Spring 2020

控制系統
Control Systems

Unit 22
Mechanical Systems – Rotational Motion

Feng-Li Lian & Ming-Li Chiang
NTU-EE

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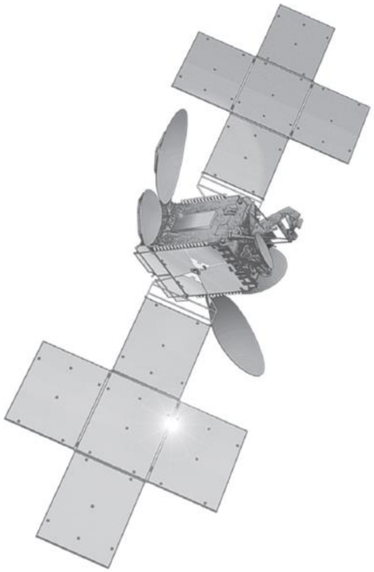
Example 2.3 (Rotational motion): Satellite Attitude Control Model

- Communication satellite



Source: Courtesy Thaicom PLC and Space Systems/Loral

- The purpose is to control the attitude of the satellite, such as
  - ✓ Antennas point toward earth
  - ✓ Solar panels orient toward the sun

▪ Model (Equations of Motion: Rotational motion)

$$M = I\,\alpha$$

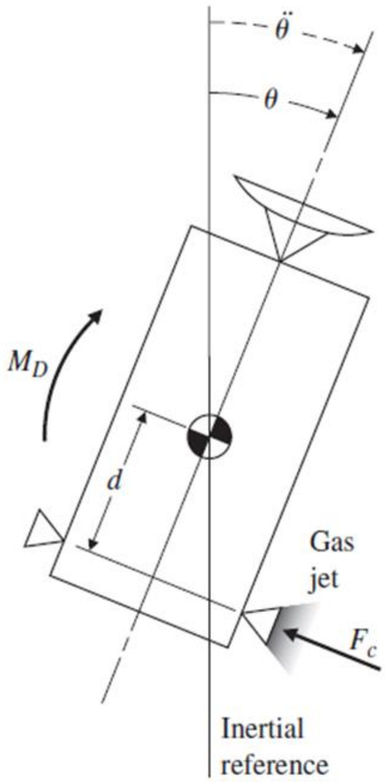
- M $(N \cdot m^2)$: the sum of all external moments about the center of mass,
- I $(Kg \cdot m^2)$: the body's mass moment of inertia about its center of mass,
- α (rad/sec^2): the angular acceleration of the body

Model (Equations of Motion)

