Spring 2020

控制系統
Control Systems

Unit 31
Laplace Transforms

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- The two-sided (or bilateral) Laplace Transform

$$F(s) \triangleq \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

- The one-sided (or unilateral) Laplace Transform

$$F(s) \triangleq \int_{0^-}^{\infty} f(t) e^{-st} dt$$

- The Fourier Transform $\quad s = \sigma + jw$

$$F(jw) \triangleq \int_{-\infty}^{\infty} f(t) e^{-jwt} dt$$

$$\mathcal{L}\left\{f(t)\right\} = \mathcal{F}\left\{f(t)e^{-\sigma t}\right\}$$

$$F(s)\Big|_{s=jw} = \mathcal{L}\left\{f(t)\right\}\Big|_{s=jw} = \mathcal{F}\left\{f(t)\right\} = F(jw)$$

- The Inverse Laplace Transform

$$f(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s) e^{st} ds$$

- LT of Step Functions

$$\mathcal{L}\left\{a\ 1(t)\right\} = \int_0^\infty a\ 1(t)\ e^{-st}dt = \frac{a}{s},\quad \mathcal{Re}\{s\} > 0$$

- LT of Ramp Functions

$$\mathcal{L}\left\{b\ t\ 1(t)\right\} = \int_0^\infty b\ t\ 1(t)\ e^{-st}dt = \frac{b}{s^2},\quad \mathcal{Re}\{s\} > 0$$

- LT of Impulse Functions

$$\mathcal{L}\left\{\delta(t)\right\} = \int_0^\infty \delta(t)\ e^{-st}dt = 1$$

- LT of Sinusoid Functions

$$\mathcal{L}\left\{\sin(wt)\right\} = \int_0^\infty \sin(wt)\ e^{-st}dt = \frac{w}{s^2 + w^2}, \mathcal{Re}\{s\} > 0$$

$$\mathcal{L}\left\{\cos(wt)\right\} = \int_0^\infty \cos(wt)\ e^{-st}dt = \frac{s}{s^2 + w^2}, \mathcal{Re}\{s\} > 0$$

Some Laplace Transform Pairs

Table of Laplace Transforms

Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$1/s$	$1(t)$
3	$1/s^2$	t
4	$2!/s^3$	t^2
5	$3!/s^4$	t^3
6	$m!/s^{m+1}$	t^m
7	$\frac{1}{s+a}$	e^{-at}
8	$\frac{1}{(s+a)^2}$	te^{-at}
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at-1+e^{-at})$

Continued

Number	$F(s)$	$f(t), t \geq 0$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bt}-ae^{-at}$
17	$\frac{a}{s^2+a^2}$	$\sin at$
18	$\frac{s}{s^2+a^2}$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin bt$
21	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$

Some Laplace Transform Pairs

TABLE 9.2 LAPLACE TRANSFORMS OF ELEMENTARY FUNCTIONS

Transform pair	Signal	Transform	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\Re e\{s\} > 0$
3	$-u(-t)$	$\frac{1}{s}$	$\Re e\{s\} < 0$
4	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$
5	$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\Re e\{s\} < 0$
6	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} > -\alpha$
7	$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} < -\alpha$
8	$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} > -\alpha$
9	$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} < -\alpha$
10	$\delta(t-T)$	e^{-sT}	All s
11	$[\cos \omega_0 t]u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
12	$[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
13	$[e^{-\alpha t}\cos \omega_0 t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\alpha$
14	$[e^{-\alpha t}\sin \omega_0 t]u(t)$	$\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\alpha$
15	$u_n(t) = \frac{d^n\delta(t)}{dt^n}$	s^n	All s
16	$u_{-n}(t) = \underbrace{u(t) * \cdots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$

信號與系統 Oppenheim et al. 1997

Some Laplace Transform Pairs

	$f(t)$	$\mathscr{L}\{f(t)\} = F(s)$
1.	1	$\dfrac{1}{s}$
2.	t	$\dfrac{1}{s^2}$
3.	t^n	$\dfrac{n!}{s^{n+1}}$, n a positive integer
4.	$t^{-1/2}$	$\sqrt{\dfrac{\pi}{s}}$
5.	$t^{1/2}$	$\dfrac{\sqrt{\pi}}{2s^{3/2}}$
6.	t^{α}	$\dfrac{\Gamma(\alpha+1)}{s^{\alpha+1}}$, $\alpha > -1$
7.	$\sin kt$	$\dfrac{k}{s^2+k^2}$
8.	$\cos kt$	$\dfrac{s}{s^2+k^2}$
9.	$\sin^2 kt$	$\dfrac{2k^2}{s(s^2+4k^2)}$
10.	$\cos^2 kt$	$\dfrac{s^2+2k^2}{s(s^2+4k^2)}$
11.	e^{at}	$\dfrac{1}{s-a}$
12.	$\sinh kt$	$\dfrac{k}{s^2-k^2}$
13.	$\cosh kt$	$\dfrac{s}{s^2-k^2}$
14.	$\sinh^2 kt$	$\dfrac{2k^2}{s(s^2-4k^2)}$
15.	$\cosh^2 kt$	$\dfrac{s^2-2k^2}{s(s^2-4k^2)}$
16.	te^{at}	$\dfrac{1}{(s-a)^2}$
17.	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$, n a positive integer

