Spring 2020

控制系統
Control Systems

Unit 55
Design Examples Using the Root Locus

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NTU-EE

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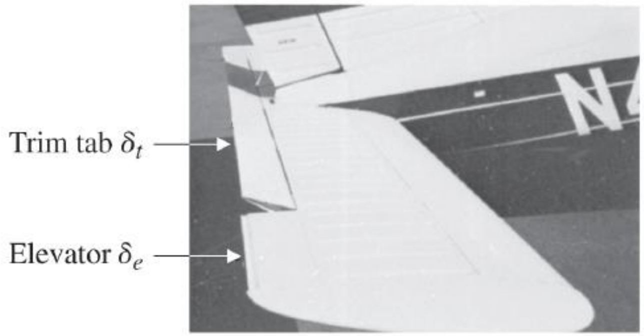
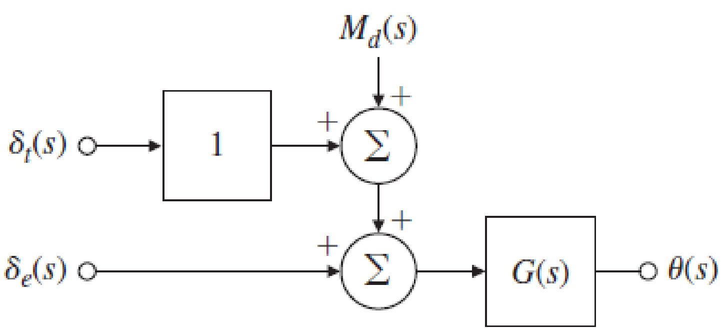
Example 5.12: Control of Small Airplane

- Autopilot design in the Piper Dakota, showing elevator and trim tab

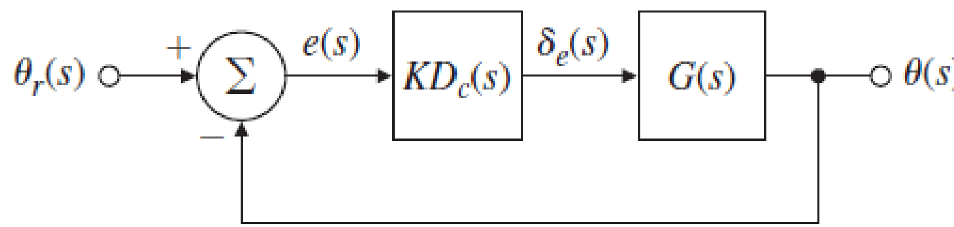


(b)

(a)



(a)



(b)

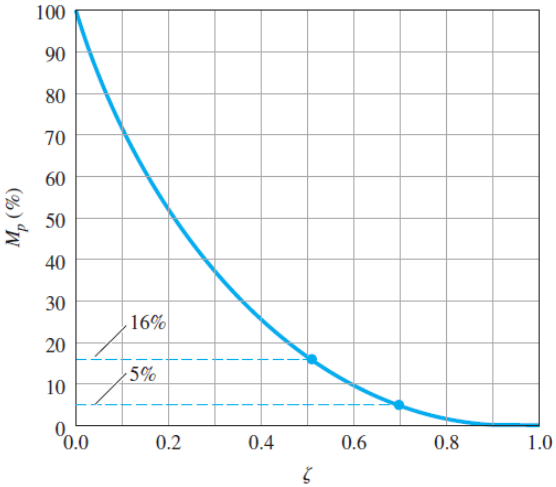
- Transfer Function between elevator input and pitch attitude:

$$G(s) = \frac{\theta(s)}{\delta_e(s)} = \frac{160\,(s+2.5)\,(s+0.7)}{(s^2+5s+40)\,(s^2+0.03s+0.06)}$$

- Example 5.12: Control of Small Airplane
- Part A:
  - Design an autopilot so that the response to a step elevator input has
    - a rise time of 1 sec or less and
    - an overshoot less than 10%.
- For ideal 2nd-order system:

