Spring 2020

控制系統
Control Systems

Unit 43
PID Control

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NTU-EE

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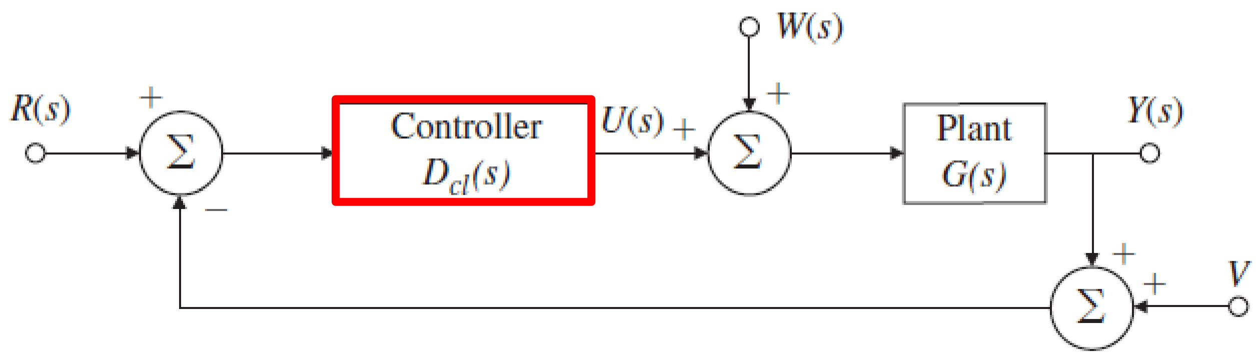
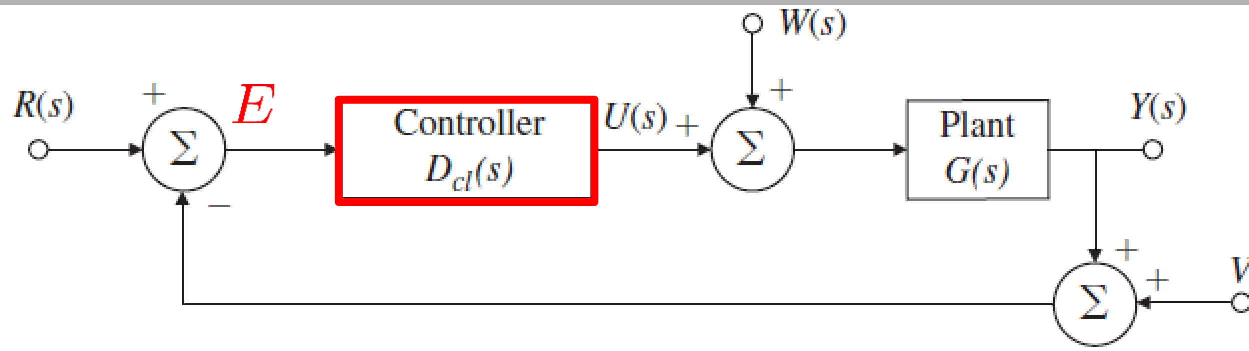
PID Control (Proportional, Integral, Derivative)

- PID Control:
  - Proportional: Simple Proportional Feedback
  - Integral: Eliminating Bias Offset
  - Derivative: Anticipatory

$$D_c(s) = k_P + \frac{k_I}{s} + k_D\, s$$



- Proportional Control



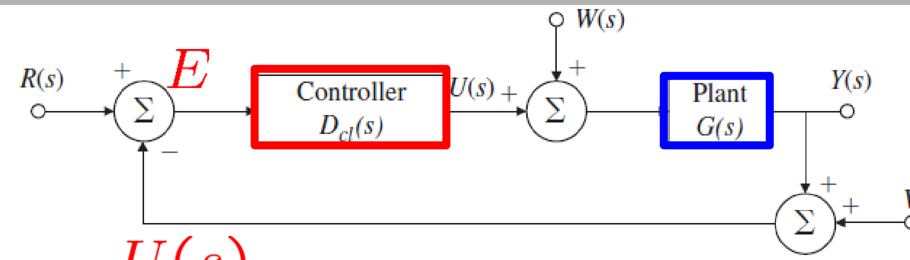
$$u(t) = k_P \, e(t)$$

$$\frac{U(s)}{E(s)} = D_{cl}(s) = k_P$$

- No dynamics, a static controller
- k_P : Proportional Gain
- An amplifier with a “knob” that can be adjusted up or down

Proportional Control

- Consider a 2nd-order plant:



$$G(s) = \frac{A}{s^2 + a_1 s + a_2}$$

$$\frac{U(s)}{E(s)} = D_{cl}(s) = k_P$$

- For the closed-loop system, the characteristic equation is:

$$\mathcal{T}(s) = \frac{Y(s)}{R(s)} = \frac{k_P\, G}{1 + k_P\, G}$$

$$1 + k_P\, G(s) = 0$$

$$1 + k_P \frac{A}{s^2 + a_1 s + a_2} = 0$$

$$s^2 + a_1 s + a_2 + k_P A = 0$$

$$s^2 + a_1 s + (a_2 + k_P A) = 0$$

$$H(s) = \frac{w_n^2}{s^2 + 2\,\zeta\, w_n\, s + w_n^2}$$

$\Rightarrow$ $(\, a_2 + k_P\, A\,)$

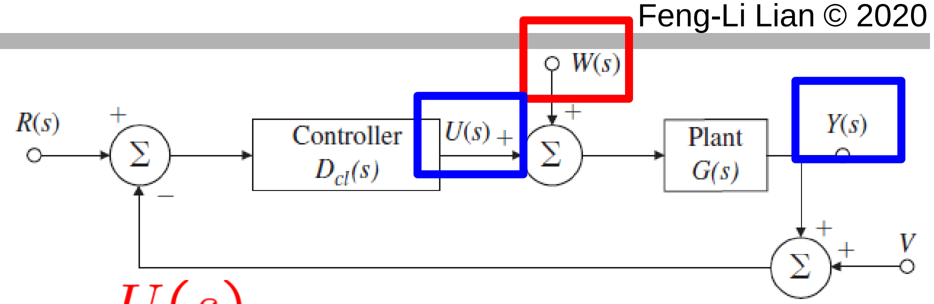
$\Rightarrow$ for natural frequency, not damping