	$f(t)$	$\mathscr{L}\{f(t)\} = F(s)$
18.	$e^{at}\sin kt$	$\dfrac{k}{(s-a)^2+k^2}$
19.	$e^{at}\cos kt$	$\dfrac{s-a}{(s-a)^2+k^2}$
20.	$e^{at}\sinh kt$	$\dfrac{k}{(s-a)^2-k^2}$
21.	$e^{at}\cosh kt$	$\dfrac{s-a}{(s-a)^2-k^2}$
22.	$t\sin kt$	$\dfrac{2ks}{(s^2+k^2)^2}$
23.	$t\cos kt$	$\dfrac{s^2-k^2}{(s^2+k^2)^2}$
24.	$\sin kt + kt\cos kt$	$\dfrac{2ks^2}{(s^2+k^2)^2}$
25.	$\sin kt - kt\cos kt$	$\dfrac{2k^3}{(s^2+k^2)^2}$
26.	$t\sinh kt$	$\dfrac{2ks}{(s^2-k^2)^2}$
27.	$t\cosh kt$	$\dfrac{s^2+k^2}{(s^2-k^2)^2}$
28.	$\dfrac{e^{at}-e^{bt}}{a-b}$	$\dfrac{1}{(s-a)(s-b)}$
29.	$\dfrac{ae^{at}-be^{bt}}{a-b}$	$\dfrac{s}{(s-a)(s-b)}$
30.	$1-\cos kt$	$\dfrac{k^2}{s(s^2+k^2)}$
31.	$kt-\sin kt$	$\dfrac{k^3}{s^2(s^2+k^2)}$
32.	$\dfrac{a\sin bt - b\sin at}{ab(a^2-b^2)}$	$\dfrac{1}{(s^2+a^2)(s^2+b^2)}$
33.	$\dfrac{\cos bt-\cos at}{a^2-b^2}$	$\dfrac{s}{(s^2+a^2)(s^2+b^2)}$
34.	$\sin kt\sinh kt$	$\dfrac{2k^2 s}{s^4+4k^4}$
35.	$\sin kt\cosh kt$	$\dfrac{k(s^2+2k^2)}{s^4+4k^4}$
36.	$\cos kt\sinh kt$	$\dfrac{k(s^2-2k^2)}{s^4+4k^4}$
37.	$\cos kt\cosh kt$	$\dfrac{s^3}{s^4+4k^4}$

	$f(t)$	$\mathscr{L}\{f(t)\} = F(s)$
38.	$J_0(kt)$	$\dfrac{1}{\sqrt{s^2+k^2}}$
39.	$\dfrac{e^{bt}-e^{at}}{t}$	$\ln\dfrac{s-a}{s-b}$
40.	$\dfrac{2(1-\cos kt)}{t}$	$\ln\dfrac{s^2+k^2}{s^2}$
41.	$\dfrac{2(1-\cosh kt)}{t}$	$\ln\dfrac{s^2-k^2}{s^2}$
42.	$\dfrac{\sin at}{t}$	$\arctan\left(\dfrac{a}{s}\right)$
43.	$\dfrac{\sin at\cos bt}{t}$	$\dfrac{1}{2}\arctan\dfrac{a+b}{s}+\dfrac{1}{2}\arctan\dfrac{a-b}{s}$
44.	$\dfrac{1}{\sqrt{\pi t}}e^{-a^2/4t}$	$\dfrac{e^{-a\sqrt{s}}}{\sqrt{s}}$
45.	$\dfrac{a}{2\sqrt{\pi t^3}}e^{-a^2/4t}$	$e^{-a\sqrt{s}}$
46.	$\operatorname{erfc}\left(\dfrac{a}{2\sqrt{t}}\right)$	$\dfrac{e^{-a\sqrt{s}}}{s}$
47.	$2\sqrt{\dfrac{t}{\pi}}e^{-a^2/4t}-a\operatorname{erfc}\left(\dfrac{a}{2\sqrt{t}}\right)$	$\dfrac{e^{-a\sqrt{s}}}{s\sqrt{s}}$
48.	$e^{ab}e^{b^2t}\operatorname{erfc}\left(b\sqrt{t}+\dfrac{a}{2\sqrt{t}}\right)$	$\dfrac{e^{-a\sqrt{s}}}{\sqrt{s}(\sqrt{s}+b)}$
49.	$-e^{ab}e^{b^2t}\operatorname{erfc}\left(b\sqrt{t}+\dfrac{a}{2\sqrt{t}}\right)+\operatorname{erfc}\left(\dfrac{a}{2\sqrt{t}}\right)$	$\dfrac{be^{-a\sqrt{s}}}{s(\sqrt{s}+b)}$
50.	$e^{at}f(t)$	$F(s-a)$
51.	$\mathscr{U}(t-a)$	$\dfrac{e^{-as}}{s}$
52.	$f(t-a)\mathscr{U}(t-a)$	$e^{-as}F(s)$
53.	$g(t)\mathscr{U}(t-a)$	$e^{-as}\mathscr{L}\{g(t+a)\}$
54.	$f^{(n)}(t)$	$s^nF(s)-s^{(n-1)}f(0)-\cdots-f^{(n-1)}(0)$
55.	$t^n f(t)$	$(-1)^n\dfrac{d^n}{ds^n}F(s)$
56.	$\displaystyle\int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$
57.	$\delta(t)$	1
58.	$\delta(t-t_0)$	e^{-st_0}

