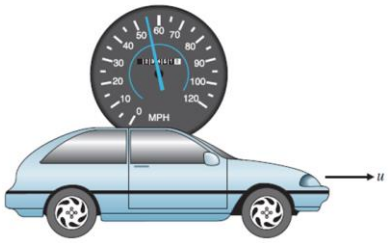
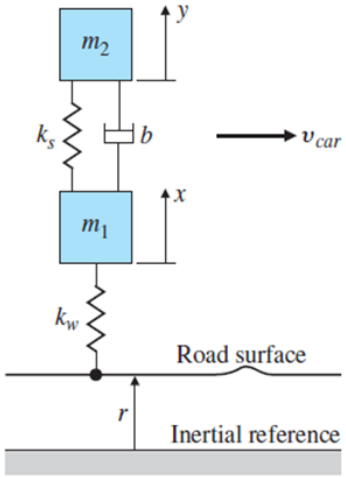
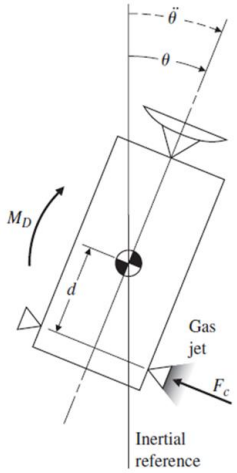
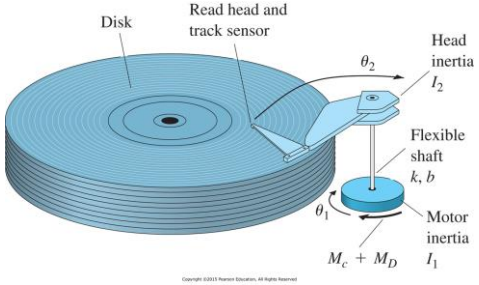
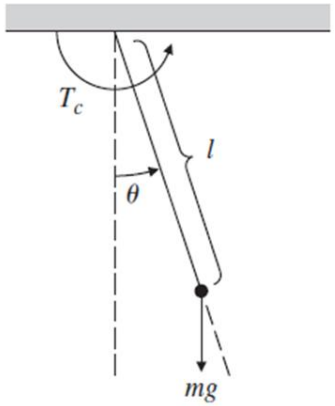
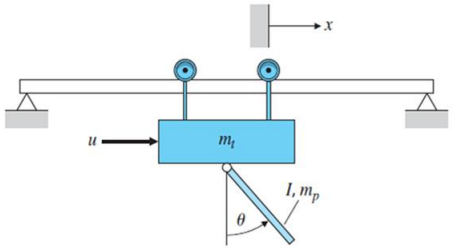
Summary of U2 :Mechanical Systems

System	Differential Equation	Transfer Function
E.g.2.1: Cruise control 	$\dot{v} + \frac{b}{m} v = \frac{u}{m}$	$\frac{V(s)}{U(s)} = \frac{\frac{1}{m}}{s + \frac{b}{m}}$
E.g. 2.2: Two-Mass System 	$\ddot{x} + \frac{b}{m_1}(\dot{x} - \dot{y}) + \frac{k_s}{m_1}(x - y) + \frac{k_w}{m_1}(x) = \frac{k_w}{m_1}(r)$ $\ddot{y} + \frac{b}{m_2}(\dot{y} - \dot{x}) + \frac{k_s}{m_2}(y - x) = 0$	$\frac{Y(s)}{R(s)}$, $Y(s) = \frac{k_w}{m_1}\frac{b}{m_2}(s + \frac{k_s}{b})$ $R(s) = s^4 + (\frac{b}{m_1} + \frac{b}{m_2})s^3 + (\frac{k_s}{m_1} + \frac{k_s}{m_2} + \frac{k_w}{m_1})s^2 + \frac{k_w b}{m_1 m_2}s + \frac{k_w k_s}{m_1 m_2}$