Proportional Control

- Consider a 2nd-order plant:

$$G(s) = \frac{1}{s^2 + 1.4s + 1}$$

$$r(t) = 1(t)$$



$$\frac{U(s)}{E(s)} = D_{cl}(s) = k_P$$

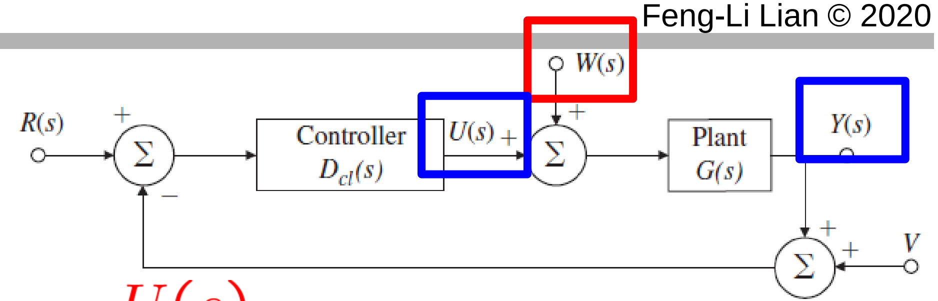
```
zeta=0.707; wn=1; A=1;

k=1.5;
sysy = k*A/(s^2+2*zeta*wn*s+wn^2+k*A);
[y,t] = step( sysy, t );

k=6;
sysy = k*A/(s^2+2*zeta*wn*s+wn^2+k*A);
[y,t] = step( sysy, t );
```

$$\frac{Y(s)}{W(s)} = \frac{G}{1 + k_P G}$$

$$\frac{U(s)}{W(s)} = \frac{k_P G}{1 + k_P G}$$

Proportional Control

- Consider a 2nd-order plant:



$$G(s) = \frac{1}{s^2 + 1.4s + 1}$$

$$r(t) = 1(t)$$

$$\frac{U(s)}{E(s)} = D_{cl}(s) = k_P$$

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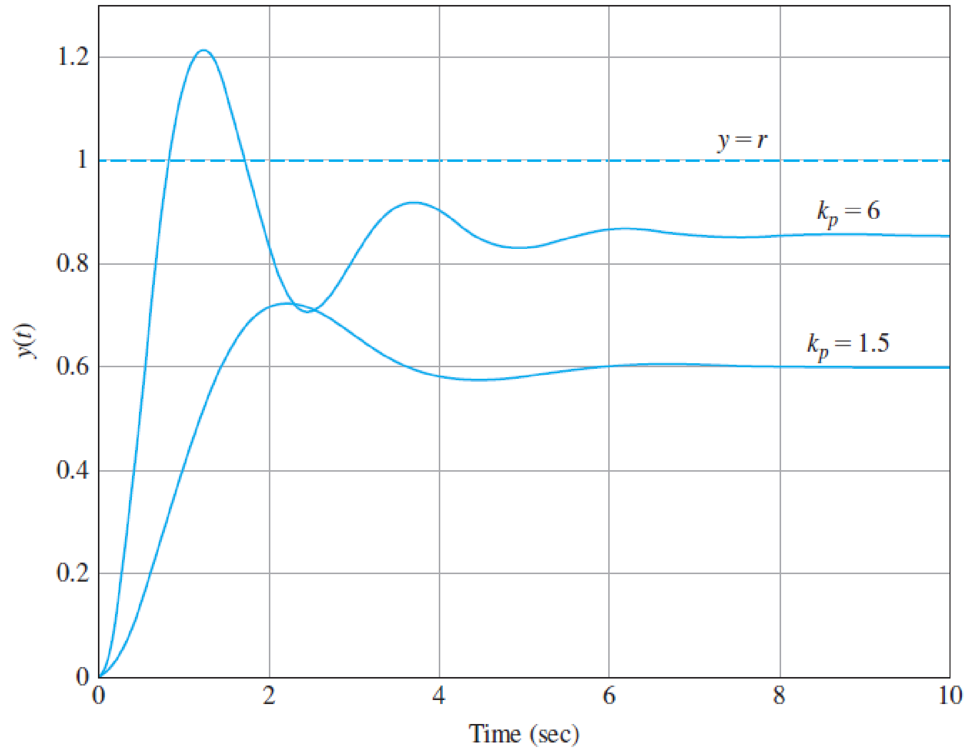
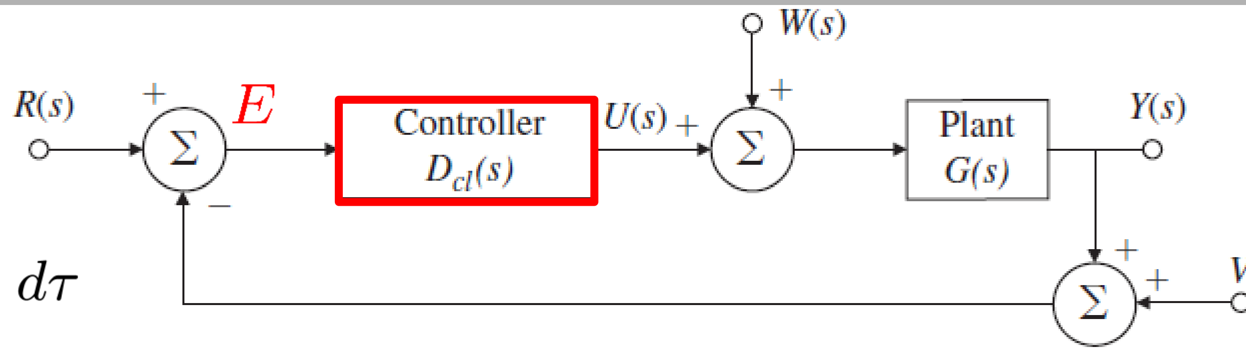
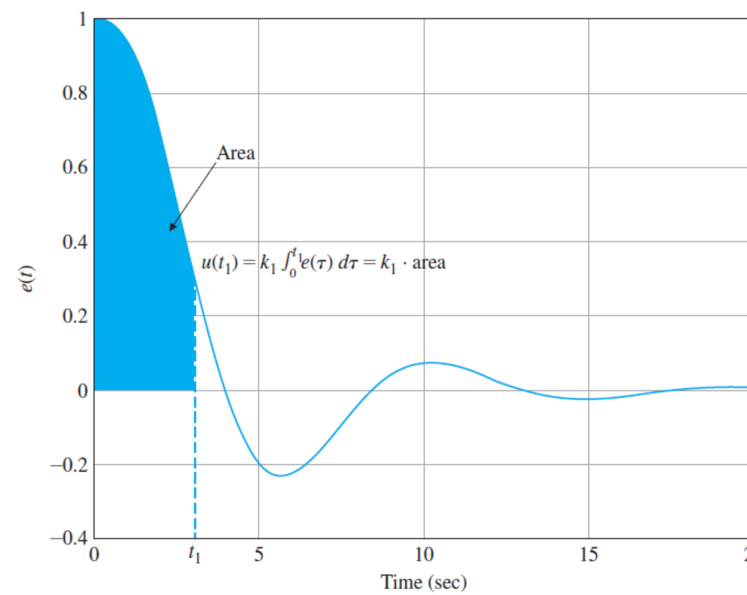