微分方程 Zill 2009

Properties of Laplace Transforms

- Superposition
- Time Delay
- Time Scaling
- Shift in Frequency
- Differentiation
- Integration
- Convolution
- Time Product
- Multiplication by Time

- Initial-Value Theorem
- Final-Value Theorem

Properties of Laplace Transforms

Number	Laplace Transform	Time Function	Comment
—	$F(s)$	$f(t)$	Transform pair
1	$\alpha F_1(s) + \beta F_2(s)$	$\alpha f_1(t) + \beta f_2(t)$	Superposition
2	$F(s)e^{-s\lambda}$	$f(t-\lambda)$	Time delay ($\lambda \geq 0$)
3	$\frac{1}{\lvert a\rvert}F\left(\frac{s}{a}\right)$	$f(at)$	Time scaling
4	$F(s+a)$	$e^{-at}f(t)$	Shift in frequency
5	$s^m F(s) - s^{m-1}f(0) - s^{m-2}\dot{f}(0) - \cdots - f^{(m-1)}(0)$	$f^{(m)}(t)$	Differentiation
6	$\frac{1}{s}F(s)$	$\int_0^t f(\zeta)\,d\zeta$	Integration
7	$F_1(s)F_2(s)$	$f_1(t) * f_2(t)$	Convolution
8	$\lim_{s\to\infty} sF(s)$	$f(0^+)$	Initial Value Theorem
9	$\lim_{s\to 0} sF(s)$	$\lim_{t\to\infty} f(t)$	Final Value Theorem
10	$\frac{1}{2\pi j}\int_{\sigma_c-j\infty}^{\sigma_c+j\infty} F_1(\zeta)F_2(s-\zeta)d\zeta$	$f_1(t)f_2(t)$	Time product
11	$\frac{1}{2\pi}\int_{-j\infty}^{+j\infty} Y(-j\omega)U(j\omega)\,d\omega$	$\int_0^\infty y(t)u(t)\,dt$	Parseval's Theorem
12	$-\frac{d}{ds}F(s)$	$tf(t)$	Multiplication by time

控制系統 Franklin 2015

Properties of Laplace Transforms

- Superposition
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- Initial-Value Theorem
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TABLE 9.1 PROPERTIES OF THE LAPLACE TRANSFORM

Section	Property	Signal	Laplace Transform	ROC
		$x(t)$ $x_1(t)$ $x_2(t)$	$X(s)$ $X_1(s)$ $X_2(s)$	R R_1 R_2
9.5.1	Linearity	$ax_1(t)+bx_2(t)$	$aX_1(s)+bX_2(s)$	At least $R_1 \cap R_2$
9.5.2	Time shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	R
9.5.3	Shifting in the s-Domain	$e^{s_0t}x(t)$	$X(s-s_0)$	Shifted version of R (i.e., s is in the ROC if $s-s_0$ is in R)
9.5.4	Time scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
9.5.5	Conjugation	$x^*(t)$	$X^*(s^*)$	R
9.5.6	Convolution	$x_1(t)*x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
9.5.7	Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
9.5.8	Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
9.5.9	Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\Re e\{s\} > 0\}$

Initial- and Final-Value Theorems

9.5.10 If $x(t)=0$ for $t<0$ and $x(t)$ contains no impulses or higher-order singularities at $t=0$, then

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

If $x(t)=0$ for $t<0$ and $x(t)$ has a finite limit as $t \longrightarrow \infty$, then

