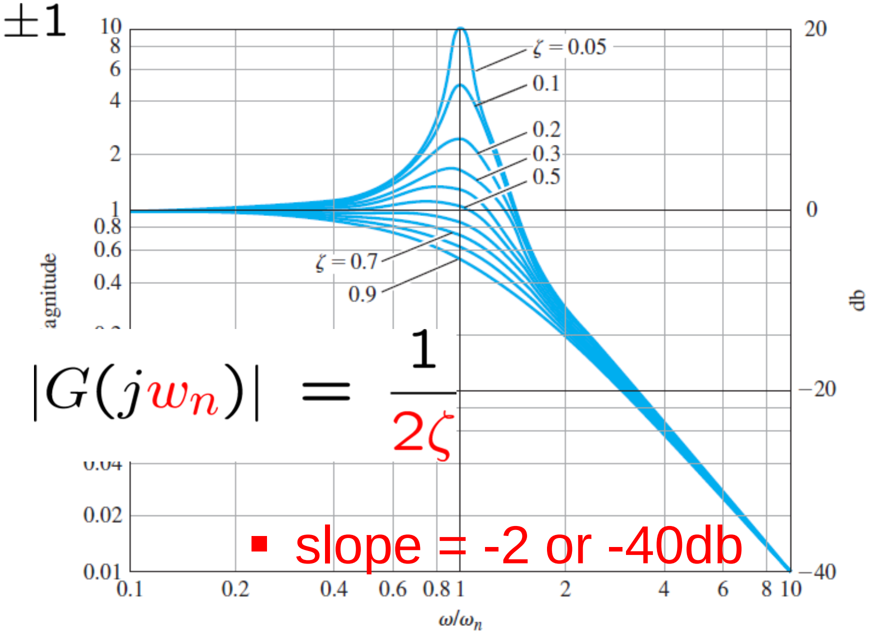
$$G(s) = 10\,s + 1 \qquad G(s) = \frac{1}{10\,s + 1}$$



Classes of Terms of Transfer Function

- Class 3: $\left[\left(\frac{jw}{w_n} \right)^2 + 2\,\zeta\,\frac{jw}{w_n} + 1 \right]^{\pm 1}$

$$|G(s = jw)|$$

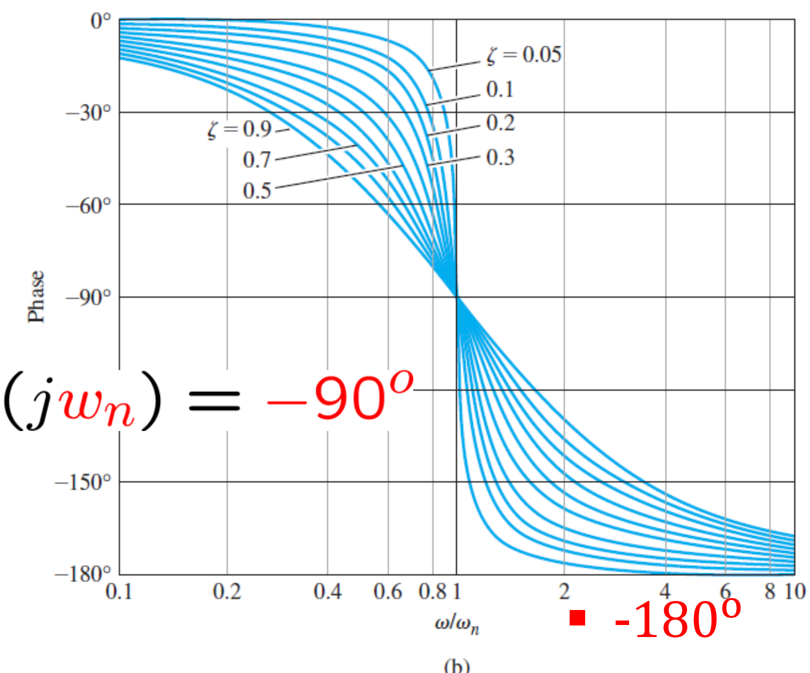


$$|G(jw_n)| = \frac{1}{2\zeta}$$

- slope = -2 or -40db

$$G(s) = \frac{1}{(s/w_n)^2 + 2\zeta(s/w_n) + 1}$$

$$\angle G(s = jw)$$



$$\angle G(jw_n) = -90^o$$

- -180^o

- Example 6.3: Bode Plot for Real Poles and Zeros

$$K\,G(s) = \frac{2000\,(s+0.5)}{s\,(s+10)\,(s+50)}$$

(1) Break points

$$K\,G(jw) = \frac{2\,[\,\frac{jw}{0.5}+1\,]}{(jw)\,[\,\frac{jw}{10}+1\,]\,[\,\frac{jw}{50}+1\,]}$$