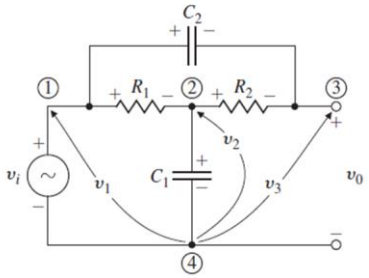
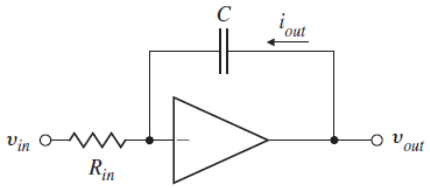
Summary of Unit2: Mechanical Systems

System	Differential Equation	Transfer Function
E.g.2.3: Satellite control 	$F_c \cdot d + M_D = I \cdot \ddot{\theta}$	$\frac{\Theta(s)}{U(s)} = \frac{1}{I} \cdot \frac{1}{s^2}$ $(F_c \cdot d + M_D = u)$
E.g. 2.4: Disk read/write head 	$I_1\ddot{\theta}_1 + b(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) = M_c + M_D$ $I_2\ddot{\theta}_2 + b(\dot{\theta}_2 - \dot{\theta}_1) + k(\theta_2 - \theta_1) = 0$	$\frac{\Theta_2(s)}{M_c(s)} = \frac{k}{I_1 I_2 s^2(s^2 + \frac{k}{I_1} + \frac{k}{I_2})}$ $\frac{\Theta_1(s)}{M_c(s)} = \frac{I_2 s^2 + k}{I_1 I_2 s^2(s^2 + \frac{k}{I_1} + \frac{k}{I_2})}$

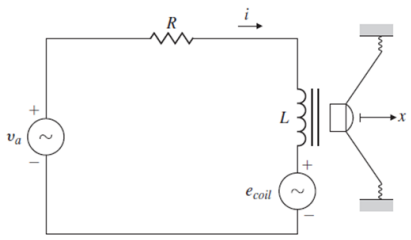
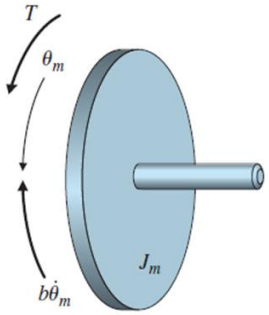
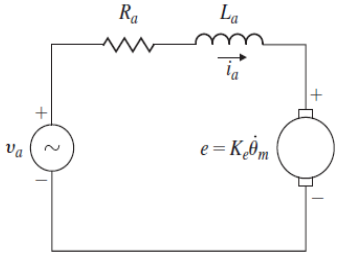
Summary of Unit2: Mechanical Systems

System	Differential Equation	Transfer Function
E.g. 2.5: Pendulum 	$\ddot{\theta} + \frac{g}{l}\theta = \frac{T_c}{ml^2}$	$\frac{\Theta(s)}{T_c(s)} = \frac{\frac{1}{ml^2}}{s^2 + \frac{g}{l}}$
Example 2.7: Hanging Crane 	$(I + m_p l^2)\ddot{\theta} + m_p g l\theta = -m_p l\ddot{x}$ $(m_t + m_p)\ddot{x} + b\dot{x} + m_p l\ddot{\theta} = u$	$\frac{\Theta(s)}{U(s)}$, $\Theta(s) = -m_p l$ $U(s) = s^2\left((I + m_p l^2)(m_t + m_p) - m_p^2 l^2\right) + m_p g l(m_t + m_p)$