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

信號與系統 Oppenheim et al. 1997

- Inverse Laplace Transforms by Partial-Fraction Expansion

$$F(s) = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}{s^n + a_1 s^{n-1} + \cdots + a_n}$$

$$= K \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

$s = z_i$, zero

$s = p_i$, pole

$$= K \frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{i=1}^{n}(s-p_i)}$$

$$= \frac{C_1}{s-p_1} + \frac{C_2}{s-p_2} + \cdots + \frac{C_n}{s-p_n}$$

$$f(t) = C_1 e^{p_1 t} 1(t) + C_2 e^{p_2 t} 1(t) + \cdots + C_n e^{p_n t} 1(t)$$

- Example 3.11: Partial-Fraction Expansion: Distinct Real Roots

$$Y(s) = \frac{(s+2)(s+4)}{s(s+1)(s+3)}$$

$$= \frac{\frac{8}{3}}{s} + \frac{-\frac{3}{2}}{s+1} + \frac{-\frac{1}{6}}{s+3}$$

$$y(t) = \frac{8}{3}\,1(t) - \frac{3}{2}\,e^{-t}\,1(t) - \frac{1}{6}\,e^{-3t}\,1(t)$$

- Example 3.12: The Final Value Theorem

$$Y(s) = \frac{3(s+2)}{s(s^2+2s+10)}$$

$$y(\infty) = s\,Y(s)\Big|_{s=0} = \frac{3\cdot 2}{10} = 0.6$$

- Example 3.15: Homogeneous Differential Equation

$$\ddot{y}(t) + y(t) = 0 \qquad \text{where } y(0) = a, \ \dot{y}(0) = b$$

$$s^2 Y(s) - a\,s - b + Y(s) = 0$$

$$(s^2 + 1)\,Y(s) = a\,s + b$$

$$Y(s) = \frac{a\,s}{s^2 + 1} + \frac{b}{s^2 + 1}$$

$$y(t) = [\,a \cos t + b \sin t\,]\,1(t)$$

- Example 3.16: Forced Differential Equation

$$\ddot{y}(t) + 5\,\dot{y}(t) + 4\,y(t) = 3 \qquad \text{where } y(0) = a,\ \dot{y}(0) = b$$

$$[\, s^2\,Y(s) - a\,s - b\,] + 5\,[\, s\,Y(s) - a\,] + 4\,Y(s) = \frac{3}{s}$$

$$Y(s) = \frac{s\,(\,s\,a + b + 5\,a\,) + 3}{s\,(s + 1)\,(s + 4)}$$

$$= \frac{\frac{3}{4}}{s} - \frac{\frac{3-b-4a}{3}}{s + 1} + \frac{\frac{3-4a-4b}{12}}{s + 4}$$

$$y(t) = \left(\frac{3}{4} + \frac{-3 + b + 4a}{3}\,e^{-t} + \frac{3 - 4a - 4b}{12}\,e^{-4t}\right) 1(t)$$

- Example 3.17: Forced Equation Solution with zero I.C.

$$\ddot{y}(t) + 5\,\dot{y}(t) + 4\,y(t) = u(t) \qquad \text{where } y(0) = 0,\ \dot{y}(0) = 0$$

$$u(t) = 2\,e^{-2t}\,1(t)$$

$$s^2\,Y(s) + 5\,s\,Y(s) + 4\,Y(s) = \frac{2}{s+2}$$

$$Y(s) = \frac{2}{(s+2)(s+1)(s+4)}$$

$$= -\frac{1}{s+2} + \frac{\frac{2}{3}}{s+1} + \frac{\frac{1}{3}}{s+4}$$

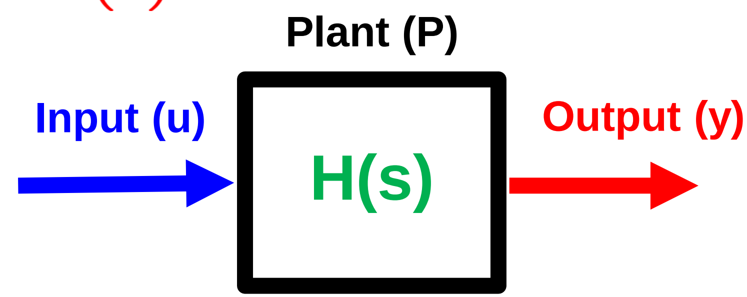
$$y(t) = \left(-1\,e^{-2t} + \frac{2}{3}\,e^{-t} + \frac{1}{3}\,e^{-4t}\right)1(t)$$