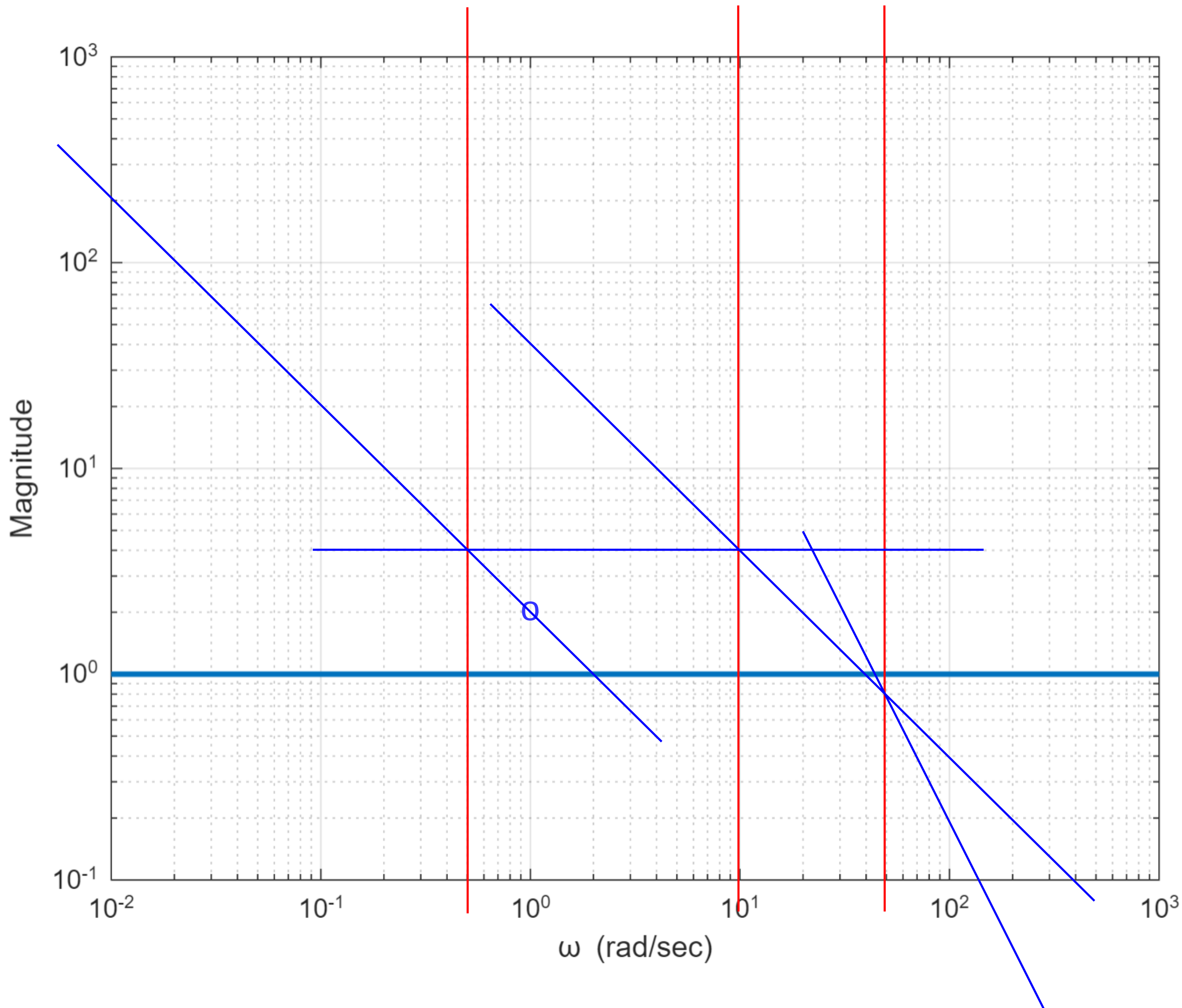
- Break points: 0.5, 10, 50

(2) Asymptotes

- Low-Frequency Asymptote: $K\,G(jw) = \frac{2}{(jw)}$ for $w < 0.1$

ω << 0.5:	slope = -1	(or -20 db/decade)
0.5 < ω < 10:	slope = 0	(or 0 db/decade)
10 < ω < 50:	slope = -1	(or -20 db/decade)
50 < ω:	slope = -2	(or -40 db/decade)