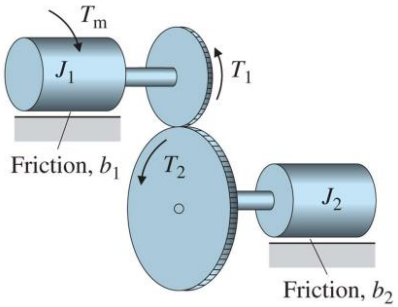
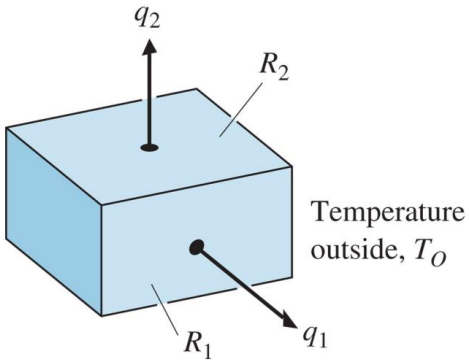
Summary of Unit2: Electric Circuits

System	Differential Equation	Transfer Function
Example 2.8: the Bridged Tee Circuit 	$-\frac{v_1 - v_2}{R_1} + \frac{v_2 - v_3}{R_2} + C_1\frac{dv_2}{dt} = 0$ $\frac{v_3 - v_2}{R_2} + C_2\frac{d(v_3 - v_1)}{dt} = 0$	$\frac{V_3(s)}{V_1(s)} =$
Example 2.11: Integrator 	$v_{out} = -\frac{1}{R_{in}C}\int_0^t v_{in}(\tau)d\tau$ $+v_{out}(0)$	$V_{out}(s) = -\frac{1}{s}\frac{V_{in}(s)}{R_{in}C}$

Summary of Unit2: Electromechanical Systems

System	Differential Equation	Transfer Function
Example 2.13: Loudspeaker 	$L\frac{di}{dt} + Ri = v_a - e_{coil}$ $= v_i$	$\frac{I(s)}{V_i(s)} =$
Example 2.14: Motors   (a)	$J_m\ddot{\theta}_m + b\dot{\theta}_m = T = K_t \cdot i_a$ $L_a\frac{di_a}{dt} + R_a i_a = v_a - K_e \cdot \dot{\theta}_m$	$\frac{\Theta_m(s)}{V_a(s)} =$ $\frac{K_t}{s\left[(J_m s + b)(L_a s + R_a) + K_t K_e\right]}$

Summary of Unit 2:

System	Differential Equation	Transfer Function
Example: Gears 	$J_1\ddot{\theta}_1 + b_1\dot{\theta}_1 = T_m - T_1$ $J_2\ddot{\theta}_2 + b_2\dot{\theta}_2 = T_2$	$\frac{\Theta_2(s)}{T_m(s)} = \frac{n}{J_{eq}s^2 + b_{eq}s}$
Example: Heat flow 	$\dot{T}_I = \frac{1}{C_I}(\frac{1}{R_1} + \frac{1}{R_2})(T_O - T_I)$	$\frac{T_I(s)}{T_o(s)} =$

Summary of Unit 2: Key equations

Key Equations for Dynamic Models

System	Important Laws or Relationships	Associated Equations	Equation Number(s)
Mechanical	Translational motion (Newton's law)	$F = m\mathbf{a}$	(2.1)
	Rotational motion	$M = I\alpha$	(2.14)
Electrical	Operational amplifier		(2.46), (2.47)
Electromechanical	Law of motors	$F = Bli$	(2.53)
	Law of generators	$e = Blv$	(2.56)
	Torque developed in a rotor	$T = K_t i_a$	(2.60)
Back emf	Voltage generated as a result of rotation of a rotor	$e = K_e\dot{\theta}_m$	(2.61)
Gears	Effective inertia	$J_{eq} = J_2 + J_1 n^2$	(2.80)
Heat flow	Heat-energy flow	$q = {}^1/_R(T_1 - T_2)$	(2.81)
	Temperature as a function of heat-energy flow	$\dot{T} = \frac{1}{C}q$	(2.82)
	Specific heat	$C = mc_v$	(2.83)
Fluid flow	Continuity relation (conservation of matter)	$\dot{m} = w_{in} - w_{out}$	(2.88)
	Force of a fluid acting on a piston	$f = pA$	(2.90)
	Effect of resistance to fluid flow	$w = {}^1/_R(p_1 - p_2)^{1/\alpha}$	(2.91)