Poles and Zeros

- Rational Transfer Function

$$H(s) = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}{s^n + a_1 s^{n-1} + \cdots + a_n} = \frac{N(s)}{D(s)}$$

$$= K \frac{\prod_{i=1}^{m}(s - z_i)}{\prod_{i=1}^{n}(s - p_i)}$$



- Zeros $\quad s = z_i,\;$ zero $\qquad H(s)\Big|_{s=z_i} = 0$

- Poles $\quad s = p_i,\;$ pole $\qquad \Big|H(s)\Big|_{s=p_i} = \infty$

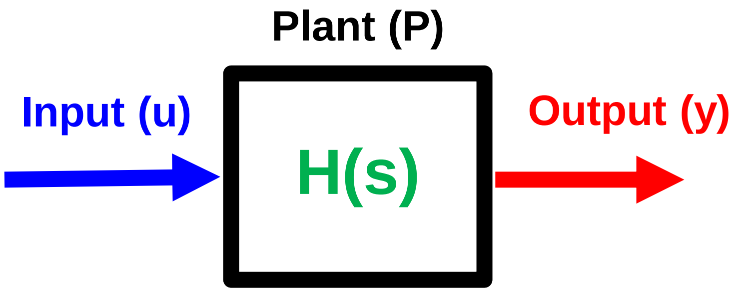
- The Poles are the Modes of the System

- The Poles of the system determine
  the Stability properties and the natural or unforced behavior.

Example 3.18: Cruise Control Transfer Function

$$H(s) = \frac{0\,s^2 + 0\,s + 0.001}{s^2 + 0.05\,s + 0} = \frac{0.001}{s\,(\,s + 0.05\,)}$$

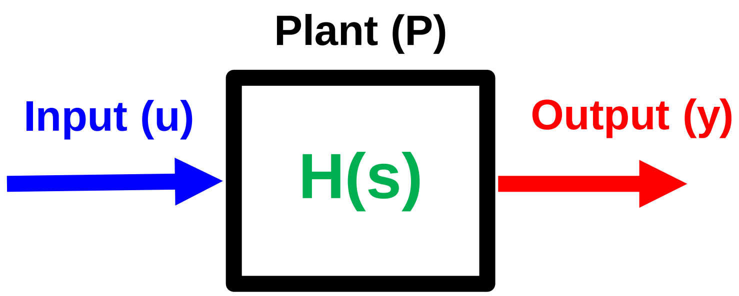
```
num = [ 0 0 0.001 ]
den = [ 1 0.05 0 ]
[ z, p, k ] = tf2zp( num, den )

%z = [ ]
%p = [ 0 -0.05 ]'
%K = 0.001
```



Example 3.19: DC Motor Transfer Function

$$H(s) = \frac{100}{s^3 + 10.1\, s^2 + 101\, s} = \frac{100}{s\,(\,s^2 + 10.1\, s + 101\,)}$$

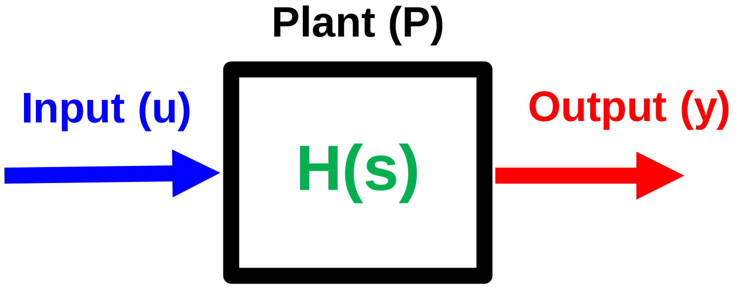
```
numb = [ 0 0 100 ];
denb = [ 1 10.1 101 ];
[ z, p, k ] = tf2zp( numb, denb )

%z = [ ]
%p = [ -5.0500+8.6889j -5.0500-8.6889j ]'
%K = 100
```