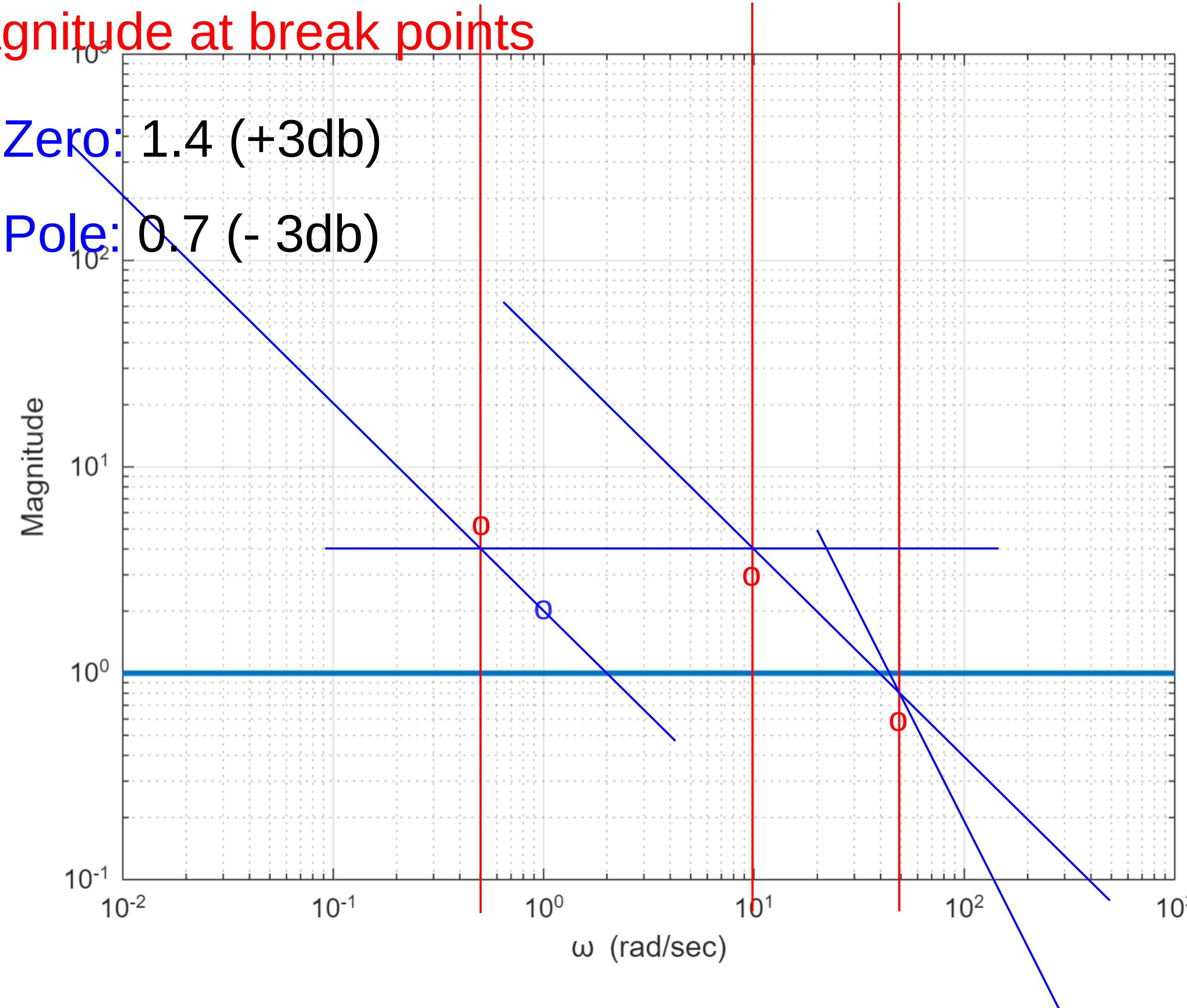
- Example 6.3: Bode Plot for Real Poles and Zeros



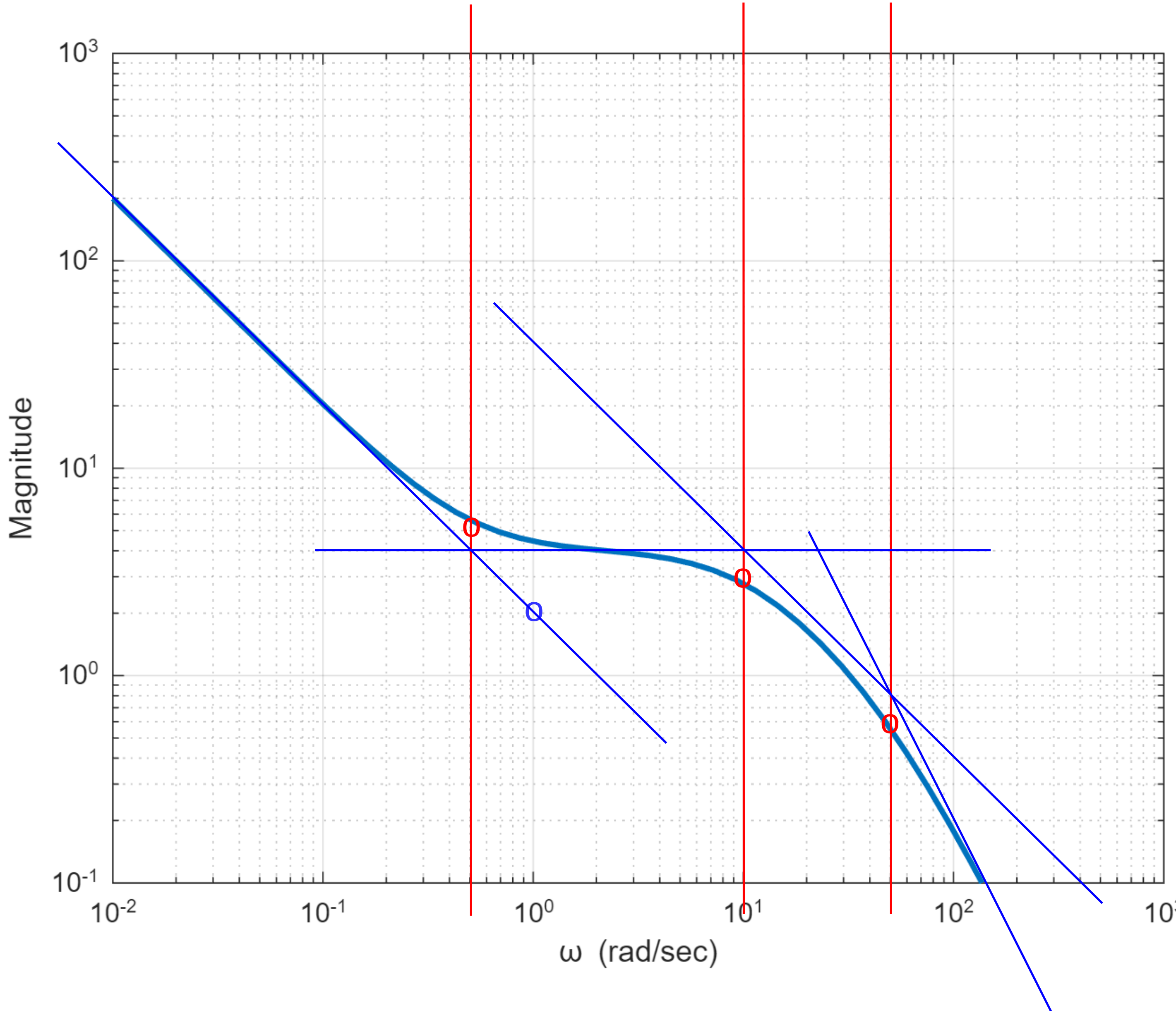
- Example 6.3: Bode Plot for Real Poles and Zeros