- Integral Control



$$u(t) = k_I \int_{t_0}^{t} e(\tau)\, d\tau$$

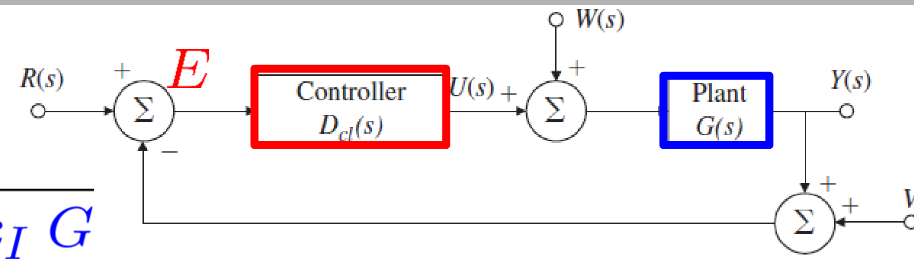
$$\frac{U(s)}{E(s)} = D_{cl}(s) = \frac{k_I}{s}$$



- k_I : Integral Gain
- Sum of all past values of the tracking error
- Based on the “history” of the system error
- A dynamic controller

Integral Control

- The closed-loop system:



$$\frac{E(s)}{R(s)} = \frac{1}{1 + \frac{k_I}{s} G} = \frac{s}{s + k_I G}$$

$$\frac{U(s)}{E(s)} = D_{cl}(s) = \frac{k_I}{s}$$

$$\frac{U(s)}{R(s)} = \frac{\frac{k_I}{s}}{1 + \frac{k_I}{s} G} = \frac{k_I}{s + k_I G}$$

$$\mathcal{T}(s) = \frac{Y(s)}{R(s)} = \frac{\frac{k_I}{s} G}{1 + \frac{k_I}{s} G} = \frac{k_I G}{s + k_I G}$$

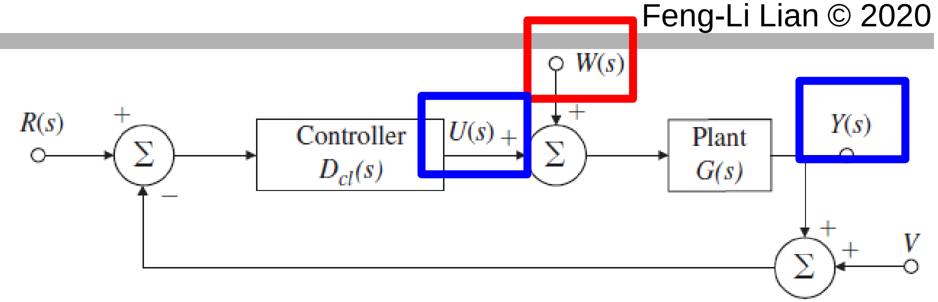
- Unit-Step Reference Input: r(t) = 1(t), R(s) = 1/s

$$y(\infty) = \lim_{s \to 0} s \frac{1}{s} \frac{k_I G}{s + k_I G} = \frac{k_I G(0)}{0 + k_I G(0)} = 1$$

$$e(\infty) = \lim_{s \to 0} s \frac{1}{s} \frac{s}{s + k_I G} = \frac{0}{0 + k_I G(0)} = 0$$

$$u(\infty) = \lim_{s \to 0} s \frac{1}{s} \frac{k_I}{s + k_I G} = \frac{k_I}{0 + k_I G(0)} = G(0)^{-1} = 1$$

