

Spring 2020

控制系統  
Control Systems

Unit 14

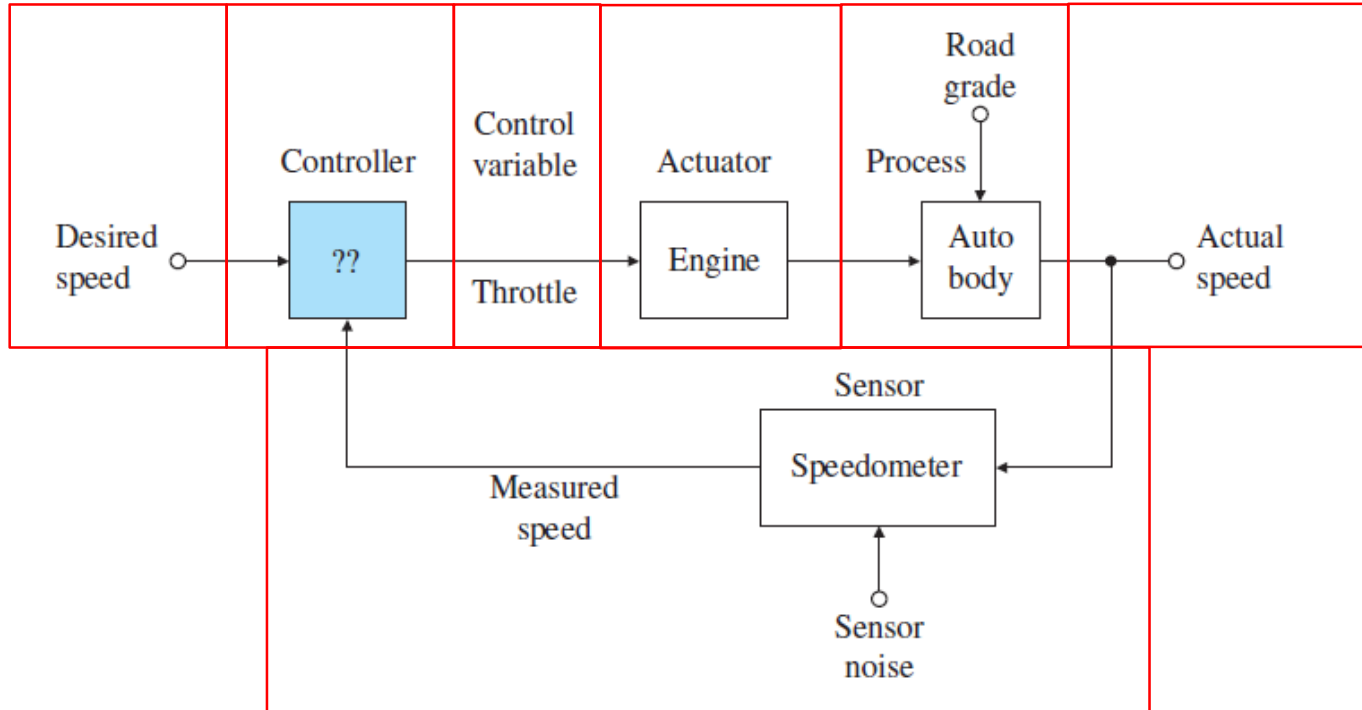
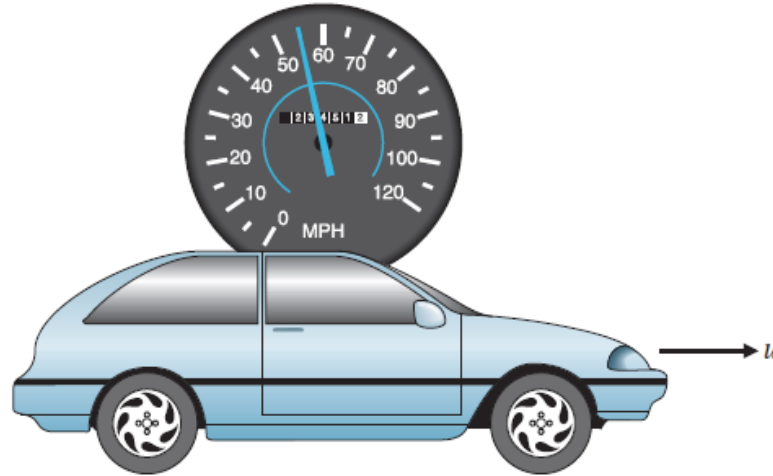
Motivating Example of Feedback and Control

Feng-Li Lian & Ming-Li Chiang

NTU-EE

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# Motivating Example: Cruise Control of An Automobile