(3) Magnitude at break points

- By Zero: 1.4 (+3db)
- By Pole: 0.7 (- 3db)



- Example 6.3: Bode Plot for Real Poles and Zeros



- Example 6.3: Bode Plot for Real Poles and Zeros

$$K\ G(s) = \frac{2000\ (s+0.5)}{s\ (s+10)\ (s+50)}$$

(1) Break points

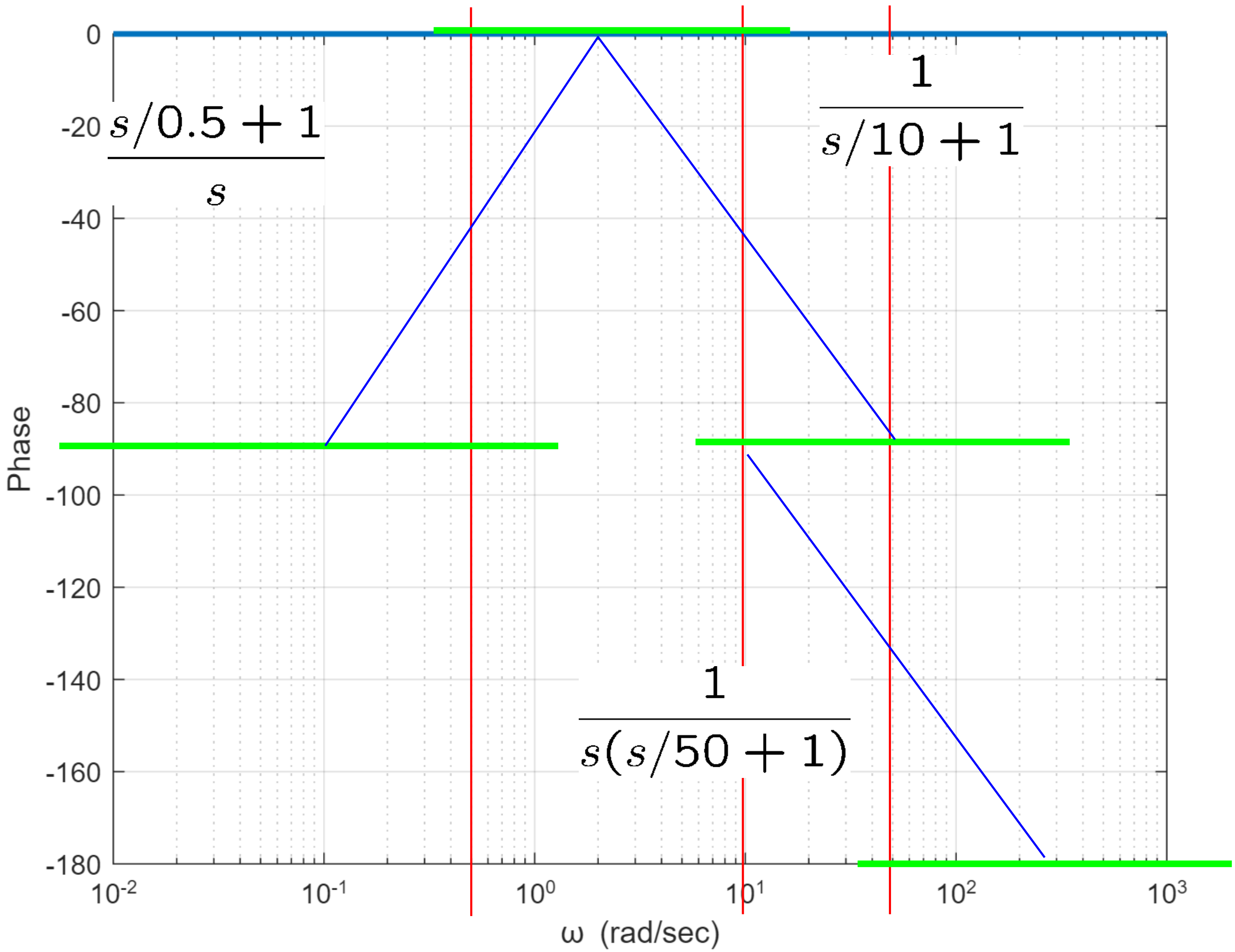
$$K\ G(jw) = \frac{2\ [\ \frac{jw}{0.5}+1\ ]}{(jw)\ [\ \frac{jw}{10}+1\ ]\ [\ \frac{jw}{50}+1\ ]}$$