Integral Control



- The closed-loop system:

$$\frac{Y(s)}{W(s)} = \frac{s\,G}{s + k_I\,G}$$

$$\frac{U(s)}{W(s)} = -\frac{k_I\,G}{s + k_I\,G}$$

- Unit-Step Disturbance Input:

  w(t) = 1(t), W(s) = 1/s

$$y(\infty) = \lim_{s\to 0} s\,\frac{1}{s}\,\frac{s\,G}{s + k_I\,G} = \frac{0\,G(0)}{0 + k_I\,G(0)} = 0$$

$$u(\infty) = \lim_{s\to 0} -s\,\frac{1}{s}\,\frac{k_I\,G}{s + k_I\,G} = -\frac{k_I\,G(0)}{0 + k_I\,G(0)} = -1$$

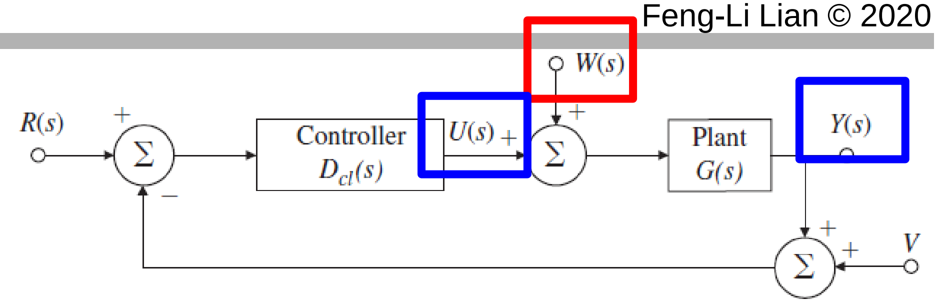
```
zeta=0.707; wn=1; A=1;

% Disturbance with integral control
ki = 0.5

sysy = s*A/(s^3+2*zeta*wn*s^2+wn^2*s+ki*A)
[y,t] = step(sysy,t);

sysu = -ki*A/(s^3+2*zeta*wn*s^2+wn^2*s+ki*A)
[u,t] = step(sysu,t);
```

Integral Control

- The closed-loop system:

$$\frac{Y(s)}{W(s)} = \frac{s\,G}{s + k_I\,G}$$

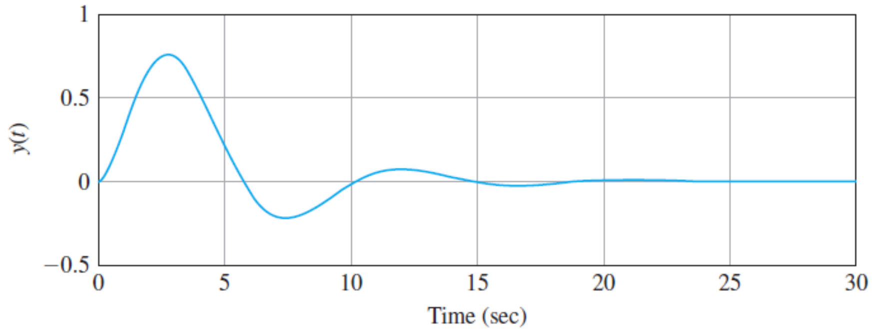
$$\frac{U(s)}{W(s)} = -\frac{k_I\,G}{s + k_I\,G}$$



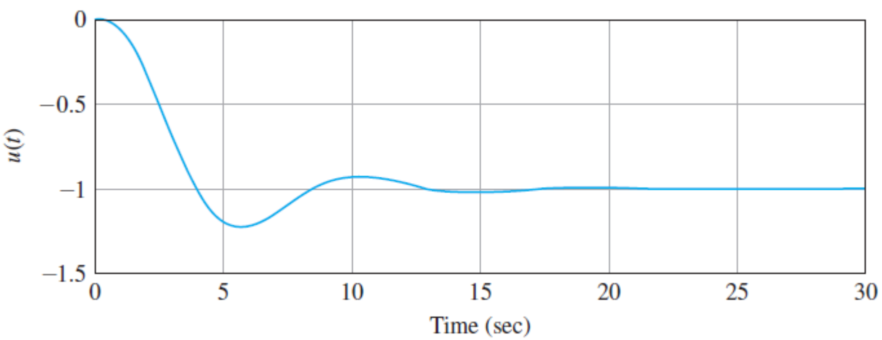
- Unit-Step Disturbance Input:
  w(t) = 1(t), W(s) = 1/s

$$y(\infty) = \lim_{s\to 0} s\,\frac{1}{s}\,\frac{s\,G}{s + k_I\,G} = \frac{0\,G(0)}{0 + k_I\,G(0)} = 0$$

$$u(\infty) = \lim_{s\to 0} -s\,\frac{1}{s}\,\frac{k_I\,G}{s + k_I\,G} = -\frac{k_I\,G(0)}{0 + k_I\,G(0)} = -1$$

