Spring 2020

控制系統
Control Systems

Unit 21
Mechanical Systems – Translational Motion

Feng-Li Lian & Ming-Li Chiang

NTU-EE

Mar 2020 – Jul 2020

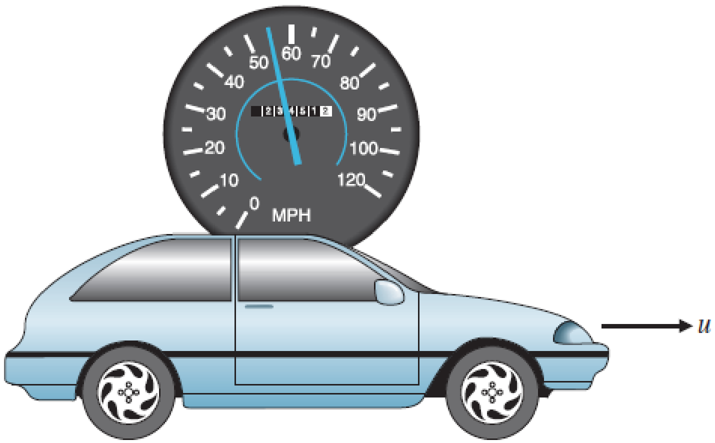
- The cornerstone for obtaining a mathematical model, or the dynamic equations, for any mechanical system is Newton's law,

$$F = m\,a$$

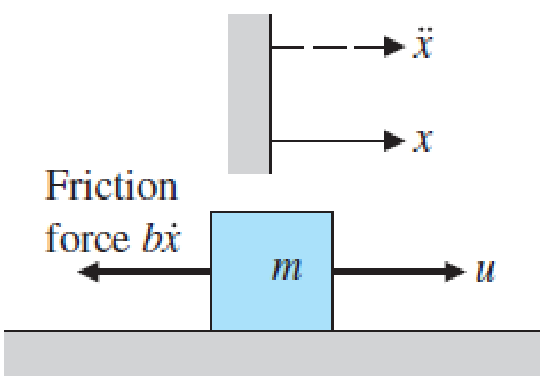
- where
  - F (newton, N)
    the vector sum of all forces applied to each body in a system,
  - a (m/sec^2)
    the vector acceleration of each body with respect to an **inertial reference frame** (that is, one that is neither accelerating nor rotating with respect to the stars); often called **inertial acceleration**.
  - m (kg)
    mass of the body.

Example 2.1: A Simple System: Cruise Control Model

- Cruise control model



- Free-body diagram for cruise control



Model (Equations of Motion)

$$F = m\,a$$

$$u - b\,\dot{x} = m\,\ddot{x}$$

$$\ddot{x} + \frac{b}{m}\dot{x} = \frac{u}{m}$$

$$\dot{v} + \frac{b}{m}v = \frac{u}{m}$$

Solution

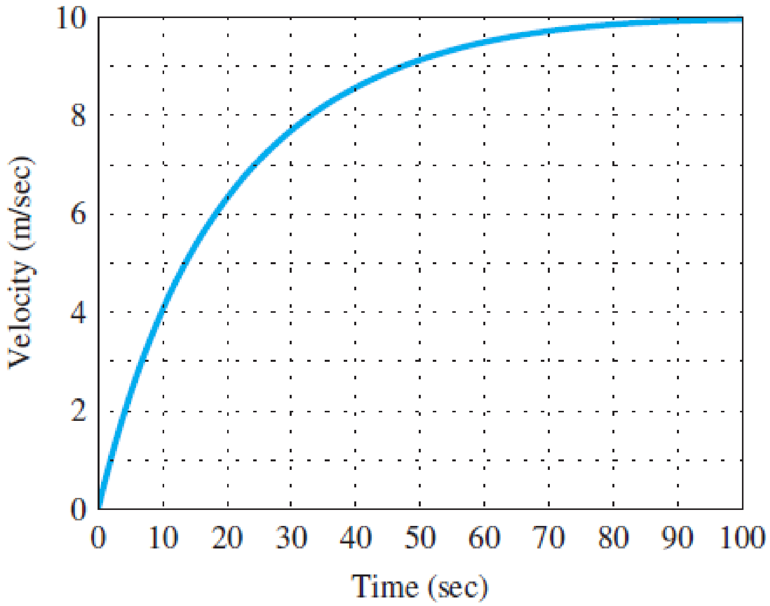
$$v(t) = V_0\, e^{st} \qquad u(t) = U_0\, e^{st}$$