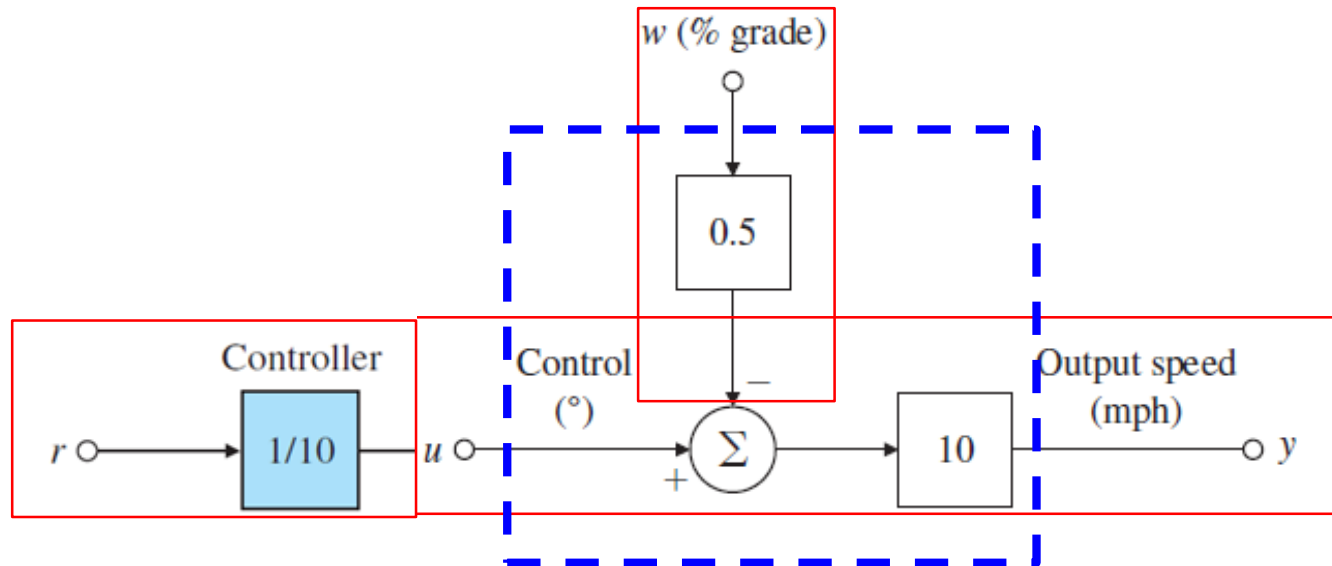


- On level road, at 65 mph,  
throttle angle = +1 (deg) ---> speed = +10 mph
- Driving up or down hills,  
grade = +1% ---> speed = -5 mph
- Mathematical Model & Block Diagram** (Signal & System):