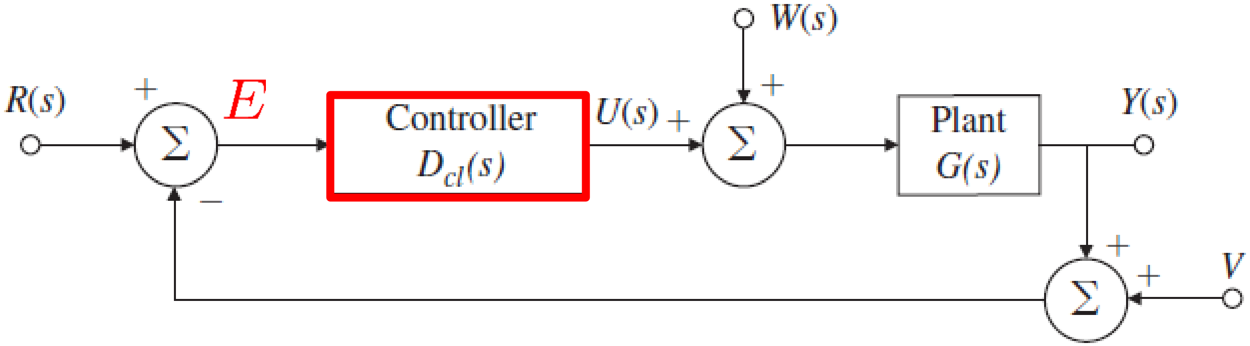


(a)



(b)

- Derivative Control



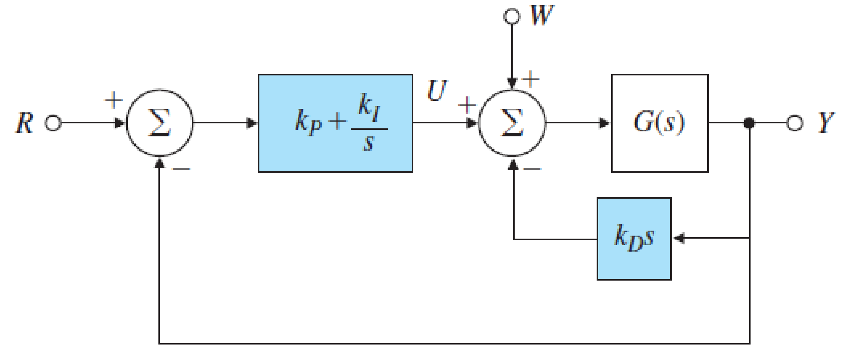
$$u(t) = k_D \, \dot{e}(t)$$

$$\frac{U(s)}{E(s)} = D_{cl}(s) = k_D \, s$$

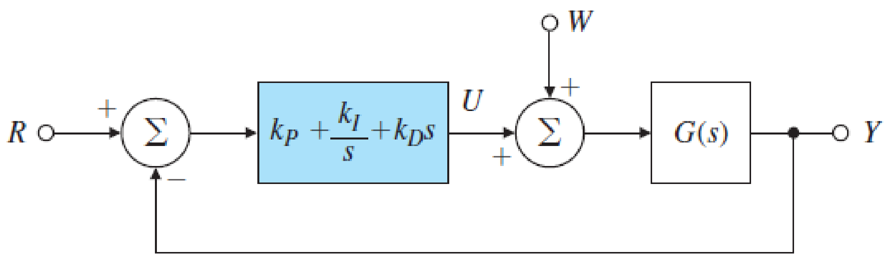
- k_D : Derivative Gain
- Rate feedback, know the slope of error signal
- Anticipatory behavior
- Improve the stability of closed-loop system
- Speed up the transient response
- Reduce overshoot
- Amplify noise

Derivative Control

- Derivative Control
  almost never used by itself.
- It is usually augmented
  by proportional control.
- Derivative Control
  does not supply desired end state.
- e(t) = constant
  ---> d e(t) = 0



(a)

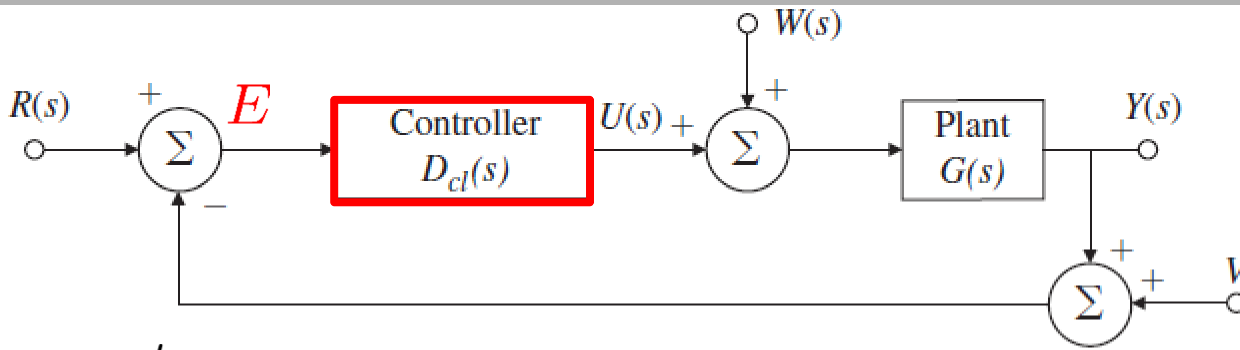


(b)

- Provide sharp response
  to suddenly changing signals
- By using velocity sensor (e.g., tachometer)
- The closed-loop characteristic equations of (a) and (b) are the same.
  But, the zeros from reference to output are different.
  Because the reference is not differentiated in (a).