- Three axes, consider one axis at a time

$$F_c \cdot d + M_D = I \cdot \ddot{\theta}$$

- $F_c \cdot d$: Moments of control force
- M_D : Moments of small disturbance



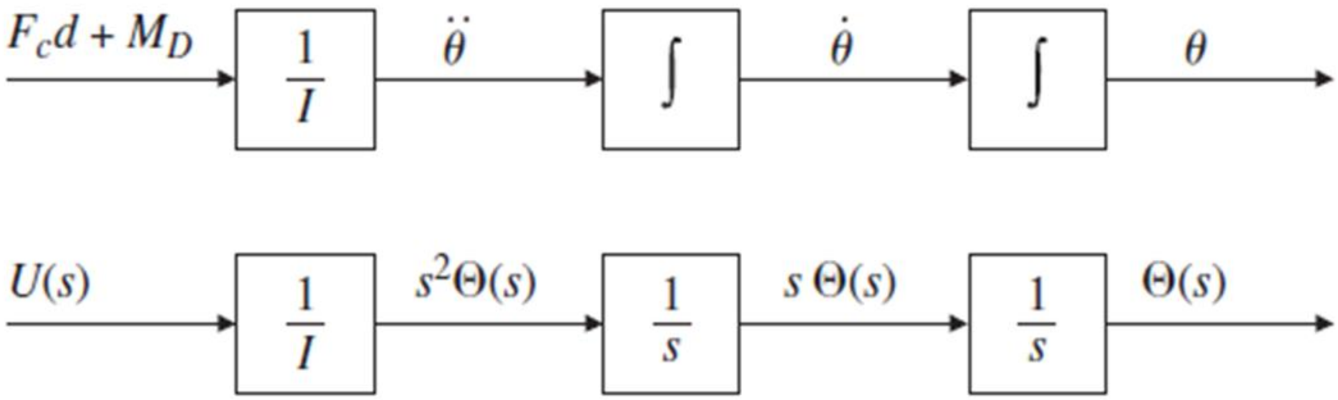
Transfer Function

- Let $F_c \cdot d + M_D = u$

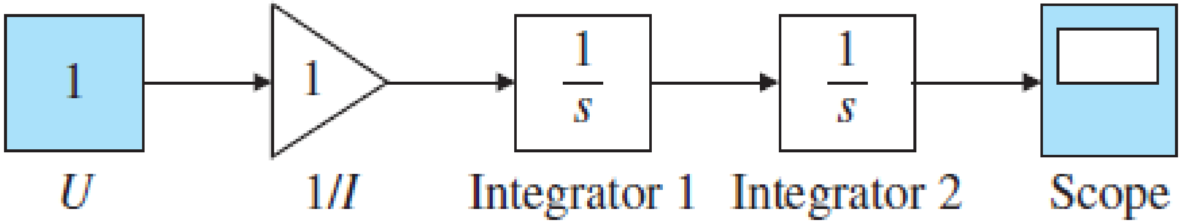
$$\frac{\Theta(s)}{U(s)} = \frac{1}{I} \cdot \frac{1}{s^2}$$ (Double-Integrator plant)

Example 2.3 (Rotational motion): Satellite Attitude Control Model

- Block diagram $$\frac{\Theta(s)}{U(s)} = \frac{1}{I} \cdot \frac{1}{s^2}$$ (Double-Integrator plant)

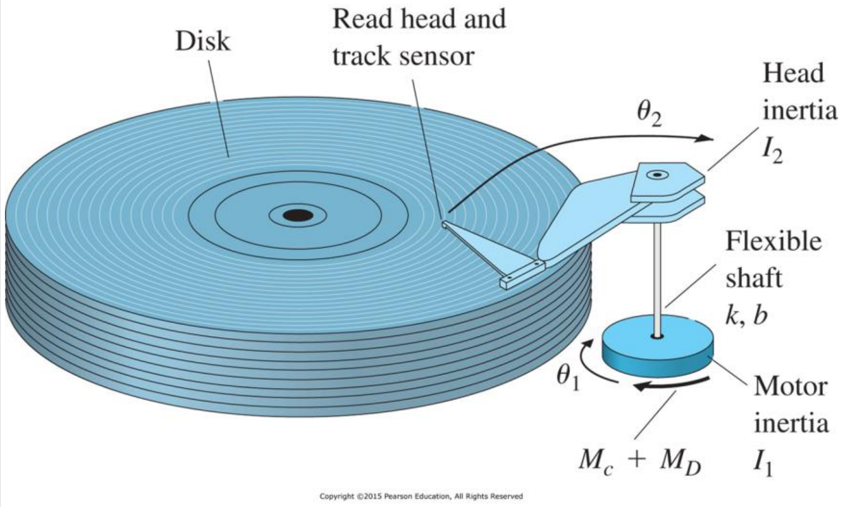


- Simulink

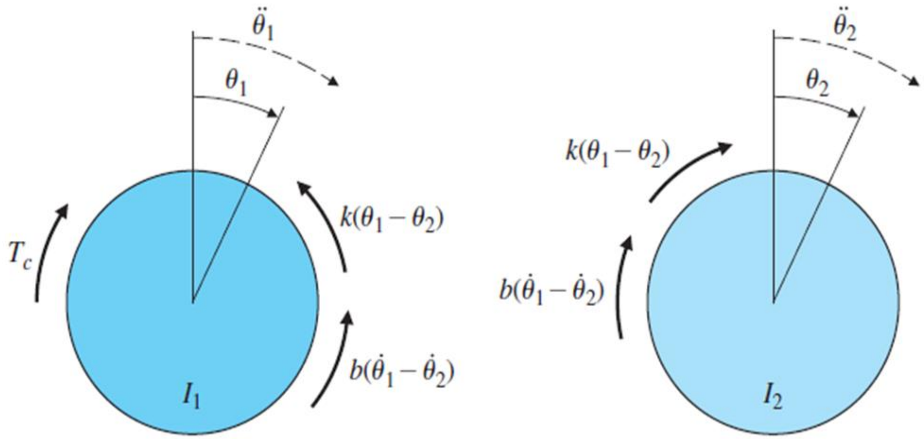


Example 2.4 Flexible Read/Write for a Disk Drive

- Disk read/write head



- The moment of each body: free body diagram



- Model (Equations of Motion: Rotational motion)

$$I_1\ddot{\theta}_1 + b(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) = M_c + M_D$$

$$I_2\ddot{\theta}_2 + b(\dot{\theta}_2 - \dot{\theta}_1) + k(\theta_2 - \theta_1) = 0$$