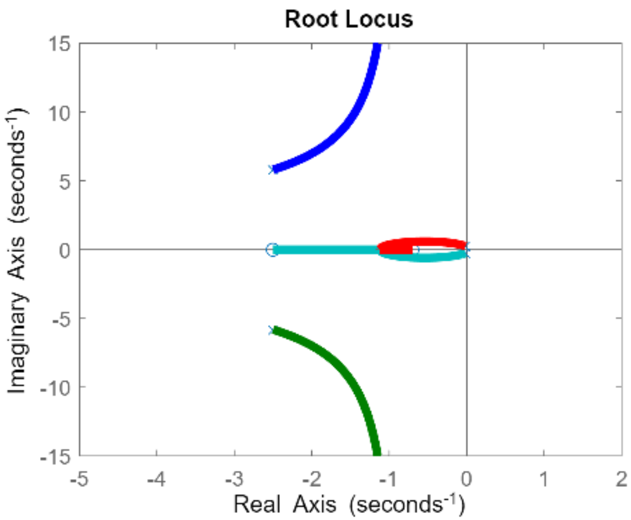
$$\Rightarrow t_r \leq 1 \qquad \Rightarrow t_r \cong \frac{1.8}{w_n} \qquad \Rightarrow w_n \geq \frac{1.8}{t_r} \qquad \Rightarrow w_n \geq 1.8$$

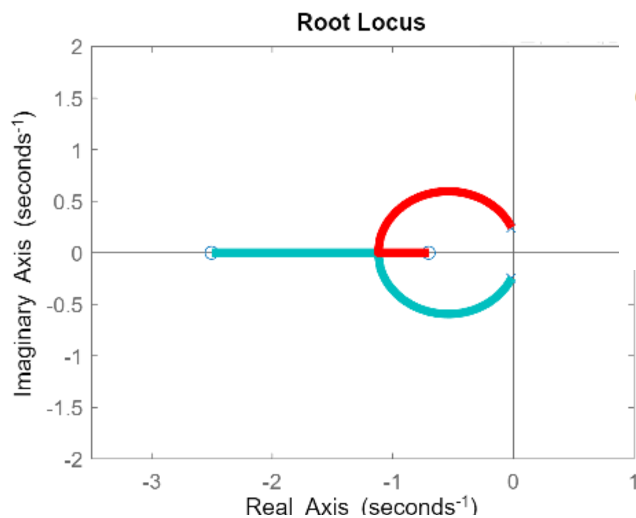
$$\Rightarrow M_p \leq 10 \qquad \Rightarrow \zeta \geq 0.6$$

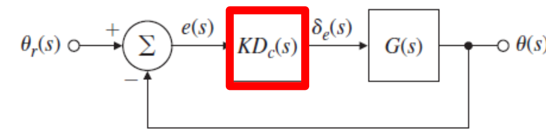


▪ Example 5.12: Control of Small Airplane

```
sysG = 160*(s+2.5)*(s+0.7)/((s^2+5*s+40)*(s^2+0.03*s+0.06));
rlocus( sysG )
```