Proportional + Integral Control (PI)

- PI Control



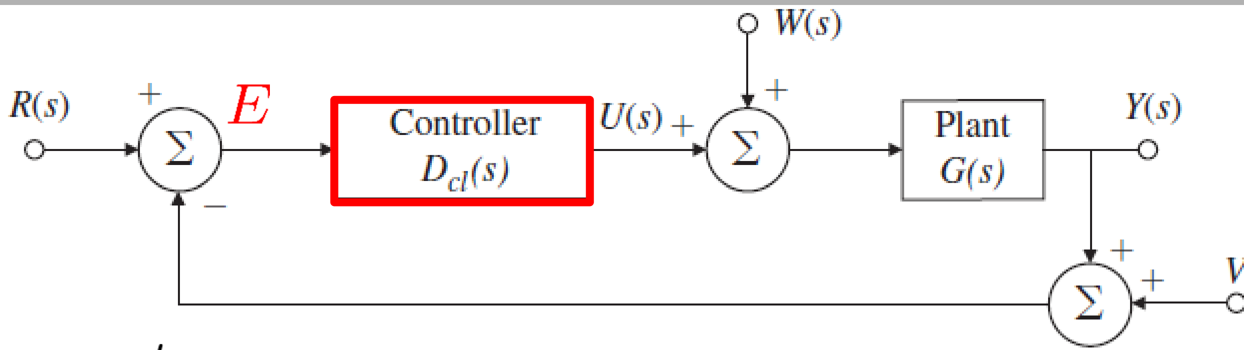
$$u(t) = k_P\, e(t) + k_I \int_{t_0}^{t} e(\tau)\, d\tau$$

$$\frac{U(s)}{E(s)} = D_{cl}(s) = k_P + \frac{k_I}{s}$$

- Allow for a fast response
- I control raises the type to Type 1 and reject constant bias disturbance

Proportional + Integral + Derivative Control (PID)

- PID Control



$$u(t) = k_P\, e(t) + k_I \int_{t_0}^{t} e(\tau)\, d\tau + k_D\, \dot{e}(t)$$

$$\frac{U(s)}{E(s)} = D_{cl}(s) = k_P + \frac{k_I}{s} + k_D\, s$$

- Characteristic Equation:

$$1 + (k_P + \frac{k_I}{s} + k_D s)\frac{A}{s^2 + a_1 s + a_2} = 0$$

$$1 + D_{cl}(s)\, G(s) = 0$$

$$s^2 + a_1 s + a_2 + A\,(\,k_P + \frac{k_I}{s} + k_D\, s\,) = 0$$

$$s^3 + a_1 s^2 + a_2 s + A\,(\,k_P s + k_I + k_D\, s^2\,) = 0$$

$$s^3 + (a_1 + A k_D)\, s^2 + (a_2 + A k_P)\, s + A\, k_I = 0$$

$\Rightarrow$ 3 roots by 3 parameters (gains)

- Example 4.5: PID Control of Motor Speed (from Ex 2.15)

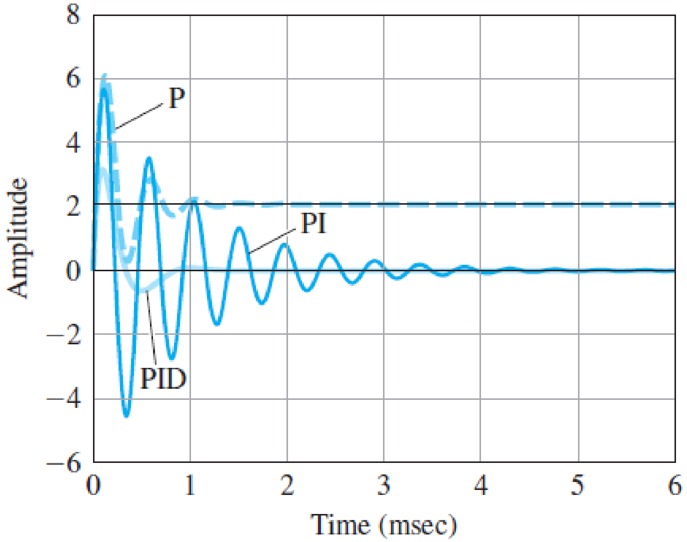
- **Parameters:**
  - J_m : 1.13 x 10^{-2} N m $\sec^2$/rad
  - *b*: 0.028 N m sec/rad
  - L_a : 10^{-1} H
  - R_a : 0.45 Ω
  - K_t : 0.67 N m/amp
  - K_e : 0.67 V sec/rad

