

Spring 2020

控制系統 Control Systems

Unit 34 Time-Domain Specifications

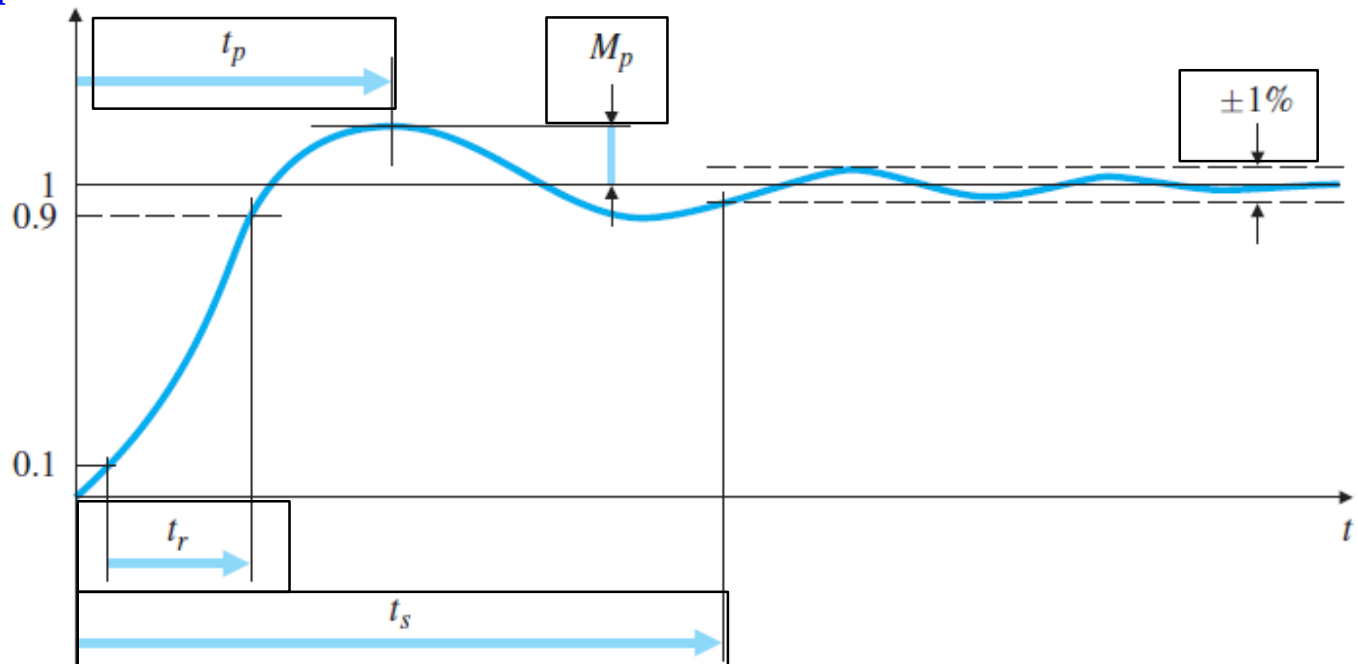
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NTU-EE

Mar 2020 – Jul 2020

- Performance specification for a control system design often involve certain requirements associated with the time response of the system.
- The requirement for a step response are expressed in terms of the standard quantities illustrated in the figure.