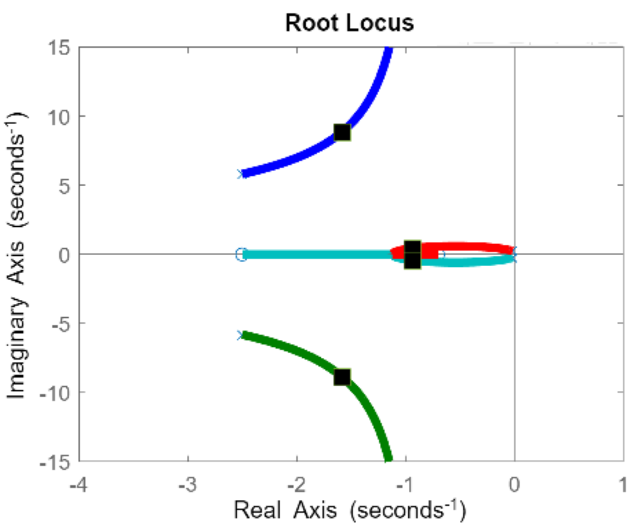


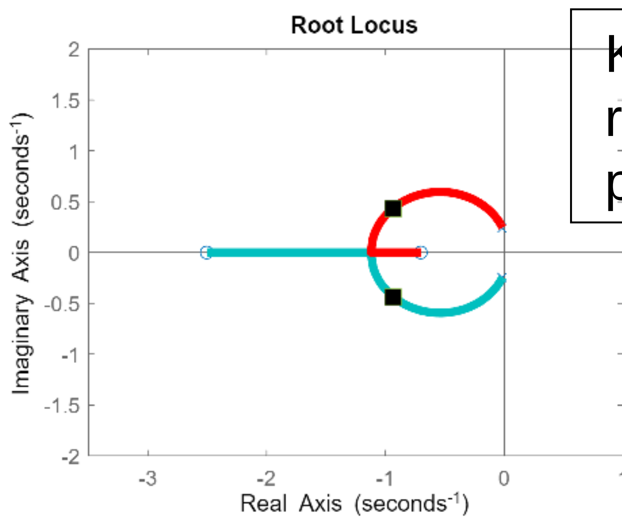




(b)

$$\Rightarrow \; KD_c(s) = K_p$$

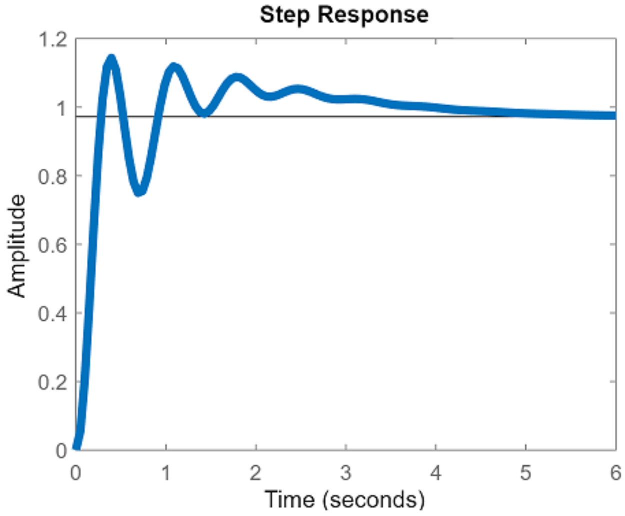




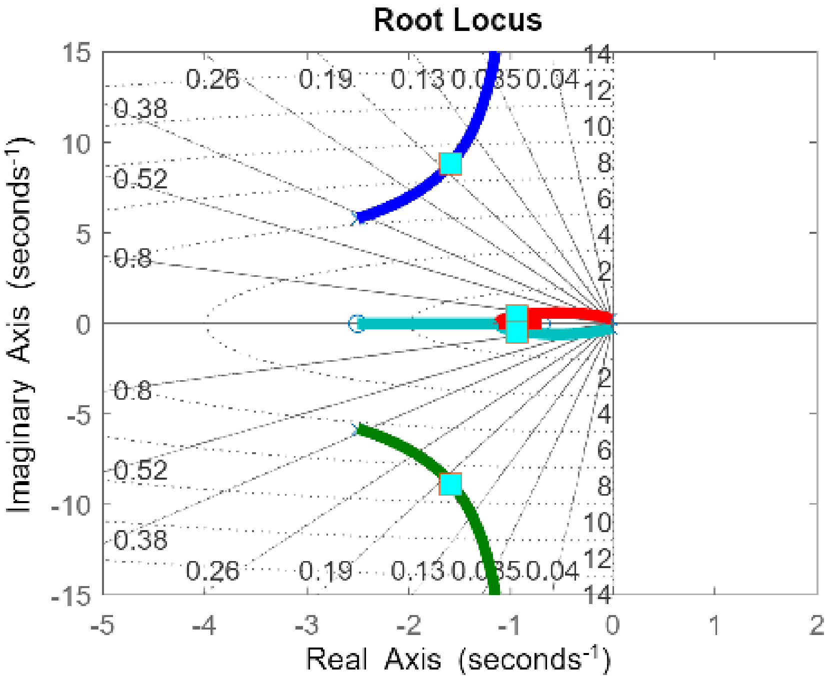

```
Kp = 0.3;
r1 = rlocus( sysG, Kp );
plot( r1, 's' )
```

Example 5.12: Control of Small Airplane

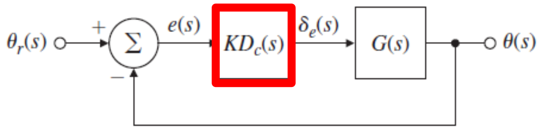
```
sysG = 160*(s+2.5)*(s+0.7)/((s^2+5*s+40)*(s^2+0.03*s+0.06));
Kp = 0.3;
sysTp = feedback(Kp*sysG,1);
step( sysTp )
```



