Spring 2020

控制系統
Control Systems

Unit 26
Electromechanical Systems – Loudspeakers, Motors, Gears

Feng-Li Lian & Ming-Li Chiang
NTU-EE

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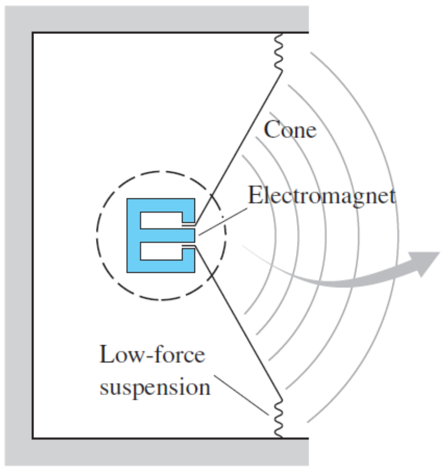
- **Law of motors**
  - A current of i amp in a conductor of l m, arranged at right angles in a magnetic field of B teslas.
  - There is a force F on the conductor at right angles to the plane of i and B, with magnitude

$$F = BliN$$



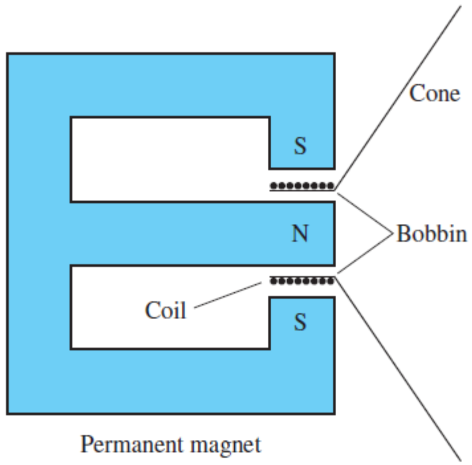
The loudspeaker configuration

- **Model (Motion of loudspeaker cone)**

$$M\ddot{x} + b\dot{x} = F$$

x : the motion of the loudspeaker cone

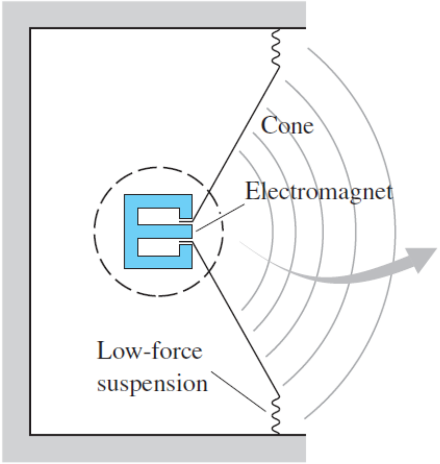
b : the friction coefficient



The electromagnet and voice coil

- Law of generators
  - A conductor of length l m is moving in a magnetic field of B teslas at a velocity of v m/sec at mutually right angles
  - an electric voltage is established across the conductor with magnitude

$$e = BlvV$$



(a)

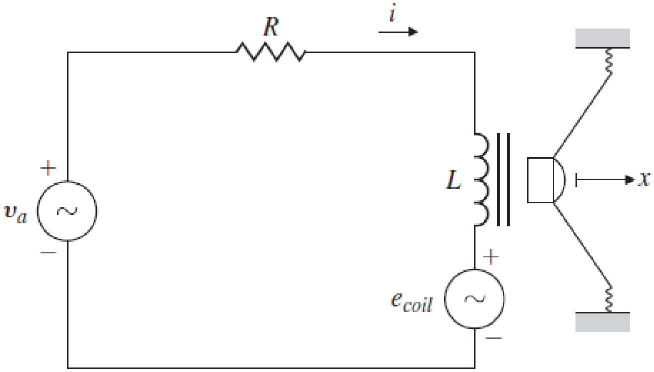
The loudspeaker configuration

- Model (Motion of electric circuit)