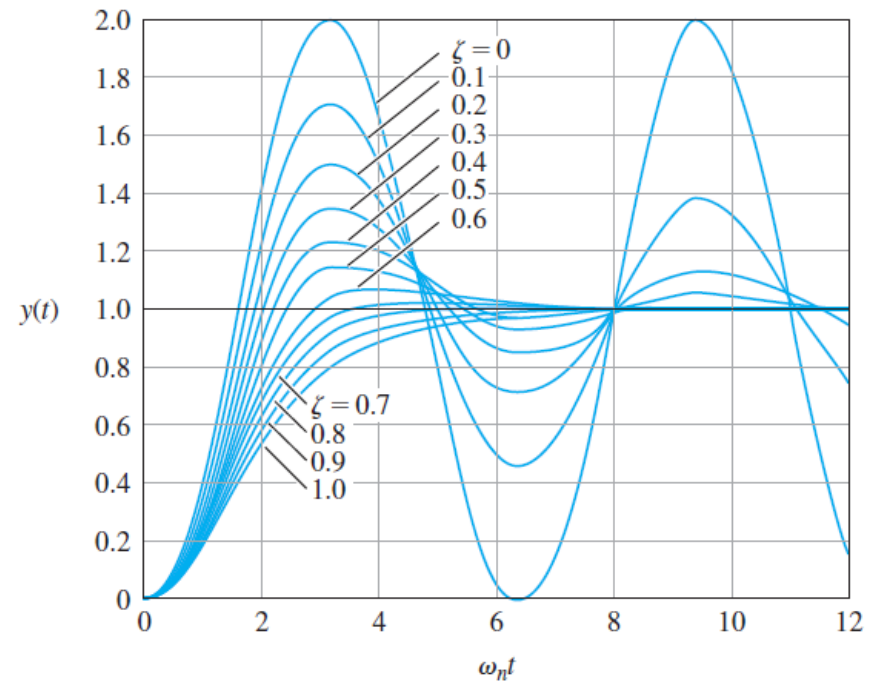
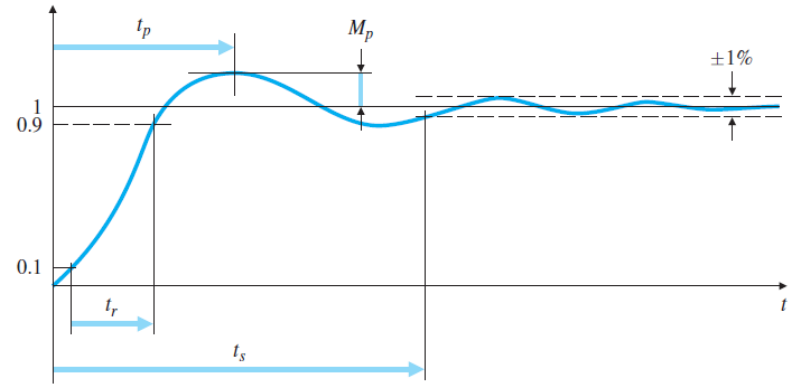
- Rise time t_r
- Settling time t_s
- Overshoot M_p
- Peak time t_p



- Rise time t_r
- The time from $y=0.1$ to $y=0.9$
- For $\zeta = 0.5$,
- Approximately,

$$\omega_n t_r = 1.8$$

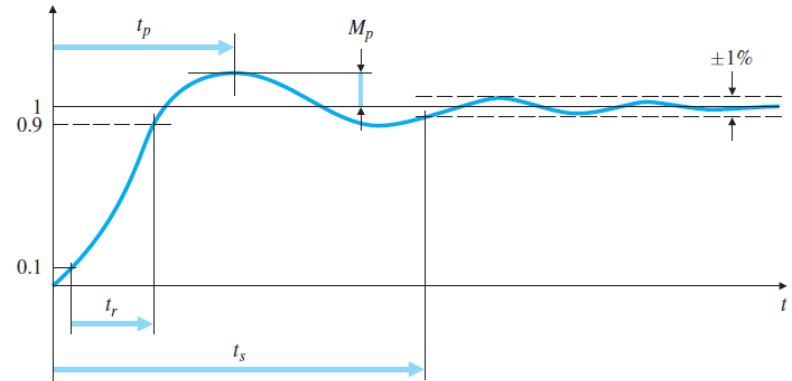
$$\Rightarrow t_r \approx \frac{1.8}{\omega_n}$$



(b)

■ Overshoot M_p and Peak time t_p

$$Y(s) = H(s) \frac{1}{s}$$



$$\Rightarrow y(t) = 1 - e^{-\sigma t} \left(\cos w_d t + \frac{\sigma}{w_d} \sin w_d t \right)$$

$$\sigma = w_n \zeta$$

$$w_d = w_n \sqrt{1 - \zeta^2}$$

$$\Rightarrow y(t) = 1 - \frac{e^{-\sigma t}}{\sqrt{1 - \zeta^2}} \cos(w_d t - \beta)$$

$$\Rightarrow \dot{y}(t_p) = 0$$

$$A \sin(\alpha) + B \cos(\beta) = C \cos(\alpha - \beta)$$

$$C = \sqrt{A^2 + B^2} = \frac{1}{\sqrt{1 - \zeta^2}}$$

$$\beta = \tan^{-1} \left(\frac{A}{B} \right)$$

$$= \tan^{-1} \left(\frac{\zeta}{\sqrt{1 - \zeta^2}} \right) = \sin^{-1}(\zeta)$$