▪ Example 3.19: DC Motor Transfer Function

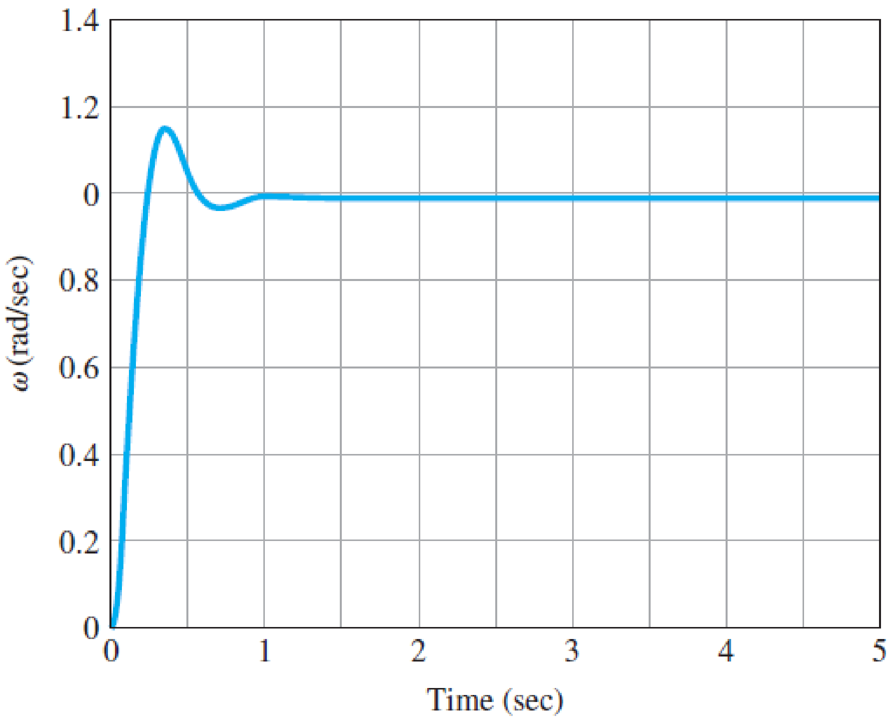
$$G(s) = \frac{100\ s}{s^3 + 10.1\ s^2 + 101\ s} = \frac{100}{(\ s^2 + 10.1\ s + 101\ )}$$

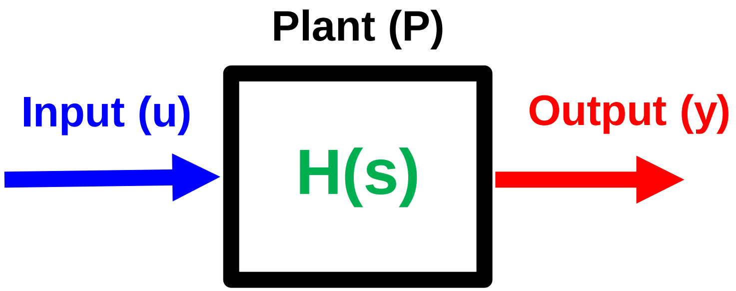
```
numb = [ 0 0 100 ];
denb = [ 1 10.1 101 ];
[ z, p, k ] = tf2zp( numb, denb )

%z = [ ]
%p = [ -5.0500+8.6889j -5.0500-8.6889j ]'
%K = 100

s = tf( 's' );
sysb = 100*s/(s^3 + 10.1*s^2 + 101*s );
t = 0:0.01:5;

y = step( sysb, t );
plot( t, y )
```



- Example 3.19: DC Motor Transfer Function

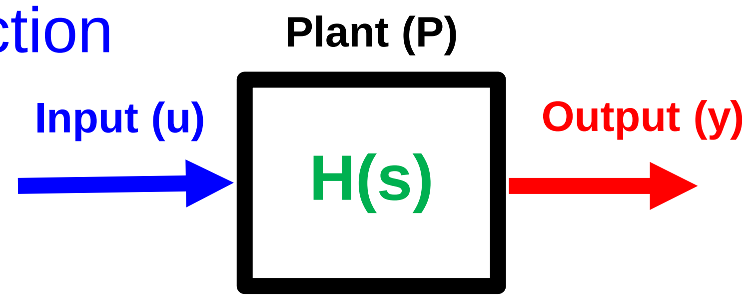
$$G(s) = \frac{100\,s}{s^3 + 10.1\,s^2 + 101\,s} = \frac{100}{(s^2 + 10.1\,s + 101)}$$