- Gain:
  - K_P : 3
  - K_I : 15
  - K_D : 0.3

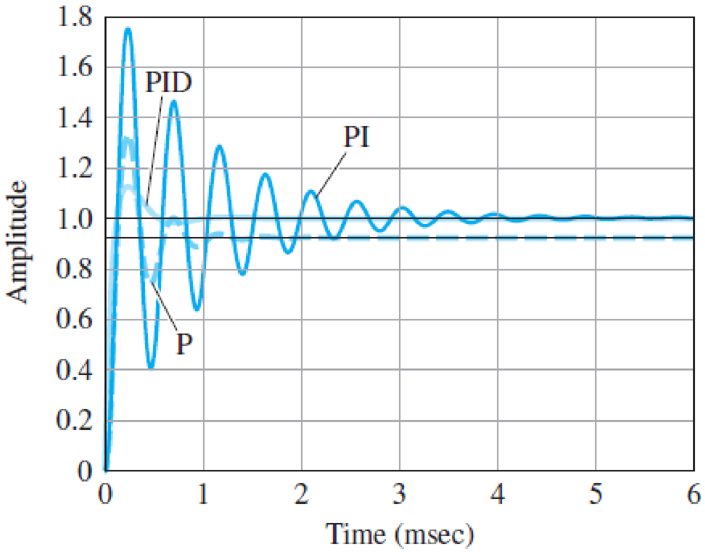
Responses of P, PI, and PID control
to: (a) step disturbance input
to: (b) step reference input

```
K=.0670; L1=0.1; J1=0.0113; R=0.45; b=0.0280;
kp=3; ki= 15; kd=0.3;
np=K;
dp=[L1*J1 R*J1+b*L1 R*b+K*K];
dclp=[L1*J1 R*J1+b*L1 R*b+K*K+K*kp];
nclp=K*kp;
nclpw=[L1 R];
dclpi=[L1*J1 R*J1+b*L1 R*b+K*K+K*kp K*ki];
nclpi=[K*kp K*ki];
nclpiw=[L1 R 0];
dclpid=[ L1*J1 R*J1+b*L1+K*kd R*b+K*K+K*kp K*ki];
nclpid=[K*kd K*kp K*ki];
nclpidw=[L1 R 0];
sysp     = tf( nclp, dclp );
syspw    = tf( nclpw, dclp );
syspi    = tf( nclpi, dclpi );
syspiw   = tf( nclpiw, dclpi );
syspid   = tf( nclpid, dclpid );
syspidw  = tf( nclpidw, dclpid );
```

- Example 4.5: PID Control of Motor Speed (from Ex 2.15)
- **Parameters:**
  - J_m : 1.13 x 10^{-2} N m $\sec^2$/rad
  - b: 0.028 N m sec/rad
  - L_a : 10^{-1} H
  - R_a : 0.45 Ω
  - K_t : 0.67 N m/amp
  - K_e : 0.67 V sec/rad
- Gain:
  - K_P : 3
  - K_I : 15
  - K_D : 0.3



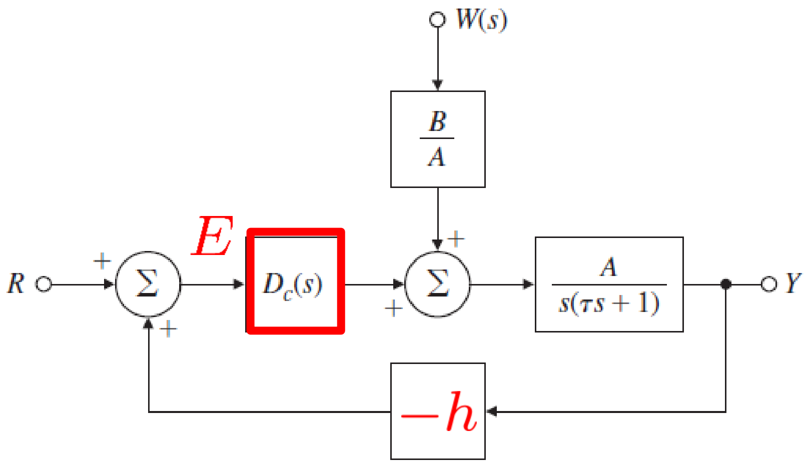
(a)



(b)

Responses of P, PI, and PID control
to: (a) step disturbance input
to: (b) step reference input

- Example 4.6: PD Control for DC Motor Position Control (From Ex 4.4)



(a) $D_c(s) = k_P$

(b) $D_c(s) = k_P + \dfrac{k_I}{s}$

(a) The closed-loop transfer function from W to E (where R = 0) is:

$$\mathcal{T}_w(s) = -\frac{B}{s(\tau s + 1) + A k_P h}$$

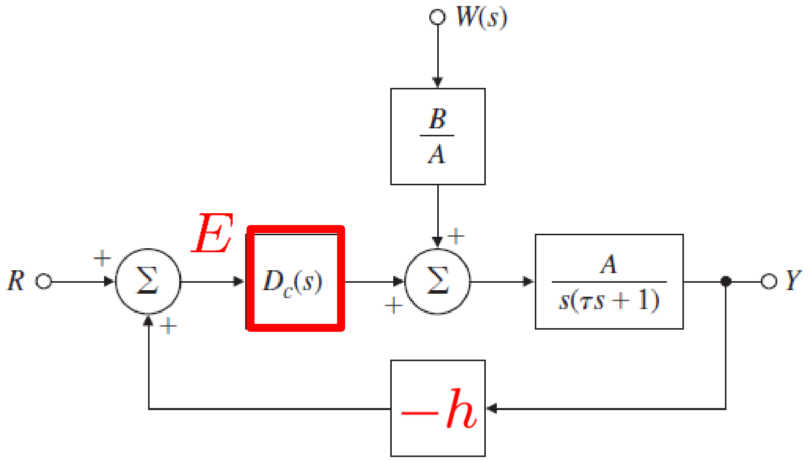
$$= s^0 \, \mathcal{T}_{w,0}(s) \qquad n = 0$$

$$K_{w,0} = -\frac{A k_P h}{B}$$

- Type 0: Steady-state error to a unit-step disturbance input is:

$$e_{ss} = -\frac{B}{A k_P h}$$

- Example 4.6: PD Control for DC Motor Position Control (From Ex 4.4)