- Break points: 0.5, 10, 50

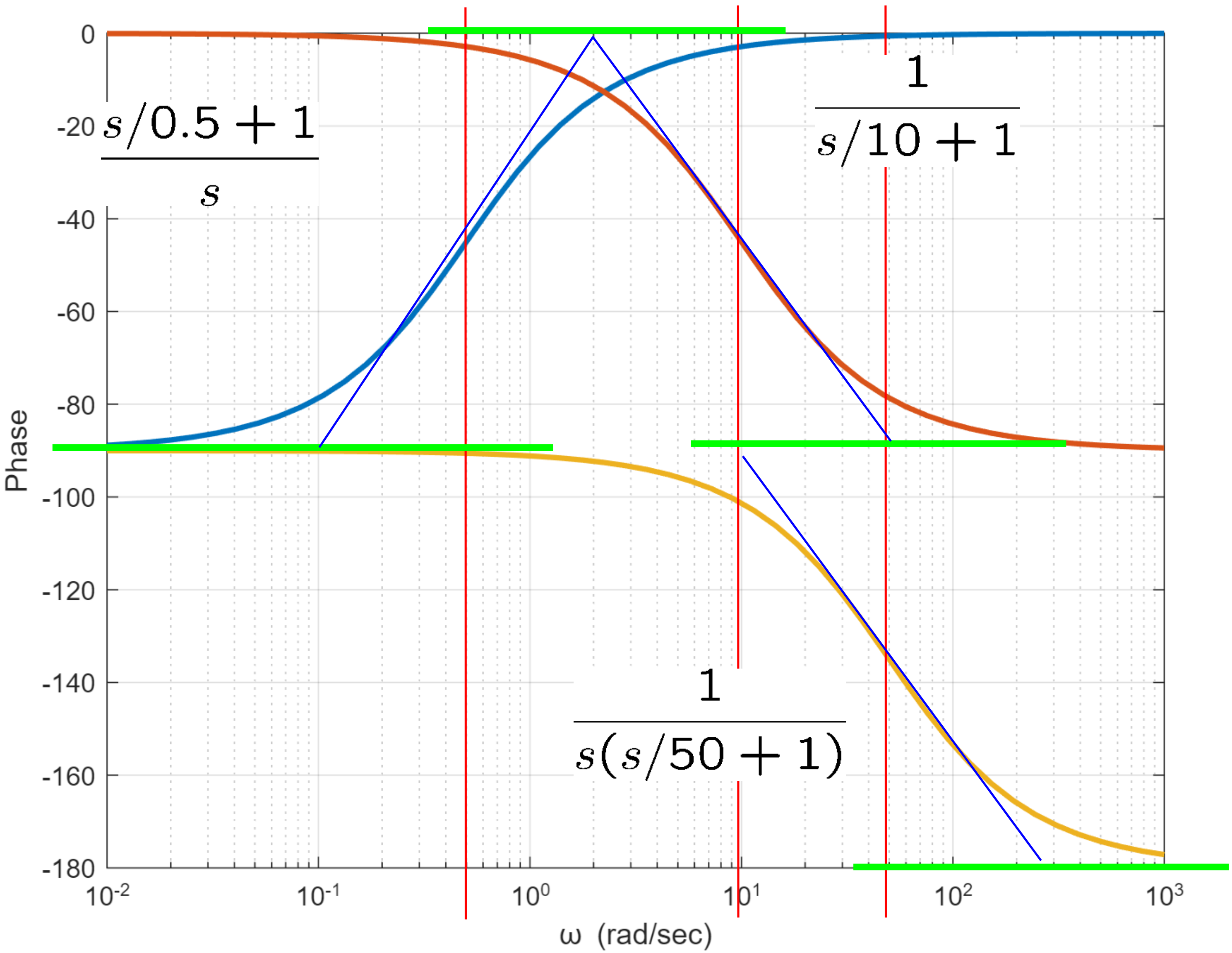
(4) Phase

- Low-Frequency Asymptote: $K\ G(jw) = \frac{2}{(jw)}$ for $w < 0.1$
- $\omega \ll 0.5$: phase = -90^o
- $0.5 < \omega < 10$: phase = 0^o
- $10 < \omega < 50$: phase = -90^o
- $50 < \omega$: phase = -180^o