- Long-term settling
- $\zeta <= 2.5/6.32 = 0.40$

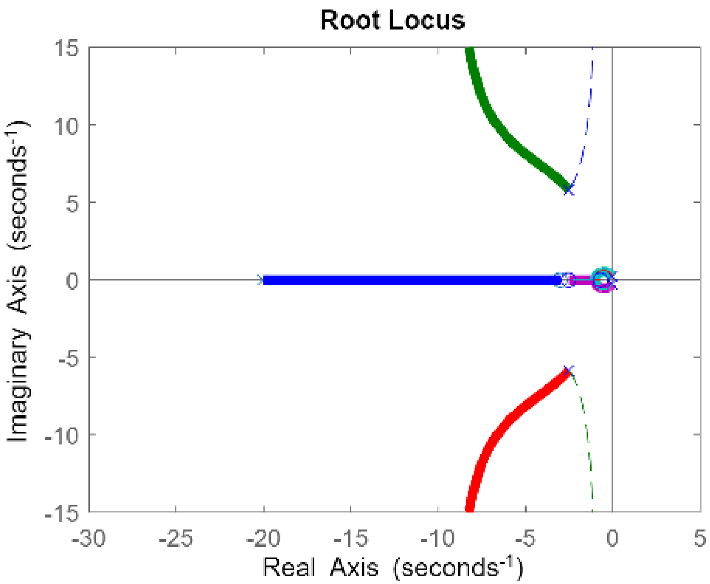


Example 5.12: Control of Small Airplane

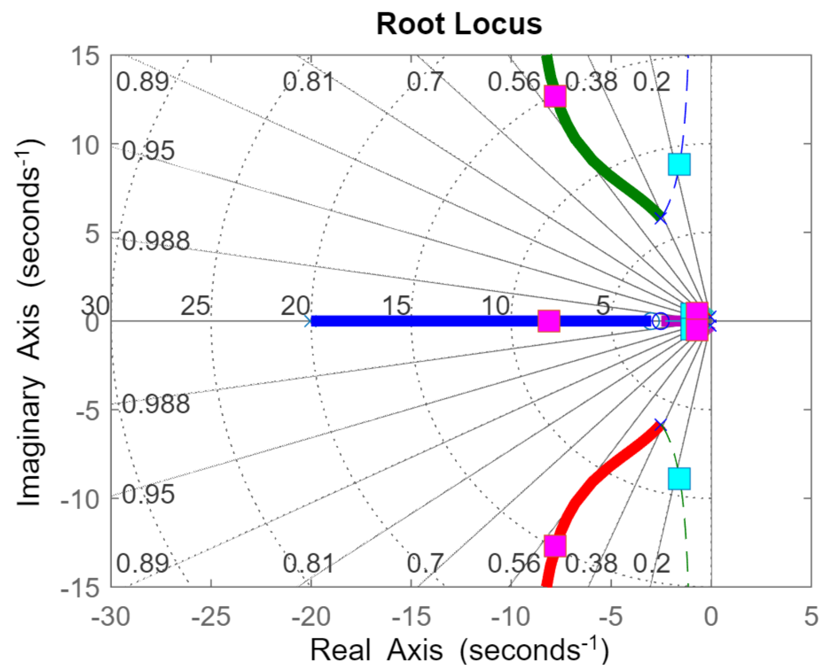


$$\Rightarrow \quad KD_c(s) = K_c \frac{s+3}{s+20}$$

```
sysG = 160*(s+2.5)*(s+0.7)/
          ((s^2+5*s+40)*(s^2+0.03*s+0.06));
sysD = (s+3)/(s+20);
sysDG = sysD*sysG;
rlocus( sysDG )
```




```
Kc = 1.5;
r2 = rlocus( sysDG, Kc );
plot( r2, 's' )
```