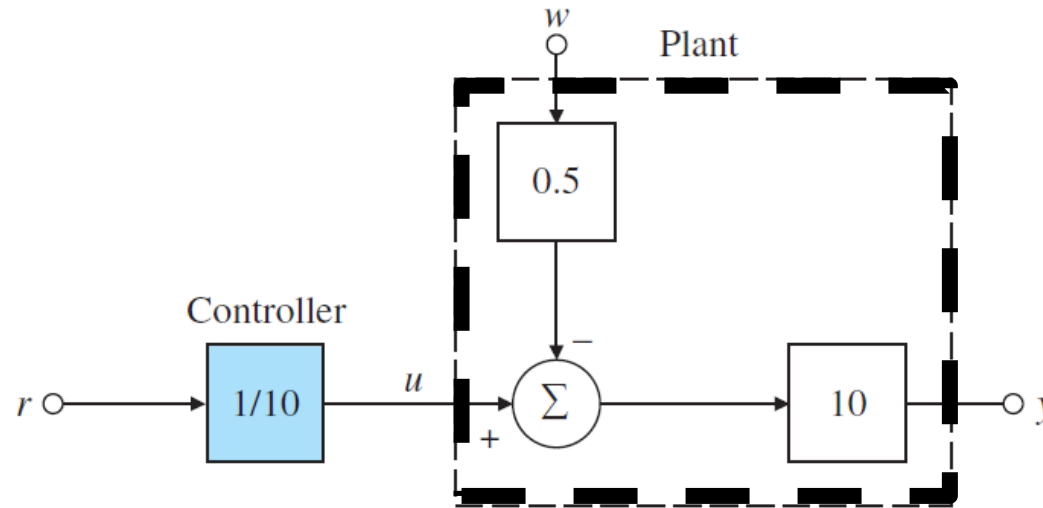
$$y = 10 u$$

$$u = \frac{1}{10} r$$

$$y = r$$



## Open-Loop



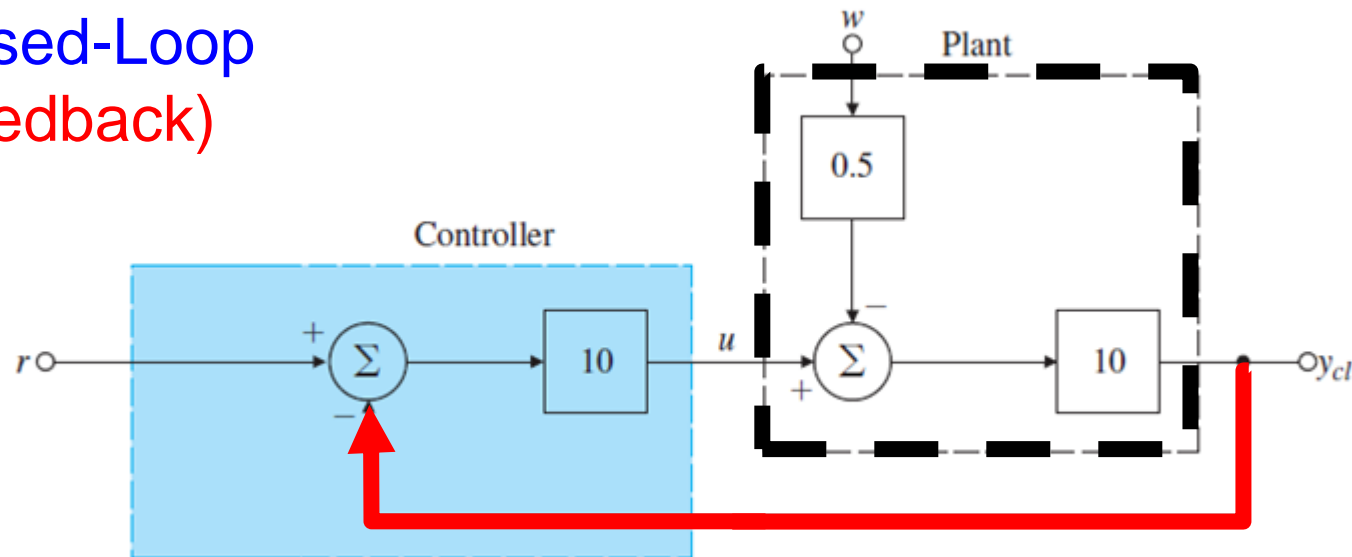
$$\begin{aligned}
 y_{ol} &= 10 ( u - 0.5 w ) \\
 &= 10 \left( \frac{1}{10} r - 0.5 w \right) \\
 &= r - 5 w
 \end{aligned}$$

$$\begin{aligned}
 e_{ol} &= r - y_{ol} \\
 &= 5 w
 \end{aligned}$$

$$\text{\% error} = \frac{5 w}{r} \times 100$$

<b>r = 65</b>	<b>w</b>	<b>error</b>	<b>% error</b>
	0	0	0
	1	5	7.69%
	2	10	15.38%

## ■ Closed-Loop (Feedback)



$$u = 10 ( r - y_{cl} )$$

$$y_{cl} = 10 ( u - 0.5 w )$$

$$= 100 r - 100 y_{cl} - 5 w$$

$$e_{cl} = r - y_{cl}$$

$$101 y_{cl} = 100 r - 5 w$$

$$= \frac{1}{101} r + \frac{5}{101} w$$

$$y_{cl} = \frac{100}{101} r - \frac{5}{101} w$$

$$\% \text{ error} = \frac{100}{101} + \frac{500 w}{101 r}$$

# Motivating Example: Cruise Control of An Automobile

$w = 0, \quad y_{cl} = \frac{100}{101} r$ 
 $y_{cl} = \frac{100}{101} r - \frac{5}{101} w$   
 $e_{cl} = \frac{1}{101} r$ 
**% error  $\approx 1\%$** 
 $e_{cl} = \frac{1}{101} r + \frac{5}{101} w$

$w = 1, \quad y_{cl} = \frac{100}{101} 65 - \frac{5}{101} 1$ 
**% error =**
 $e_{cl} = \frac{1}{101} 65 + \frac{5}{101} 1$ 
 $= \frac{100}{101} + \frac{500 w}{101 r}$