$$L\frac{di}{dt} + Ri = v_a - e_{coil}$$

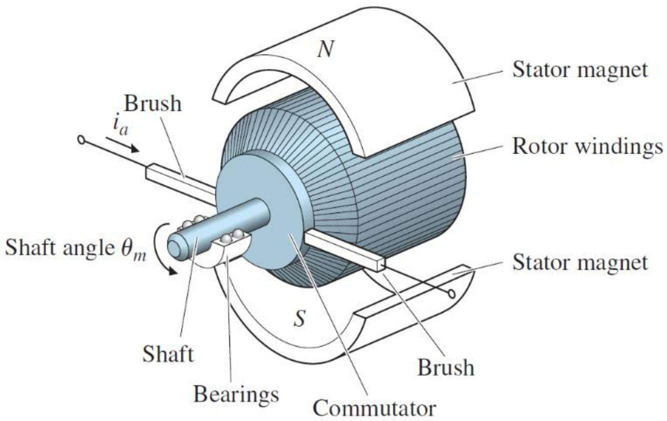
L : the inductance

R : the effective circuit resistance



The electric circuit of the loudspeaker

- The direct current (DC) motor



Motor equations

- Motor equations give the torque T on the rotor in terms of the armature current i_a
- Express the back emf voltage in terms of the shaft's rotational velocity $\dot{\theta}_m$

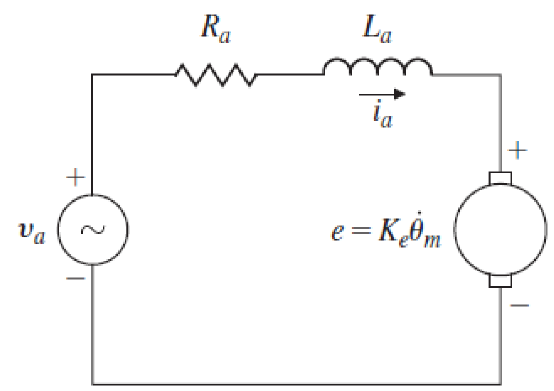
Model (Equations of Motion)

$$T = K_t \cdot i_a$$

$$e = K_e \cdot \dot{\theta}_m$$

K_t : the torque constant

K_e : the electric constant



The electric circuit of the armature

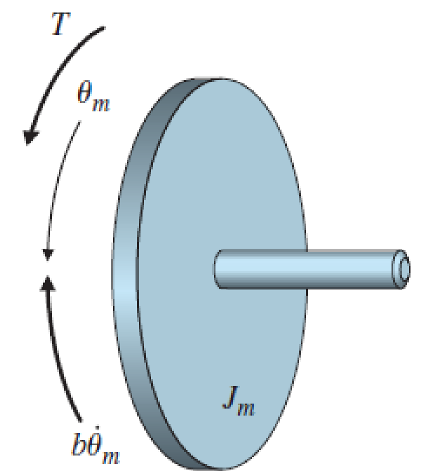
Example 2.14 Motors

- Model (DC motors)

$$J_m \ddot{\theta}_m + b\dot{\theta}_m = T = K_t \cdot i_a$$

J_m : the inertia of rotor

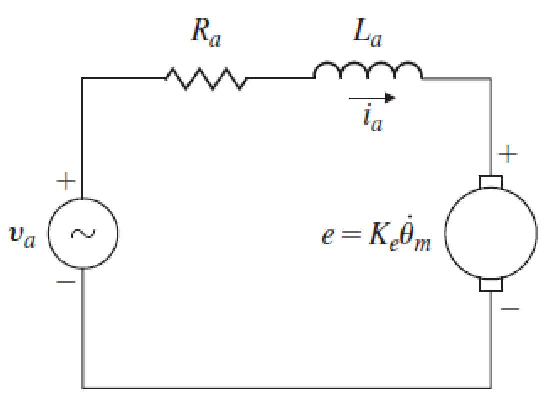
b : the viscous friction coefficient



Free-body diagram of the rotor

- Model (Electric circuit)

$$L_a \frac{di_a}{dt} + R_a i_a = v_a - K_e \cdot \dot{\theta}_m$$



The electric circuit of the armature

- Transfer function (Electric circuit)

$$\frac{\Theta_m(s)}{V_a(s)} = \frac{K_t}{s\left[(J_m s + b)(L_a s + R_a) + K_t K_e\right]}$$

Example 2.14 Motors

- **Model (DC motors)**
  - Sometimes, the effect of inductance is negligible

$$L_a \frac{di_a}{dt} + R_a i_a = v_a - K_e \cdot \dot{\theta}_m \quad \left(L_a \frac{di_a}{dt} \to 0\right)$$