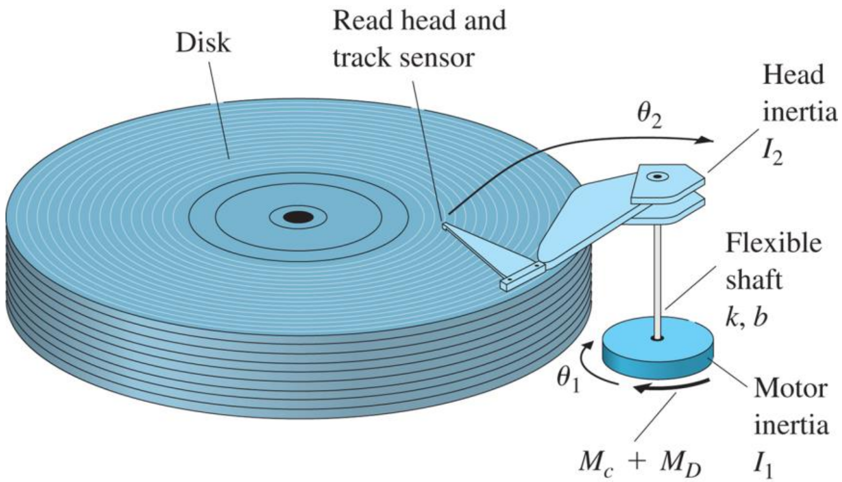
- M_c : Moments of applied control
- M_D : Moments of small disturbance

Model (Equations of Motion)

- Simplify the model, consider the case $M_D = 0, b = 0$

$$I_1\ddot{\theta}_1 + k(\theta_1 - \theta_2) = M_c$$

$$I_2\ddot{\theta}_2 + k(\theta_2 - \theta_1) = 0$$



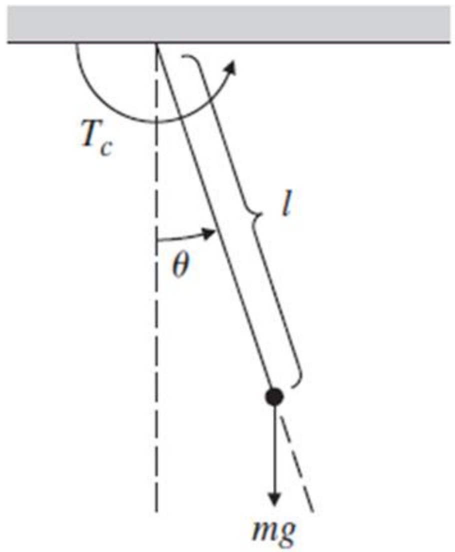
Transfer Function

$$\frac{\Theta_2(s)}{M_c(s)} = \frac{k}{I_1 I_2 s^2 (s^2 + \frac{k}{I_1} + \frac{k}{I_2})}$$

$$\frac{\Theta_1(s)}{M_c(s)} = \frac{I_2 s^2 + k}{I_1 I_2 s^2 (s^2 + \frac{k}{I_1} + \frac{k}{I_2})}$$

- **"Noncollocated case":** there is **flexibility** between the sensor and the actuator
- **"Collocated case":** the sensor and the actuator are **rigidly attached** to one another

- Pendulum



Model (Equations of Motion)

$$T_c - mgl\sin\theta = I\ddot{\theta}$$

- The moments of inertia about the pivot point is $I = ml^2$

$$\ddot{\theta} + \frac{g}{l}\sin\theta = \frac{T_c}{ml^2}$$

- The model is nonlinear due to $\sin\theta$
- When the motion is small, i.e., θ small, $\sin\theta \approx \theta$