▪ **Overshoot** M_p and **Peak time** t_p

$$y(t) = 1 - e^{-\sigma t} \left(\cos w_d t + \frac{\sigma}{w_d} \sin w_d t \right)$$

$$\Rightarrow \dot{y}(t) = \sigma e^{-\sigma t} \left(\cos w_d t + \frac{\sigma}{w_d} \sin w_d t \right) - e^{-\sigma t} (-w_d \sin w_d t + \sigma \cos w_d t)$$

$$= e^{-\sigma t} \left(\frac{\sigma^2}{w_d} + w_d \right) \sin w_d t \quad \Rightarrow \dot{y}(t_p) = 0$$

$$\Rightarrow \sin w_d t = 0$$

$$\Rightarrow y(t_p) \triangleq 1 + M_p \quad \Rightarrow t_p = \frac{\pi}{w_d}$$

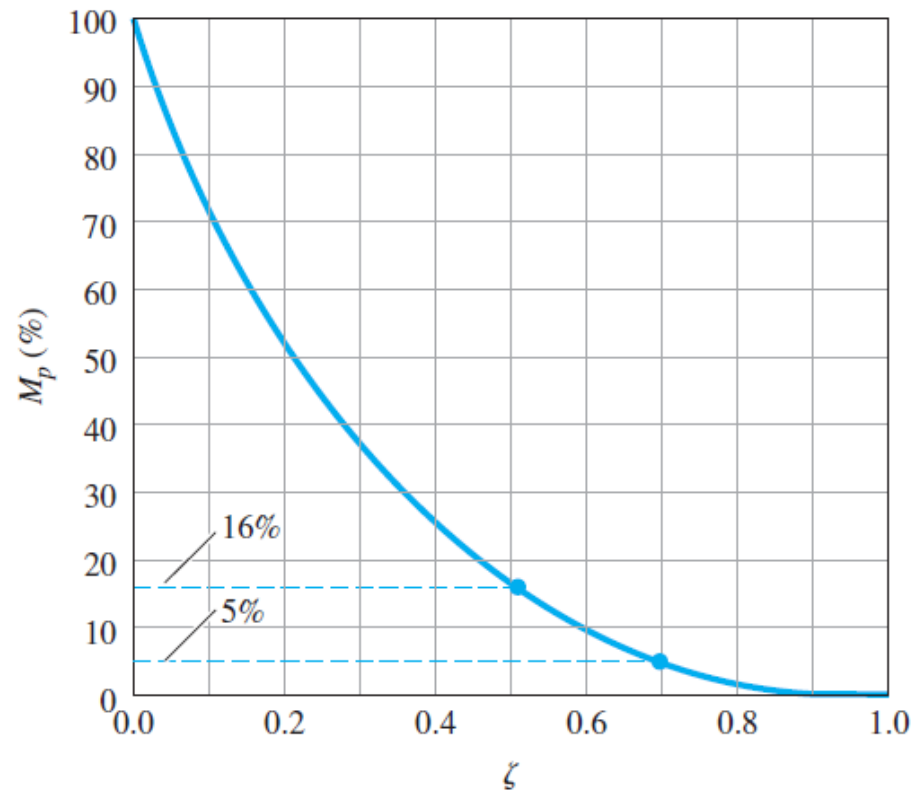
$$= 1 - e^{-\sigma \frac{\pi}{w_d}} \left(\cos \pi + \frac{\sigma}{w_d} \sin \pi \right)$$

$$= 1 + e^{-\sigma \frac{\pi}{w_d}}$$

$$\Rightarrow M_p = e^{\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}} \quad 0 \leq \zeta < 1$$

- Overshoot M_p and Peak time t_p

$$\Rightarrow M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \quad 0 \leq \zeta < 1$$