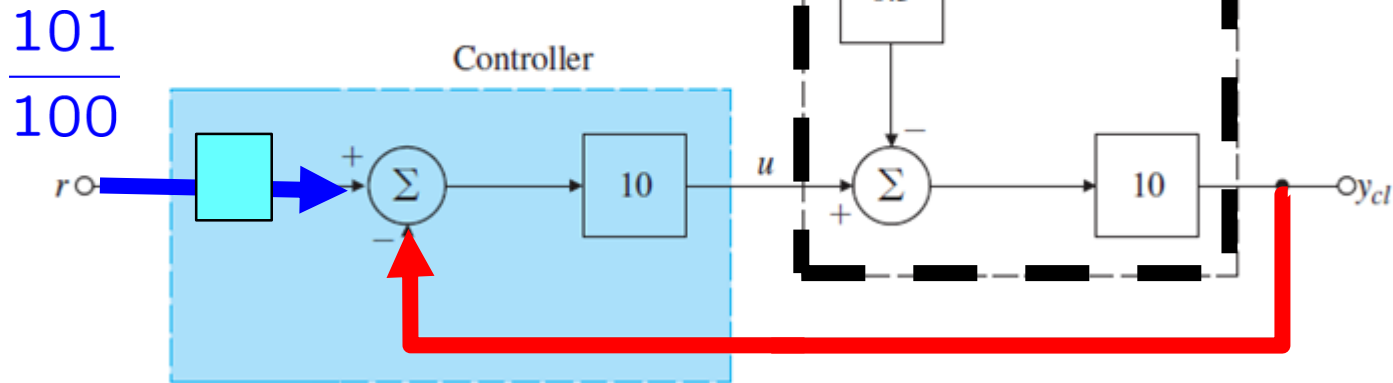
**% error**
 $= \frac{100}{101} + \frac{500 \times 1}{101 \times 65} = 0.99 + 0.0762$   
 $= 1.0662$

$w = 2, \quad y_{cl} = \frac{100}{101} 65 - \frac{5}{101} 2$   
 $e_{cl} = \frac{1}{101} 65 + \frac{5}{101} 2$

**% error**
 $= \frac{100}{101} + \frac{500 \times 2}{101 \times 65} = 0.99 + 0.1523 = 1.0423$

<b>r = 65</b>	<b>w</b>	<b>error</b>	<b>% error</b>
		<b>Open</b>	<b>Open</b>
	0	0	0
	1	5	7.69%
	2	10	15.38%

- Closed-Loop  
(Feedback + Feedforward)



$$u = 10 \left( \frac{101}{100} r - y_{cl} \right)$$

$$y_{cl} = 10 ( u - 0.5 w )$$

$$= 101 r - 100 y_{cl} - 5 w$$

$$e_{cl} = r - y_{cl}$$

$$101 y_{cl} = 101 r - 5 w$$

$$= \frac{5}{101} w$$

$$y_{cl} = r - \frac{5}{101} w$$

$$\% \text{ error} = \frac{500 w}{101 r}$$

# Motivating Example: Cruise Control of An Automobile

$$\begin{aligned} \blacksquare w = 0, \quad y_{cl} &= r & y_{cl} &= r - \frac{5}{101} w \\ e_{cl} &= 0 & \% \text{ error} &= 0\% & e_{cl} &= \frac{5}{101} w \end{aligned}$$

$$\begin{aligned} \blacksquare w = 1, \quad y_{cl} &= 65 - \frac{5}{101} 1 & \% \text{ error} &= \frac{500 w}{101 r} \\ e_{cl} &= \frac{5}{101} 1 \end{aligned}$$