- Example 3.21: Satellite Transfer Function

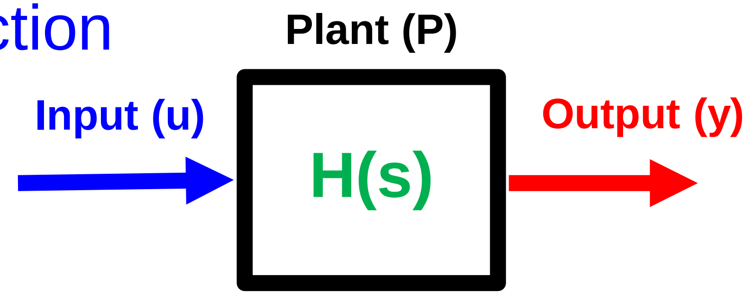
$$H(s) = \frac{0,0002}{s^2}$$



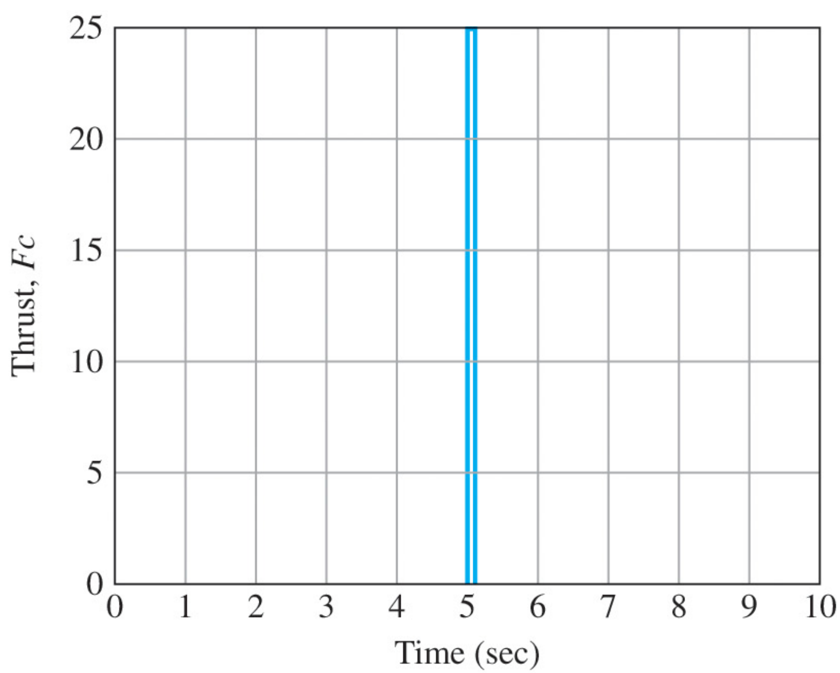

```
numG = [ 0 0 0.0002 ];
denG = [ 1 0 0 ];

s = tf( 's' )
sysG = 0.0002/(s^2)

t = 0:0.01:10;

% u1
u1 = [ zeros( 1, 500 ) 25*ones( 1, 10) zeros( 1, 491 ) ];
[ y1 ] = lsim( sysG, u1, t );
ff = 180/pi;
y1 = ff*y1;

plot( t, u1 );
plot( t, y1 );
```

▪ Example 3.21: Satellite Transfer Function

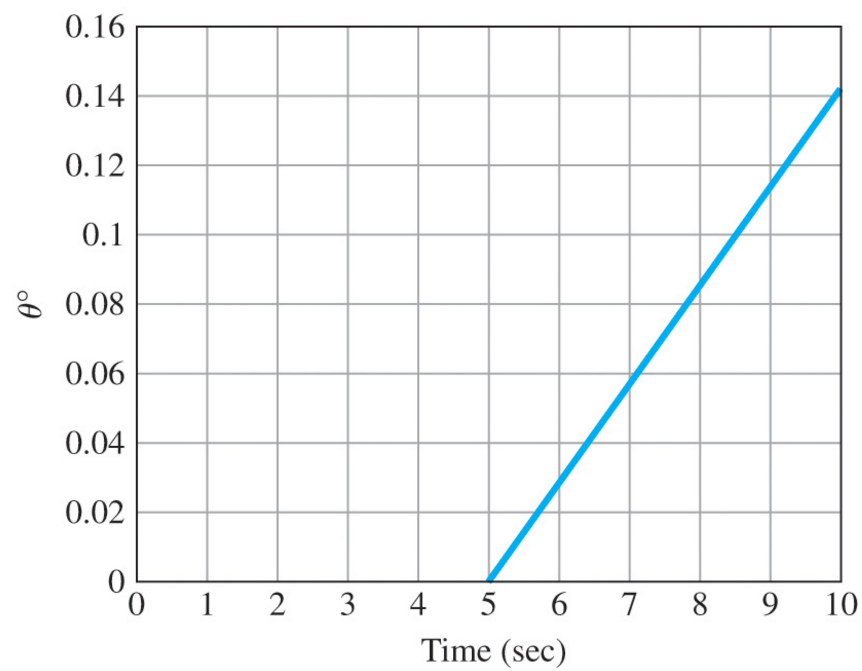
$$H(s) = \frac{0,0002}{s^2}$$



- Transient response for satellite:
  (a) thrust input (b) satellite attitude

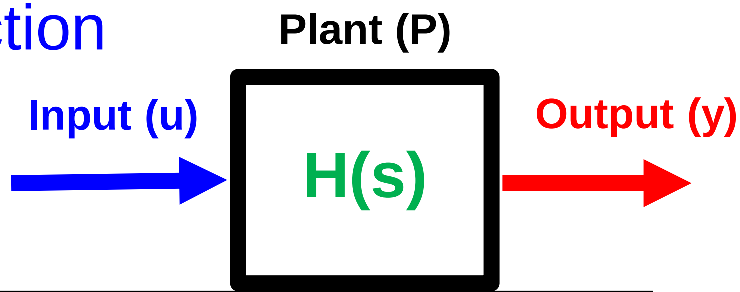


(a)