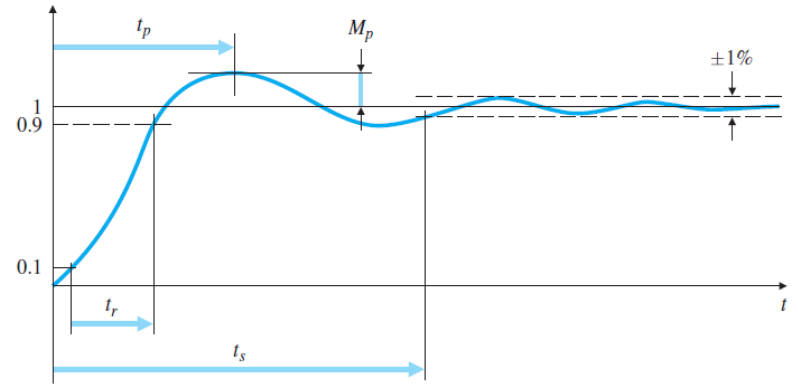


■ Settling time t_s

$$\Rightarrow e^{-\zeta \omega_n t_s} = 0.01$$

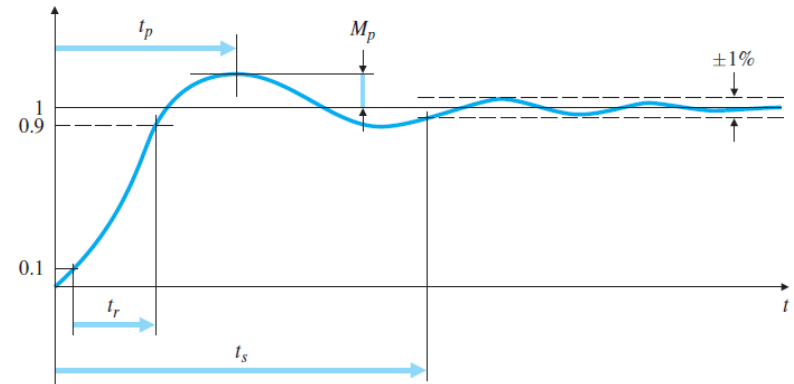
$$\Rightarrow \zeta \omega_n t_s = 4.6$$