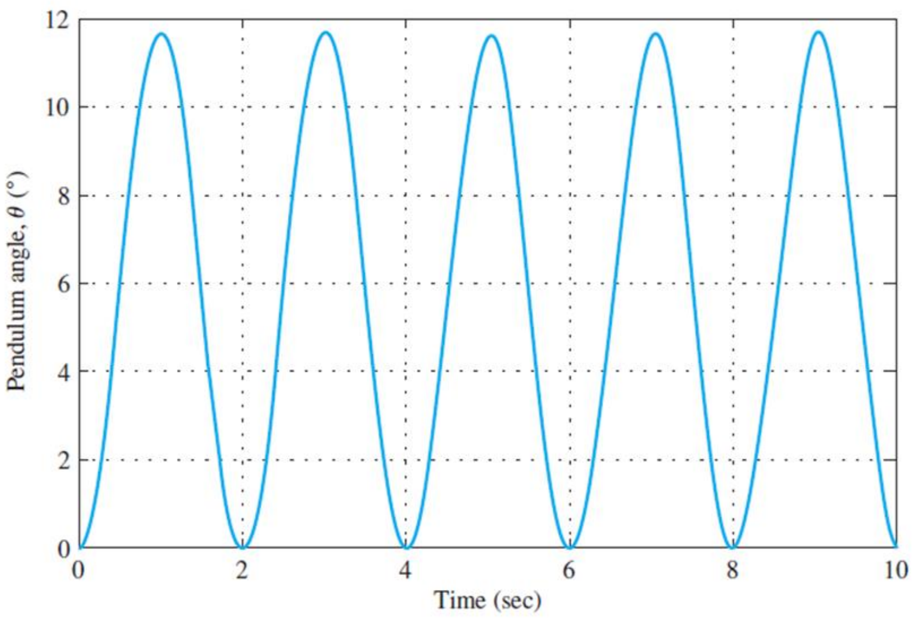
$$\ddot{\theta} + \frac{g}{l}\theta = \frac{T_c}{ml^2}$$

(Linearization model)

Example 2.5 Pendulum

- Transfer Function

$$\frac{\Theta(s)}{T_c(s)} = \frac{\frac{1}{ml^2}}{s^2 + \frac{g}{l}}$$



- Matlab code
  - t=0:0.02:10;
  - m=1; L=1; g=9.81;
  - s = tf(‘s’);
  - sys = (1/(m*L^2))/(s^2+g/L) ;
  - Y=step(sys,t);

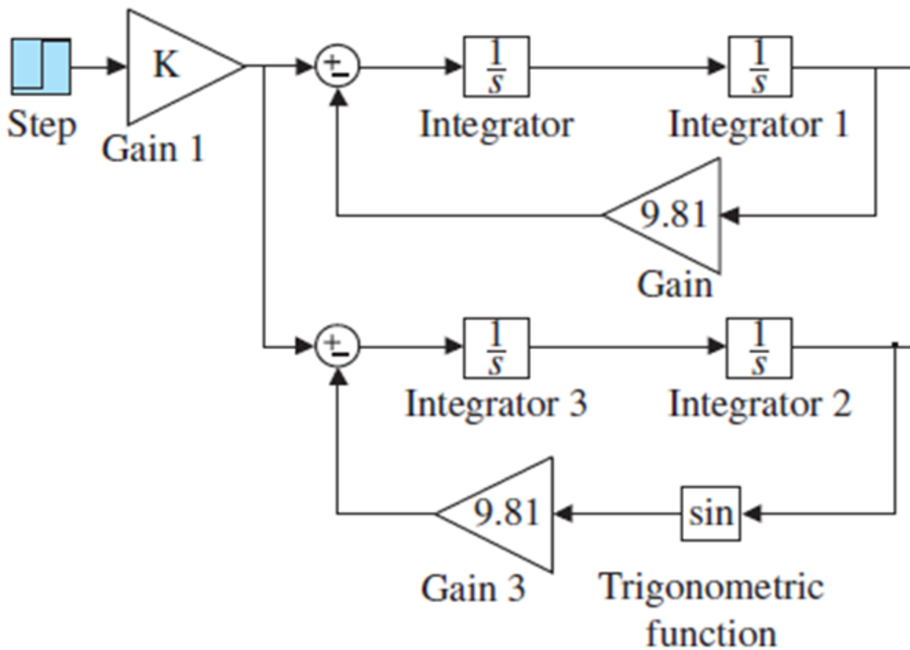
  - Rad2Deg=57.3;
  - Plot(t,Rad2Deg*y) %converts output from radians to degrees

Example 2.6 Pendulum (Simulink for nonlinear motion)