Example 5.12: Control of Small Airplane

```
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Kp = 0.3;
sysTp = feedback(Kp*sysG,1);
step( sysTp )
```
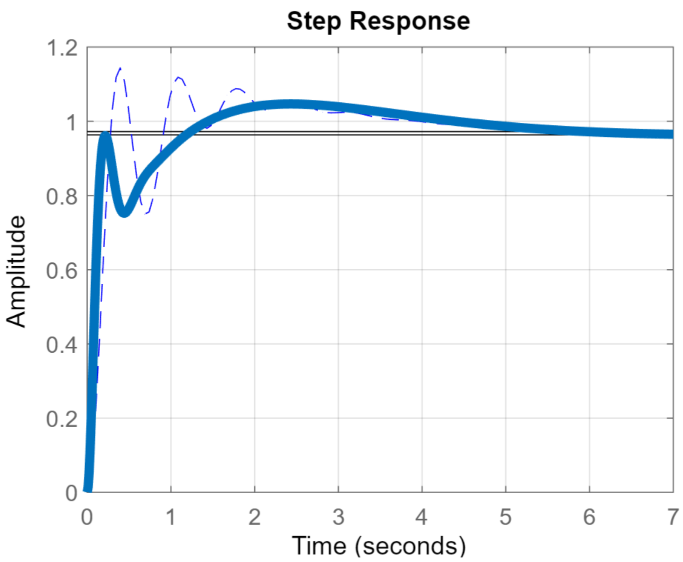
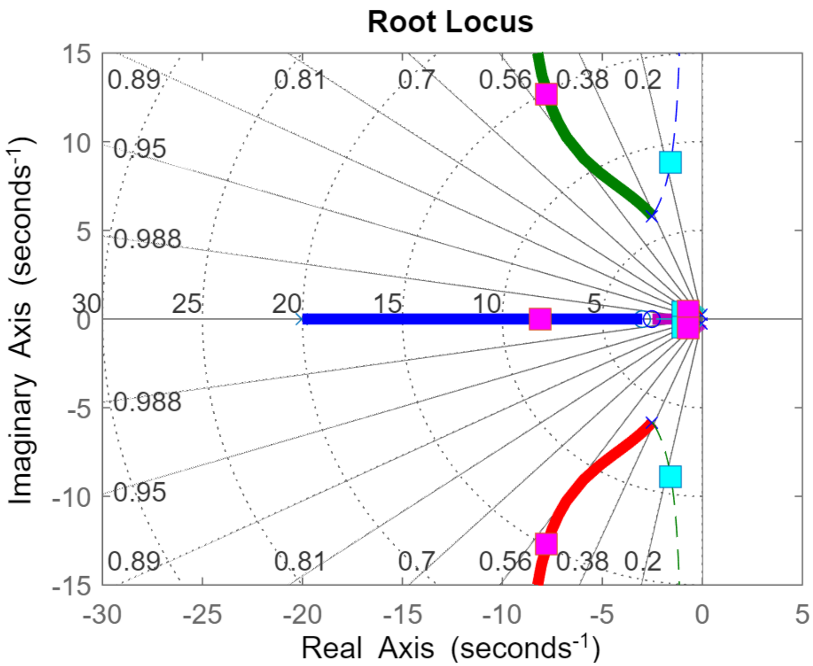
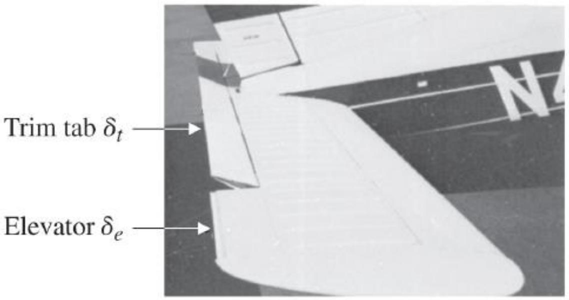



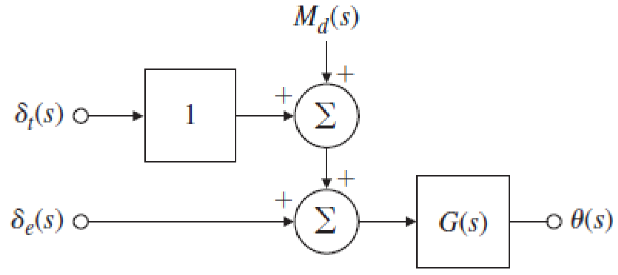
- t_r = 0.9 sec
- ζ = 0.52
- M_p = 8%
- ω_n = 15 rad/sec