(a) $D_c(s) = k_P$

(b) $D_c(s) = k_P + \frac{k_I}{s}$

(b) The closed-loop transfer function from W to E (where R = 0) is:

$$\mathcal{T}_w(s) = -\frac{Bs}{s^2(\tau s + 1) + (k_P s + k_I)Ah}$$

$$n = 1$$

$$K_{w,n} = -\frac{Ak_I h}{B}$$

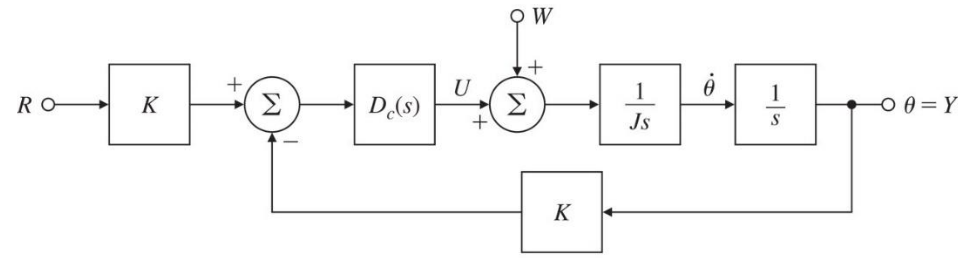
- Type 1: Steady-state error to a unit-ramp disturbance input is:

$$e_{ss} = -\frac{B}{Ak_I h}$$

- Example 4.7: Satellite Attitude Control
- J : moment of inertia
- W: disturbance torque
- K : sensor and reference gain
- D_c : compensator

(b) $D_c(s) = k_P + k_D s$

(c) $D_c(s) = k_P + \frac{k_I}{s} + k_D s$



(a)

Model of a satellite attitude control:
(a) basic system
(b) PD control
(c) PID control

- Example 4.7: Satellite Attitude Control

(b) $D_c(s) = k_P + k_D s$

$$\mathcal{T}_w(s) = \frac{1}{Js^2 + k_D s + k_P} = \mathcal{T}_{w,0}(s)$$

$n = 0 \qquad K_{w,0} = k_P$

- Type 0: $\quad e_{ss} = \dfrac{1}{k_P}$

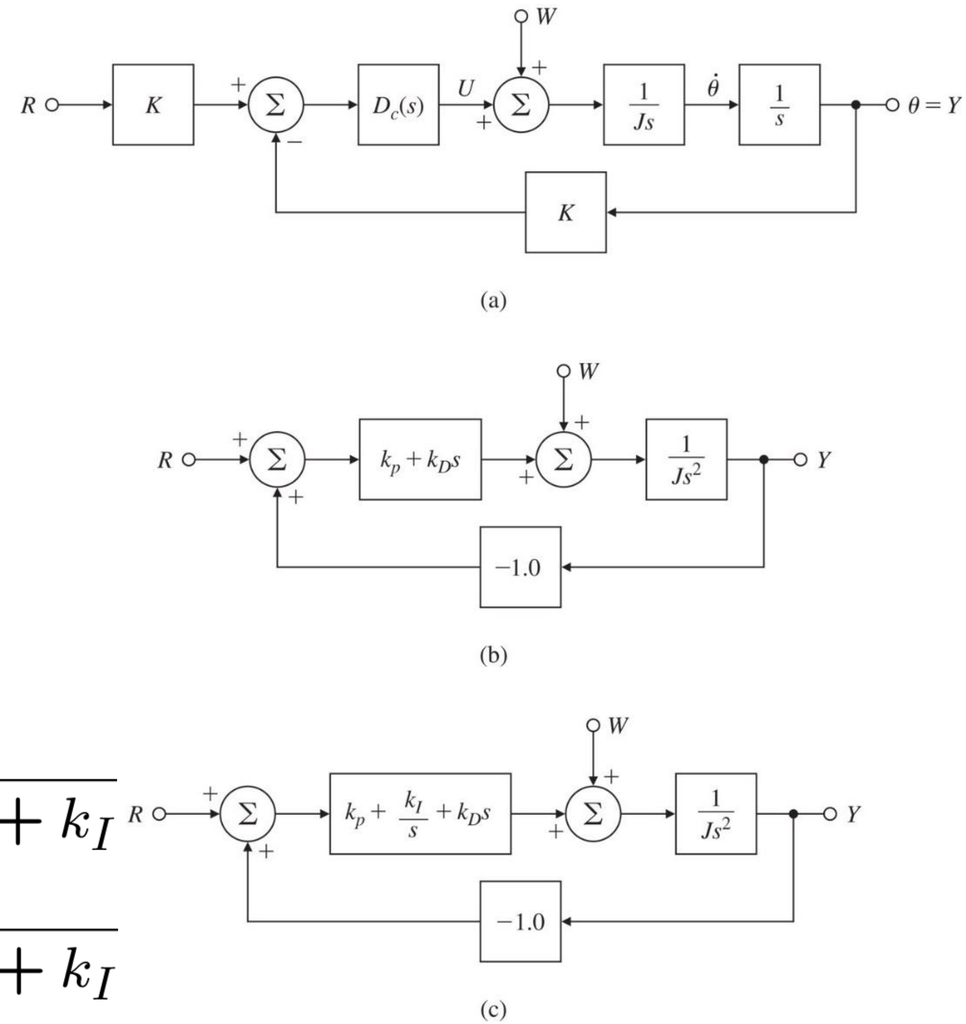
(c) $D_c(s) = k_P + \dfrac{k_I}{s} + k_D s$

$$\mathcal{T}_w(s) = \frac{s}{Js^3 + k_D s^2 + k_P s + k_I}$$

$$\mathcal{T}_{w,0}(s) = \frac{1}{Js^3 + k_D s^2 + k_P s + k_I}$$

$n = 1 \qquad K_{w,n} = k_I$

- Type 1: $\quad e_{ss} = \dfrac{1}{k_I}$



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Model of a satellite attitude control:
(a) basic system
(b) PD control
(c) PID control