- Matlab Simulink (m=1; L=1; g=9.81)

$$\ddot{\theta} + \frac{g}{l}\theta = \frac{T_c}{ml^2}$$

Linear model

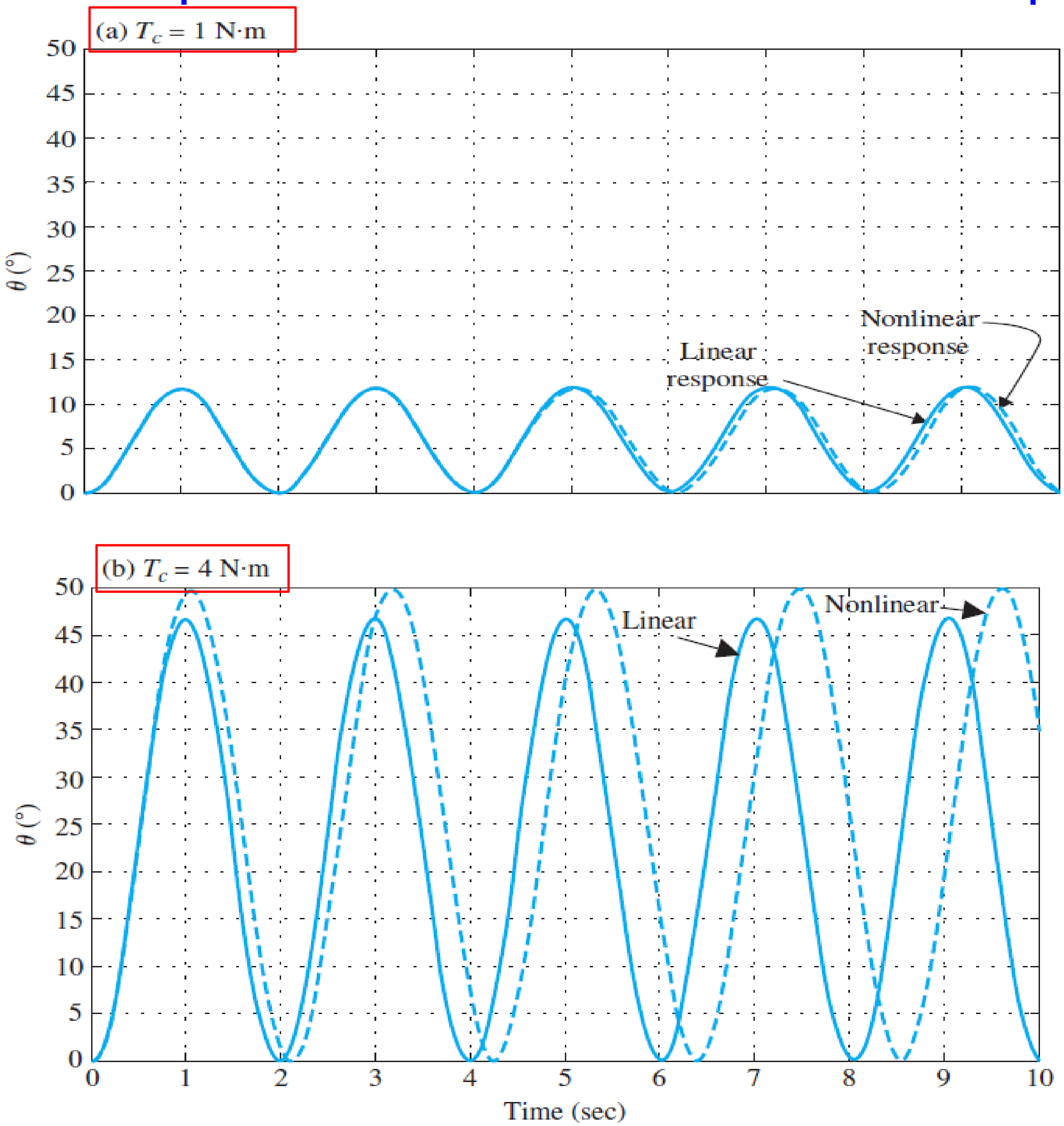


Nonlinear model

$$\ddot{\theta} + \frac{g}{l}\sin\theta = \frac{T_c}{ml^2}$$

Example 2.6 Pendulum (Simulink for nonlinear motion)

▪ Comparisons of linear & nonlinear responses



$$\ddot{\theta} + \frac{g}{l}\sin\theta = \frac{T_c}{ml^2}$$

$$\ddot{\theta} + \frac{g}{l}\theta = \frac{T_c}{ml^2}$$

- When T_c=1, the output θ remains small, thus the approximation is still good ($\sin\theta \approx \theta$)
- When T_c=4, the output θ becomes large, thus the approximation is not good ($\sin\theta \approx \theta$ does not hold)