Example 5.12: Control of Small Airplane



(b)



(a)

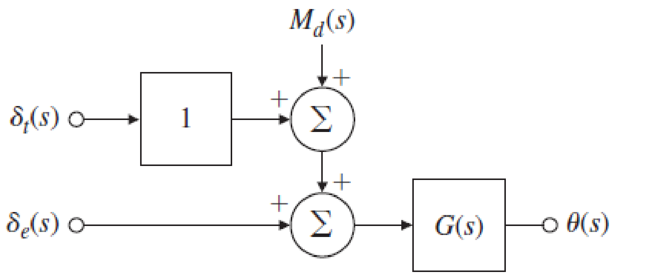
Part B:

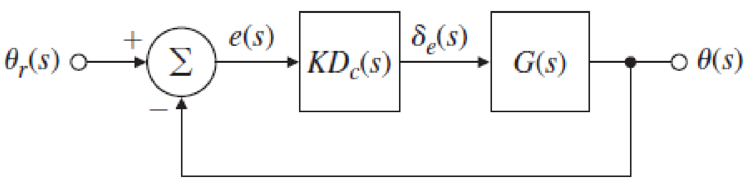
- When there is a constant disturbing moment acting on the aircraft so that the pilot must supply a constant force for steady-flight control
- It is said to be out of trim.
- The transfer function between the disturbing moment and the attitude is:

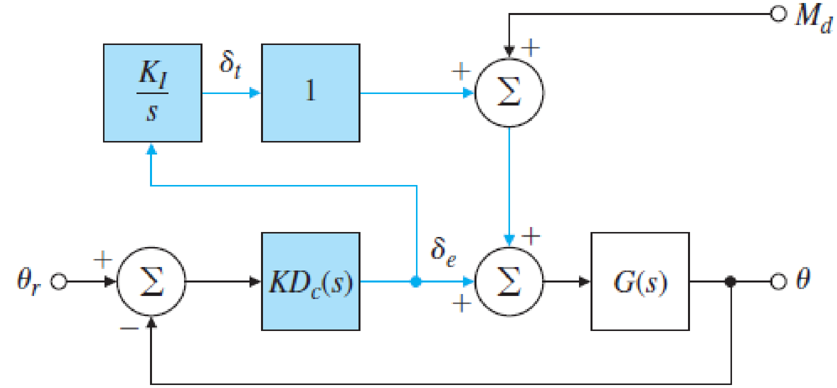
$$G(s) = \frac{\theta(s)}{M_d(s)} = \frac{160\,(s+2.5)\,(s+0.7)}{(s^2+5s+40)\,(s^2+0.03s+0.06)}$$

- No steady-state control effort for elevator, that is, $\delta_e = 0$
- Only command the trim δ_t for arbitrary constant moment M_d

▪ Example 5.12: Control of Small Airplane





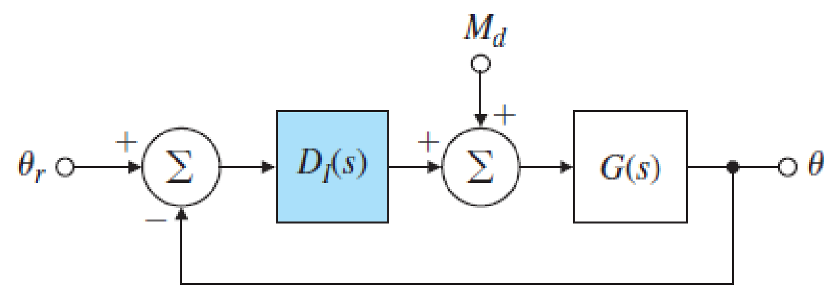


(a)