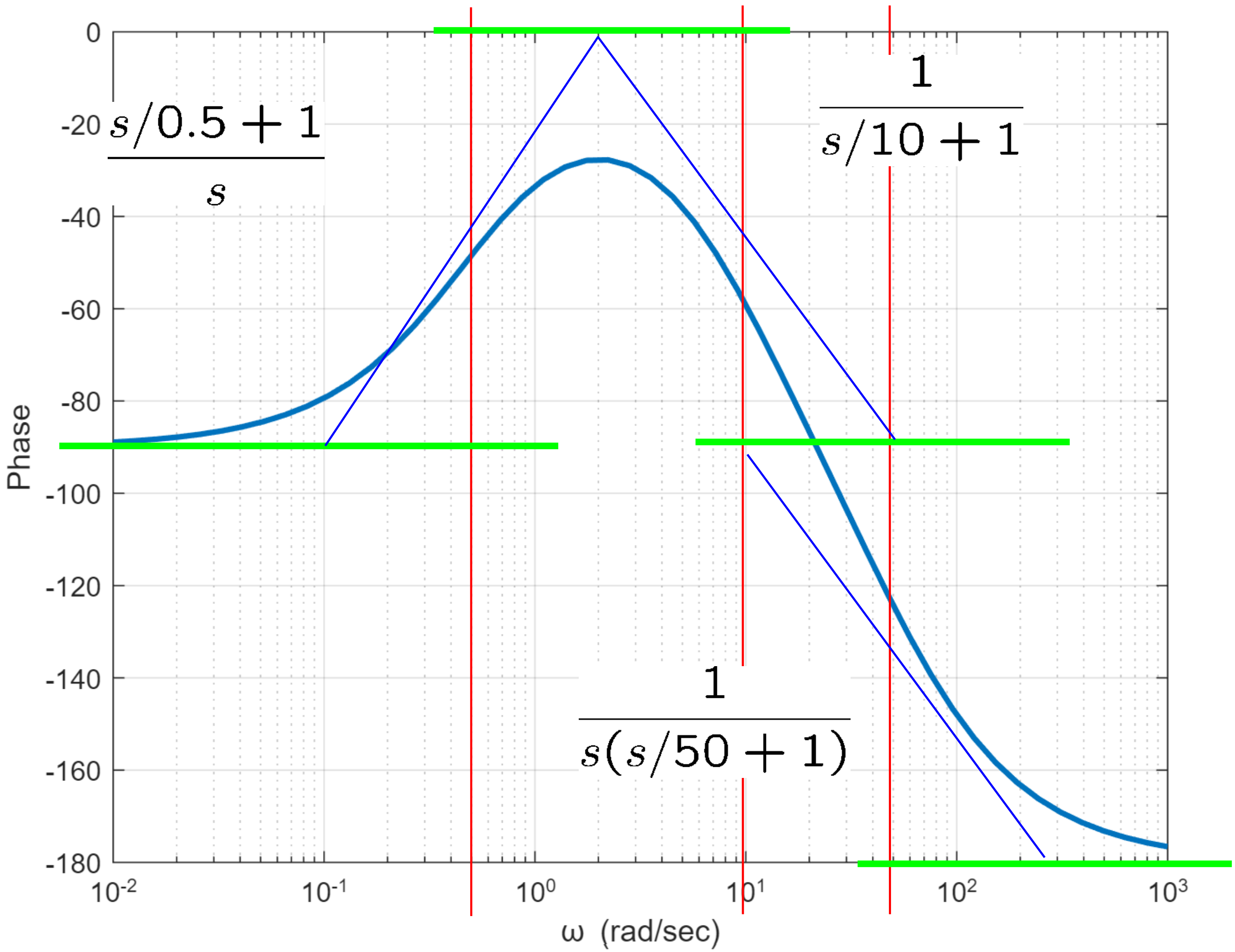
Example 6.3: Bode Plot for Real Poles and Zeros



- Example 6.3: Bode Plot for Real Poles and Zeros



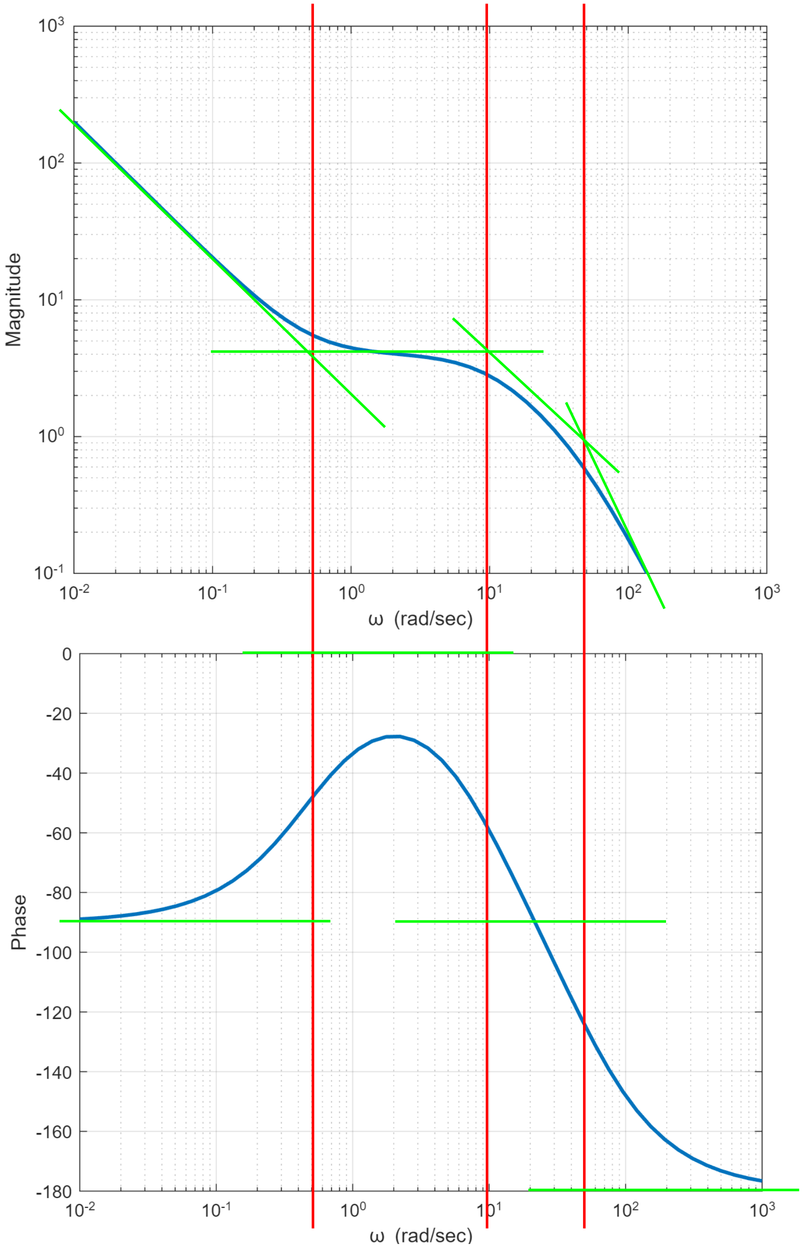
- Example 6.3: Bode Plot for Real Poles and Zeros