Transfer Function

$$\frac{V(s)}{U(s)} = \frac{\frac{1}{m}}{s + \frac{b}{m}}$$

Example 2.1: A Simple System: Cruise Control Model

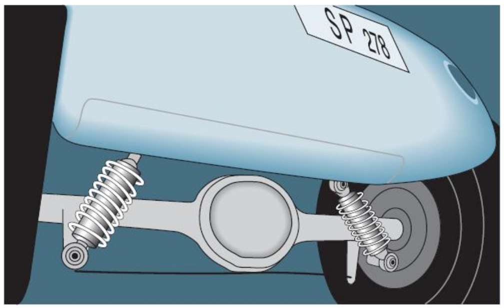
- Time Response
  - m = 1000 (kg),
  - b = 50 m sec/N,
  - u = 500 N



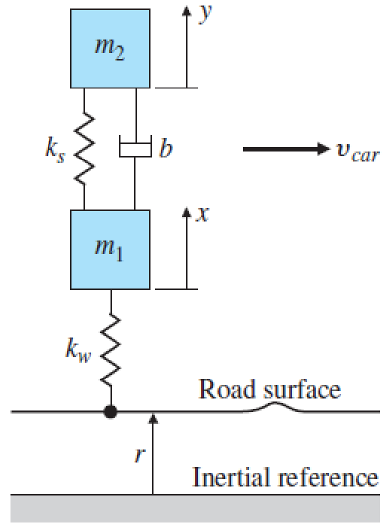
- Matlab code
  - s = tf(‘s’);
  - sys = (1/10000) / (s + 50/1000);
  - step(500*sys);

Example 2.2: A Two-Mass System: Suspension Model

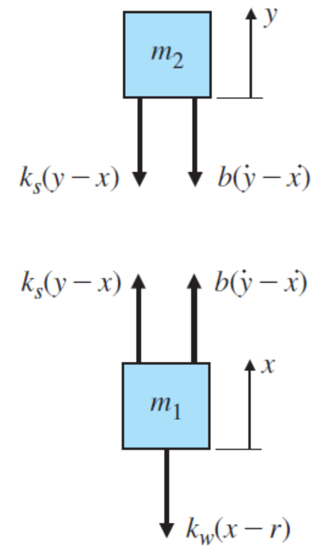
- Automobile suspension



- Quarter-car model



- Free-body diagrams for suspension system



- Model (Equations of Motion)

$$-b(\dot{y}-\dot{x}) - k_s(y-x) = m_2\,\ddot{y}$$

$$b(\dot{y}-\dot{x}) + k_s(y-x) - k_w(x-r) = m_1\,\ddot{x}$$

$$\ddot{x} + \frac{b}{m_1}(\dot{x}-\dot{y}) + \frac{k_s}{m_1}(x-y) + \frac{k_w}{m_1}(x) = \frac{k_w}{m_1}(r)$$

- Model (Equations of Motion)

$$\ddot{x} + \frac{b}{m_1}(\dot{x} - \dot{y}) + \frac{k_s}{m_1}(x - y) + \frac{k_w}{m_1}(x) = \frac{k_w}{m_1}(r)$$

$$\ddot{y} + \frac{b}{m_2}(\dot{y} - \dot{x}) + \frac{k_s}{m_2}(y - x) = 0$$

$$s^2 X(s) + s\frac{b}{m_1}(X(s) - Y(s)) + \frac{k_s}{m_1}(X(s) - Y(s)) + \frac{k_w}{m_1}X(s) = \frac{k_w}{m_1}R(s)$$

$$s^2 Y(s) + s\frac{b}{m_2}(Y(s) - X(s)) + \frac{k_s}{m_2}(Y(s) - X(s)) = 0$$

Example 2.2: A Two-Mass System: Suspension Model

- Transfer Function

$$\frac{Y(s)}{R(s)} = \frac{\frac{k_w}{m_1}\frac{b}{m_2}(s + \frac{k_s}{b})}{s^4 + (\frac{b}{m_1} + \frac{b}{m_2})s^3 + (\frac{k_s}{m_1} + \frac{k_s}{m_2} + \frac{k_w}{m_1})s^2 + \frac{k_w b}{m_1 m_2}s + \frac{k_w k_s}{m_1 m}}$$

- Parameters
  - m = 1580 kg, m1 = 20 kg, m2 = 375 kg
  - ks = 130,000 N/m, kw = 1,000,000 N/m
  - b = 9800 N sec/N

$$\frac{Y(s)}{R(s)} = \frac{1.31e06(s + 13.3)}{s^4 + (516.1)s^3 + (5.69e04)s^2 + (1.31e06)s + 1.73e07}$$