  - In this case, we have: by $\left(J_m \ddot{\theta}_m + b\dot{\theta}_m = K_t \cdot i_a \right)$

$$J_m \ddot{\theta}_m + (b + \frac{K_t K_e}{R_a})\dot{\theta}_m = \frac{K_t}{R_a} v_a$$

$$\frac{\Omega(s)}{V_a(s)} = s\frac{\Theta_m(s)}{V_a(s)} = \frac{K}{\tau s + 1}.$$

- **Transfer function (Electric circuit)**

$$\frac{\Theta_m(s)}{V_a(s)} = \frac{\frac{K_t}{R_a}}{J_m s^2 + s\left(b + \frac{K_t K_e}{R_a}\right)} = \frac{K}{s(\tau s + 1)}$$

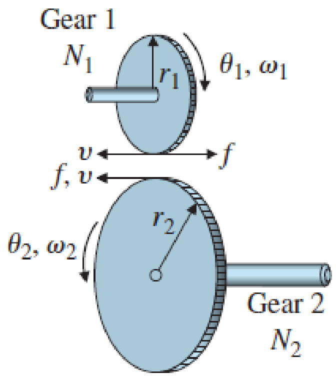
$$K := \frac{K_t}{bR_a + K_t K_e},$$

$$\tau := \frac{R_a J_m}{bR_a + K_t K_e}.$$

- **Transfer function (input v_a to output $\dot{\theta}_m := \omega$)**

$$\frac{\Omega(s)}{V_a(s)} = s\frac{\Theta_m(s)}{V_a(s)} = \frac{K}{(\tau s + 1)}$$

- Forces of gears



(a)

$$\frac{T_1}{r_1} = \frac{T_2}{r_2} = f \quad \text{(force applied to teeth)}$$

$$\frac{T_2}{T_1} = \frac{r_2}{r_1} = \frac{N_2}{N_1} := n \quad (N_i : \text{number of gears})$$

$$\omega_1 r_1 = \omega_2 r_2 = v$$

$$\frac{\omega_1}{\omega_2} = \frac{r_1}{r_2} = \frac{N_1}{N_2} = n$$

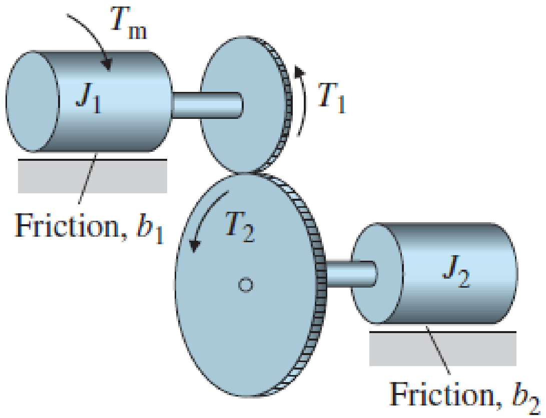
$$\frac{\theta_1}{\theta_2} = \frac{\omega_1}{\omega_2} = n$$

Model (Equations of Motion)

$$J_1\ddot{\theta}_1 + b_1\dot{\theta}_1 = T_m - T_1$$

$(T_m :$ output torque of the servo motor$)$

$$J_2\ddot{\theta}_2 + b_2\dot{\theta}_2 = T_2$$



(b)

Example: Gears

▪ Model (Equations of Motion)

$$\frac{T_2}{T_1} = \frac{r_2}{r_1} = \frac{N_2}{N_1} := n$$

$$\frac{\theta_1}{\theta_2} = \frac{\omega_1}{\omega_2} = n$$

- Substitute θ_2 for θ_1 and replace T_2 with T_1

$$(J_2 + J_1 n^2)\ddot{\theta}_2 + (b_2 + b_1 n^2)\dot{\theta}_2 = nT_m$$

▪ Transfer function (Electric circuit)

$$\frac{\Theta_2(s)}{T_m(s)} = \frac{n}{J_{eq}s^2 + b_{eq}s}$$

$$\begin{pmatrix} J_{eq} & = J_2 + J_1 n^2 \\ b_{eq} & = b_2 + B_1 n^2 \end{pmatrix}$$