- Example 6.3: Bode Plot for Real Poles and Zeros

$$\frac{2000\,(s+0.5)}{s\,(s+10)\,(s+50)}$$

$$\frac{2\,[\,\frac{jw}{0.5}+1\,]}{(jw)\,[\,\frac{jw}{10}+1\,]\,[\,\frac{jw}{50}+1\,]}$$

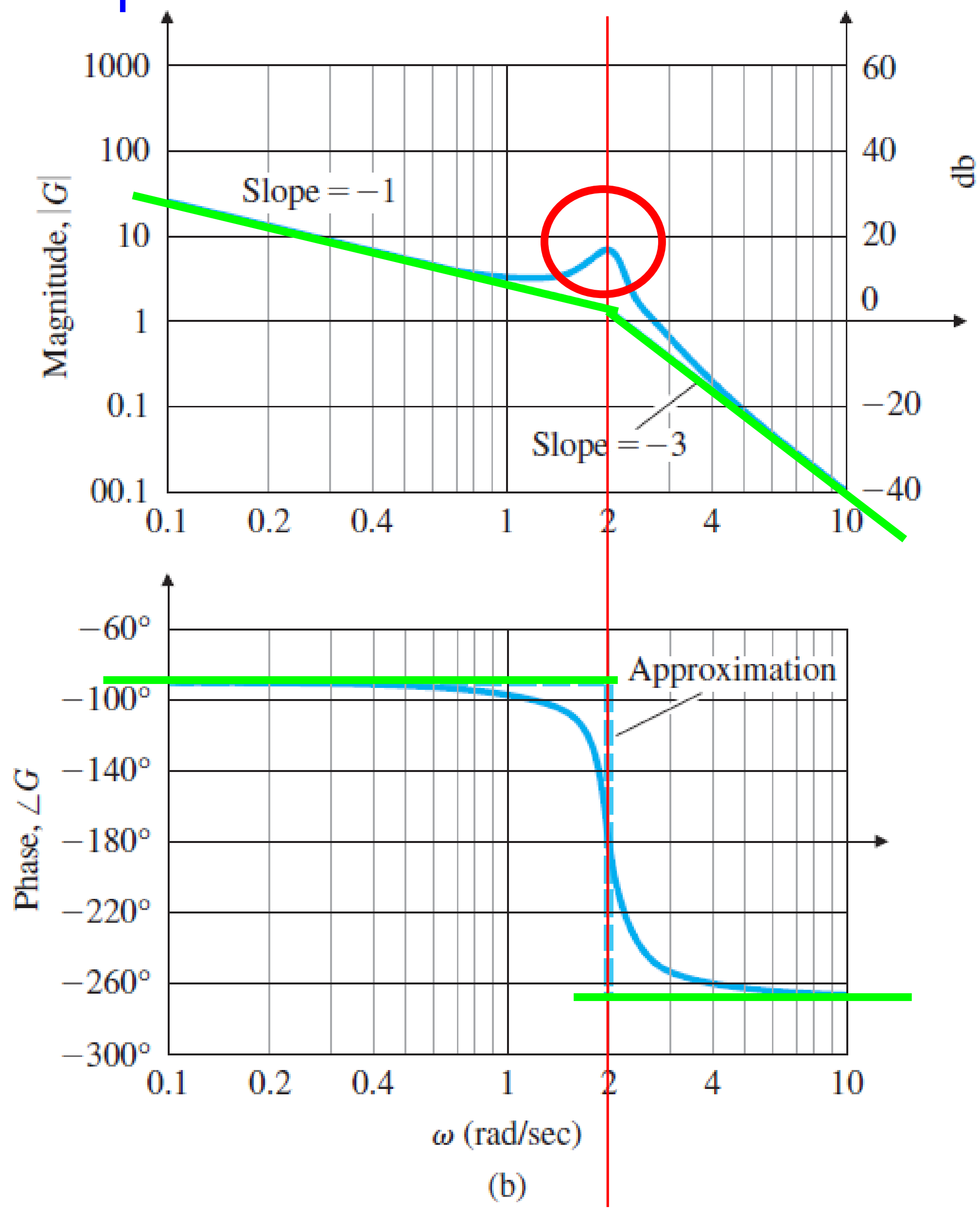


- Break points: 0.5, 10, 50

- Example 6.4: Bode Plot for Complex Poles

$$K\,G(s) = \frac{10}{s\,[s^2 + 0.4s + 4]}$$

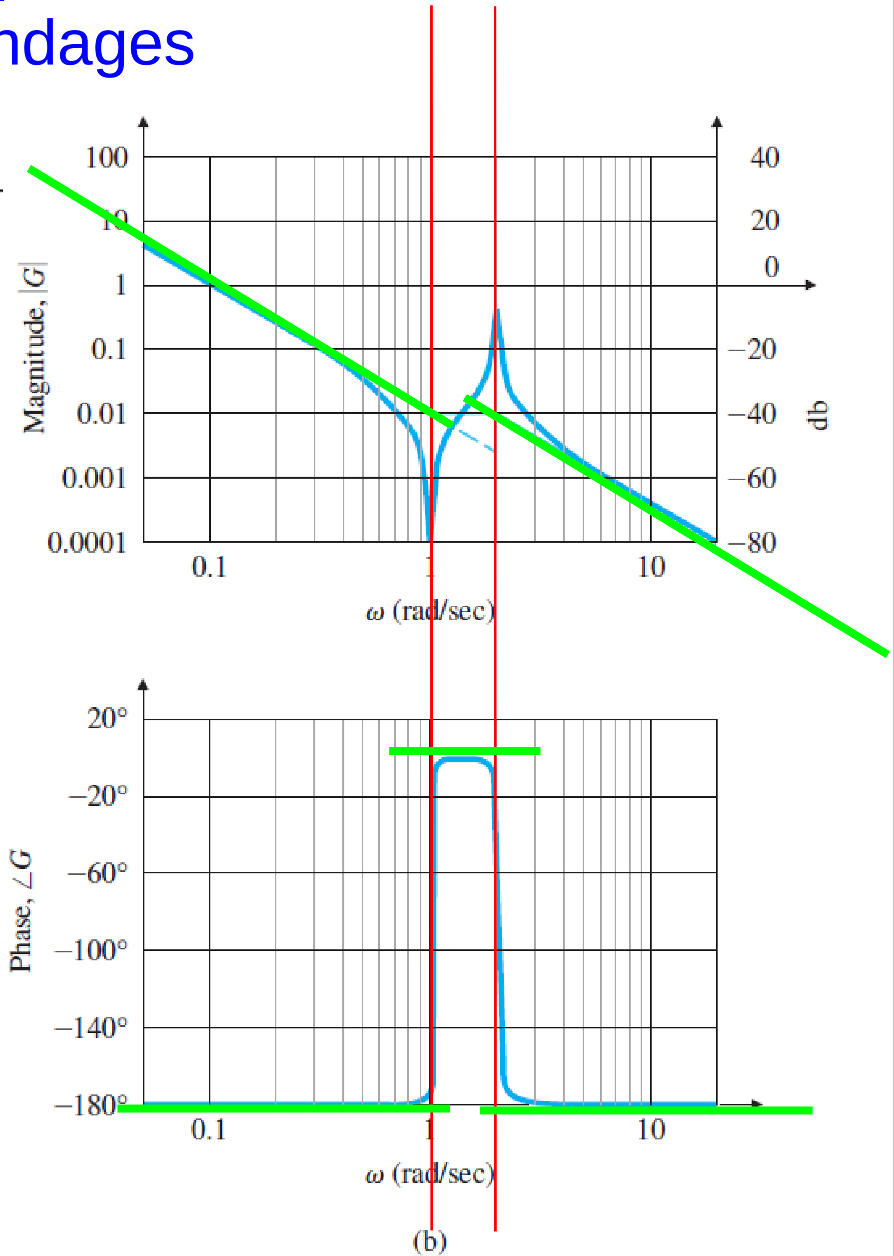
$$= \frac{10}{4}\,\frac{1}{s\,[\,\frac{s^2}{4} + 2(0.1)\frac{s}{2} + 1\,]}$$



- Break points: 2

- Example 6.5: Bode Plot for Complex Poles and Zeros: Satellite with Flexible Appendages

$$K\,G(s) = \frac{0.01\,[s^2 + 0.01s + 1]}{s^2\,[(\frac{s^2}{4}) + 0.02(\frac{s}{2}) + 1]}$$



- Break points: 1, 2

Example 6.6: Computer-Aided Bode Plot for Complex Poles and Zeros

```
num = 0.01*[1 0.01 1];
den  = conv([1 0 0],[.25 0.01 1]);

w = logspace(-2,2,1000);

[m,p] = bode( num, den, w);

subplot(2,1,1)
loglog( w, m, 'LineWidth', 2);

subplot(2,1,2)
semilogx( w, p, 'LineWidth', 2);
```