$$\Rightarrow t_s = \frac{4.6}{\zeta \omega_n} = \frac{4.6}{\sigma}$$



Time-Domain Specifications

- Rise time t_r
- Settling time t_s
- Overshoot M_p
- Peak time t_p



$$\Rightarrow t_r \approx \frac{1.8}{\omega_n}$$

$$\Rightarrow \omega_n \geq \frac{1.8}{t_r}$$

$$\Rightarrow t_s = \frac{4.6}{\zeta \omega_n} = \frac{4.6}{\sigma}$$

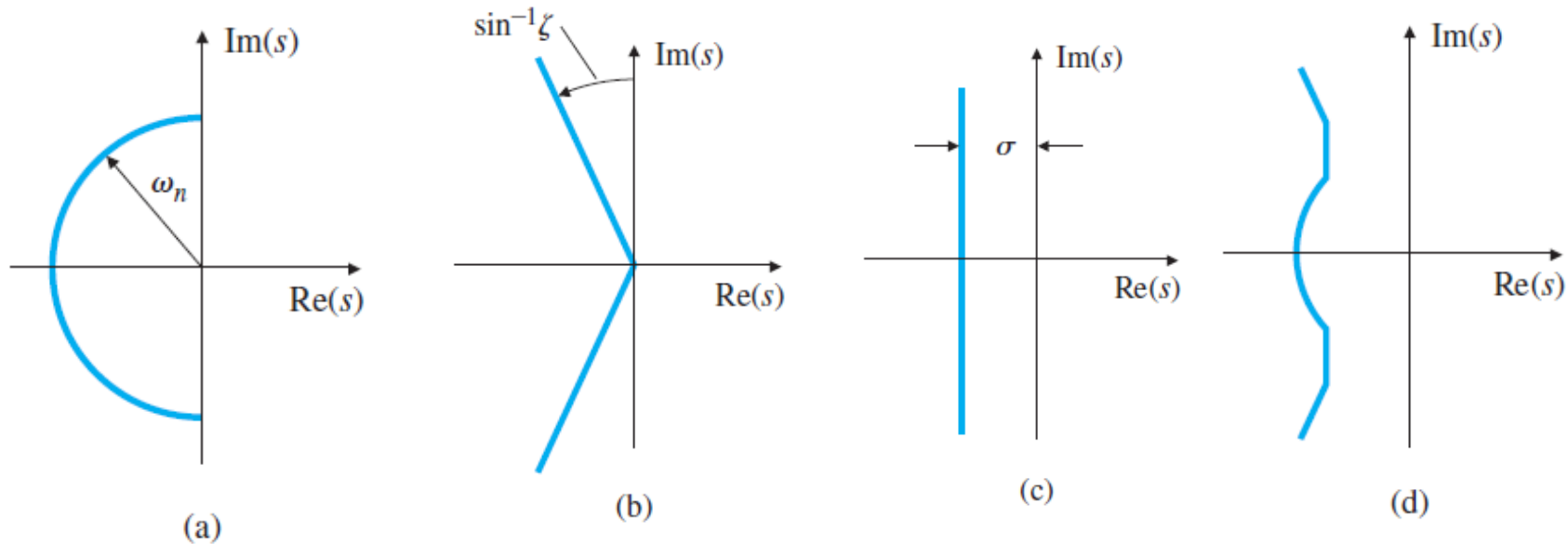
$$\Rightarrow \zeta \geq \zeta(M_p)$$

$$\Rightarrow M_p = e^{\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}} \quad 0 \leq \zeta < 1$$

$$\Rightarrow \sigma \geq \frac{4.6}{t_s}$$

- Graphs of regions in the s-plane delineated by certain transient requirements:

- (a) Rise time, (b) Overshoot, (c) Settling time,
(d) Composite of all three requirements



$$\omega_n \geq \frac{1.8}{t_r}$$

$$\zeta \geq \zeta(M_p)$$

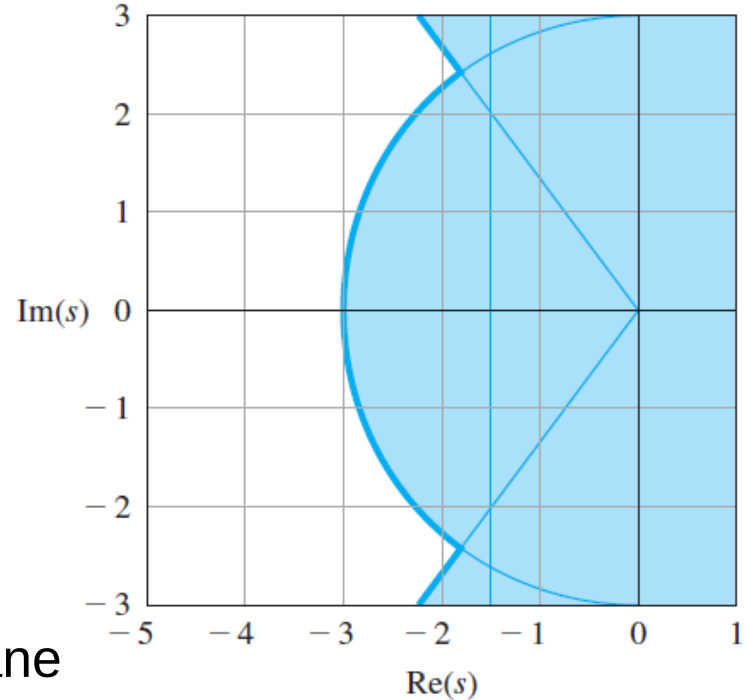
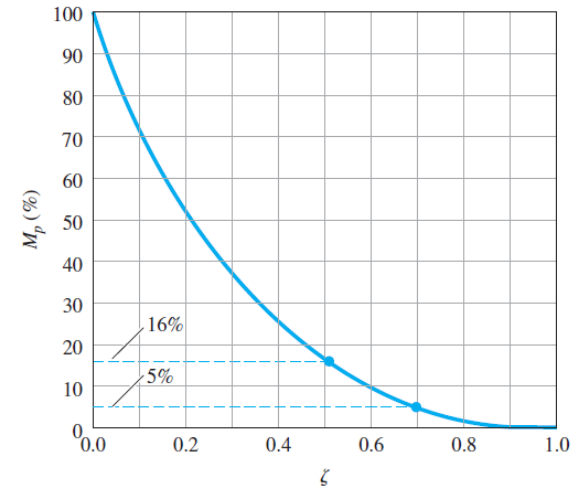
$$\sigma \geq \frac{4.6}{t_s}$$

■ Example 3.27: Transformation of specifications to s-plane

$$\Rightarrow \omega_n \geq \frac{1.8}{t_r} = 3.0 \text{ rad/sec}$$

$$\Rightarrow \zeta \geq 0.6$$

$$\Rightarrow \sigma \geq \frac{4.6}{3} = 1.5 \text{ sec}$$



- Time domain specifications region in s-plane