$$\Rightarrow D_I(s) = K\ D_c(s)\left(1 + \frac{K_I}{s}\right)$$

$$\Rightarrow 1 + G(s)\ D_I(s) = 0$$

$$\Rightarrow 1 + K\ D_c\ G + \frac{K_I}{s} K\ D_c\ G = 0$$

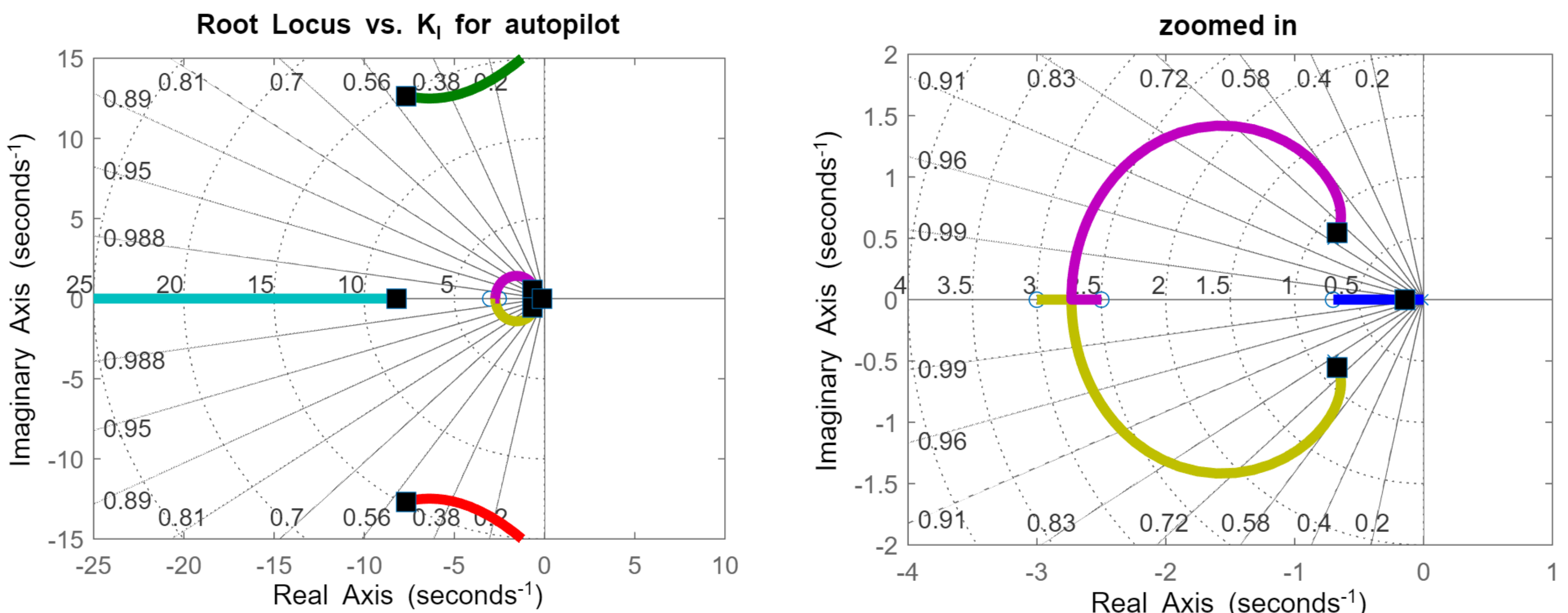


(b)

$$\Rightarrow 1 + K_I \frac{\frac{1}{s} K D_c G}{1 + K D_c G} = 0 \qquad \Rightarrow 1 + K_I\ L(s) = 0$$

Example 5.12: Control of Small Airplane

```
sysG = 160*(s+2.5)*(s+0.7)/((s^2+5*s+40)*(s^2+0.03*s+0.06));
sysD = (s+3)/(s+20);
sysDG = sysD*sysG;
Kc = 1.5;
sysT = feedback( Kc*sysDG, 1 );
sysL = sysT/s; % add integral control pole
rlocus(sysL)
KI = 0.15;
[R1] = rlocus( sysL, KI );        plot( R1,'s' )
```



Example 5.12: Control of Small Airplane

```
sysG = 160*(s+2.5)*(s+0.7)/((s^2+5*s+40)*(s^2+0.03*s+0.06));
sysD = (s+3)/(s+20);
sysDG = sysD*sysG;
Kc = 1.5;
KI = 0.15;
sysDI = Kc*sysD*(1+KI/s);
sysLICL=feedback(sysDI*sysG,1);
step(5*sysLICL,30) % X5 because a step command of 5 deg
sysFBDe=sysG*(1+KI/s);
sysDeCL=feedback(Kc*sysD,sysFBDe);
step( 5*sysDeCL,30 )
```

- Integral Term at s = -0.14
- t_s >= 4.6/0.14 = 33