(b)

▪ Example 3.21: Satellite Transfer Function

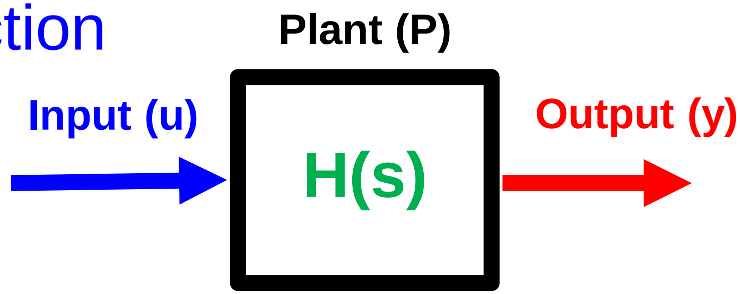


$$H(s) = \frac{0,0002}{s^2}$$

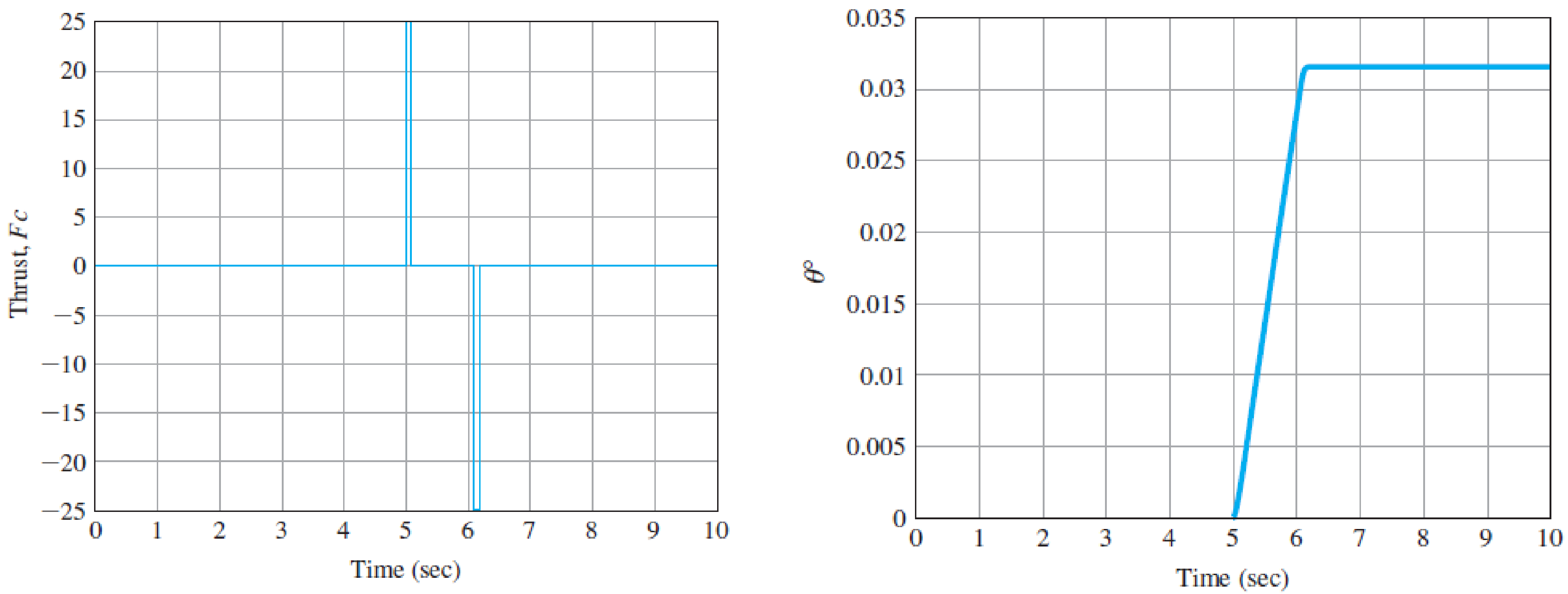
```
numG = [ 0 0 0.0002 ];
denG = [ 1 0 0 ];

s = tf( 's' )
sysG = 0.0002/(s^2)

t = 0:0.01:10;

% u2
u2 = [ zeros(1,500) 25*ones(1,10) zeros(1,100) (-25)*ones(1,10) zeros(1, 381) ];
[ y2 ] = lsim( sysG, u2, t );
ff = 180/pi;
y2 = ff*y2;

plot( t, u2 );
plot( t, y2 );
```

▪ Example 3.21: Satellite Transfer Function

$$H(s) = \frac{0,0002}{s^2}$$



- Transient response for satellite (double pulse):
  (a) thrust input (b) satellite attitude



(a) (b)