$$\% \text{ error} = \frac{500 \times 1}{101 \times 65} = 0.0762$$

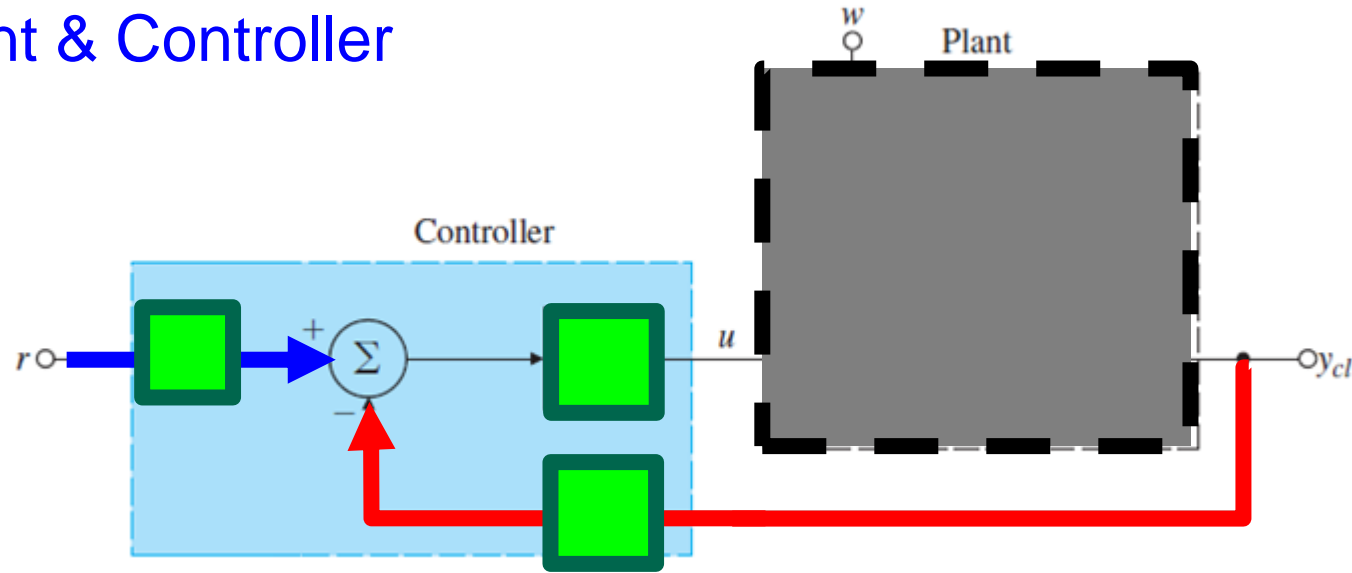
$$\begin{aligned} \blacksquare w = 2, \quad y_{cl} &= 65 - \frac{5}{101} 2 \\ e_{cl} &= \frac{5}{101} 2 \end{aligned}$$

$$\begin{aligned} \% \text{ error} &= \frac{500 \times 2}{101 \times 65} \\ &= 0.1523 \end{aligned}$$

r = 65	w	error Open	% error Open
	0	0	0
	1	5	7.69%
	2	10	15.38%

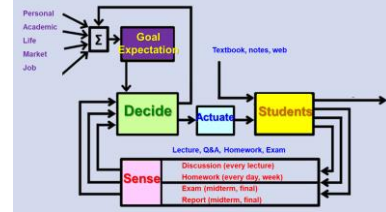


## Plant & Controller



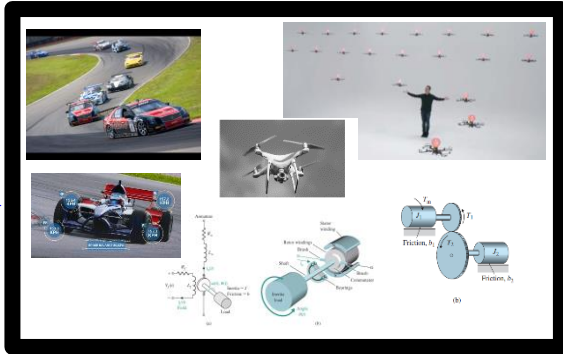
$r = 65$	error Open	% error Open	% error Closed (Feedback)	% error Feedback+Feedforward
w				
0	0	0	0.99%	0%
1	5	7.69%	1.0662%	0.0762%
2	10	15.38%	1.0423%	0.1523%

# Plant (P)



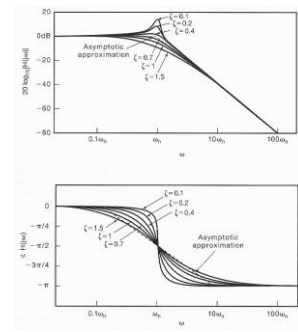
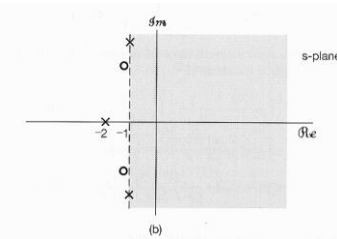
Ref (r) Input (u)

Output (y)



$$\frac{d^2y(t)}{dt^2} + 2 \frac{dy(t)}{dt} - 3y(t) = 5u(t)$$

$$P(s) = \frac{Y(s)}{U(s)} = \frac{5}{s^2 + 2s - 3}$$



**Controller**

$$\frac{d^2y(t)}{dt^2} + 4 \frac{dy(t)}{dt} + 3y(t) = 3r(t)$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{3}{s^2 + 4s + 3}$$

1. Model
2. Response
3. Analysis
4. Feedback
5. Control