Silicon Photonics
矽光子學
3 Optical Fiber Communication

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Outline

- **3.1 THE STRUCTURE OF OPTICAL FIBERS**
- **3.2 MODES OF AN OPTICAL FIBER**
- **3.3 NUMERICAL APERTURE AND ACCEPTANCE ANGLE**
- **3.4 DISPERSION IN OPTICAL FIBERS**
- **3.5 SINGLE-MODE FIBERS: MODE PROFILE, MODE-FIELD DIAMETER, AND SPOT SIZE**
- **3.6 NORMALIZED FREQUENCY, NORMALIZED PROPAGATION CONSTANT, AND CUTOFF WAVELENGTH**

3.1 THE STRUCTURE OF OPTICAL FIBERS

OPTICAL FIBERS

- **Step-Index Optical Fiber**
  - **Typically the core of a single-mode fiber will be of the order of 2-10 μm.**

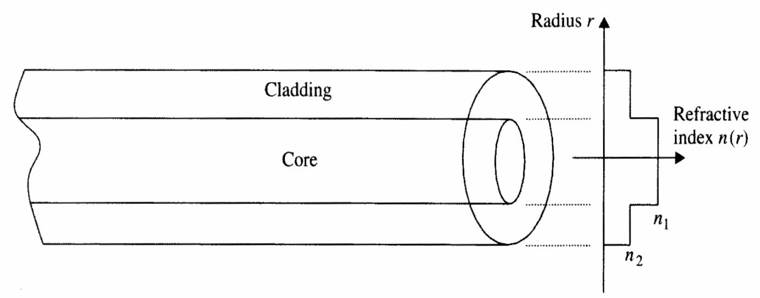


Figure 3.1 The structure of the step-index optical fiber

Graded-index Fiber

- ***Graded-Index Optical Fiber***

$$n(r) = n_1\sqrt{1-2\Delta\left(\frac{r}{a}\right)^{\alpha}} \qquad r \le a$$

$$n(r) = n_1\sqrt{1-2\Delta} \qquad r \ge a$$

$$\Delta = \frac{n_1 - n_2}{n_1}$$

  – Communications fibers fall into this category with Δ usually being less than 3 %
  – For many applications, α = 2 is the optimum (i.e. a parabolic profile)

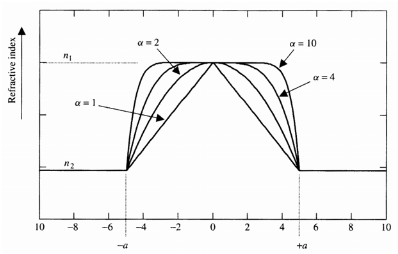


Figure 3.2 Refractive index profile of the graded-index optical fiber core

3.2 MODES OF AN OPTICAL FIBER

MODES OF AN OPTICAL FIBER

- TE_{lm} or TM_{lm} modes
  – The integer l represents the fact that there will be $2l$ field maxima around the circumference of the field distribution.
  – The integer m refers to the m field maxima along a radius.

Modes of a Step-index Fiber

- *Meridional rays*
- The propagation angles of the skew rays are such that it is possible for components of both the E and H fields to be transverse to the fiber axis.
- HE_{lm} or EH_{lm} depending on whether the E or H field dominates the transverse field.
- In ***weakly guiding fibers***, the exact modal solutions are usually approximated by ***Linearly Polarized modes***, designated LP_{lm} modes

Exact Fiber Modes

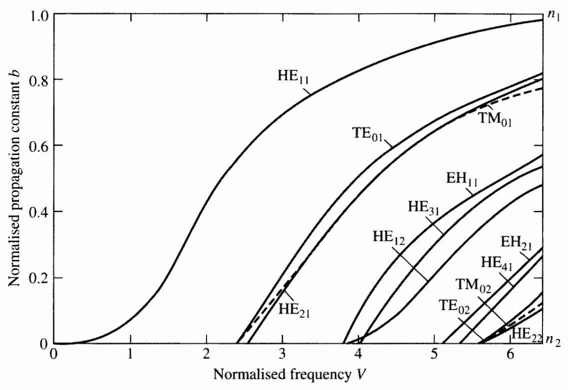


Figure 3.3 Propagation constant of the exact fiber modes plotted against normalized frequency.

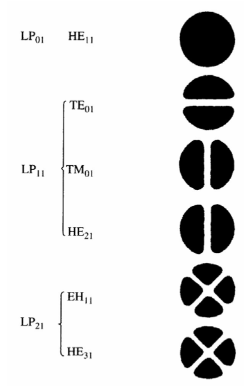


Figure 3.4 Intensity profiles of the three lowest-order LP modes.

LP modes

Table 3.1 Relationship between approximate LP modes and exact modes. Source: *Optical Fiber Communications, Principles and Practice,* 2nd edn, J M Senior, Pearson Education Limited. Reproduced by permission of Pearson Education

Linearly polarized	Exact modes
LP_{01}	HE_{11}
LP_{11}	HE_{21}, TE_{01}, TM_{01}
LP_{21}	HE_{31}, EH_{11}
LP_{02}	HE_{12}
LP_{31}	HE_{41}, EH_{21}
LP_{12}	HE_{22}, TE_{02}, TM_{02}
LP_{lm}	HE_{2m}, TE_{0m}, TM_{0m}
$LP_{lm}(l \neq 0 \text{ or } 1)$	$HE_{l+1,m}$, $EH_{l-1,m}$

Modes of a Graded-index Fiber

- ***Consider a graded-index fiber with a parabolic refractive index profile (i.e.*** $\alpha = 2$***)***

$$n(r) = n_1\sqrt{1-2\Delta\left(\frac{r}{a}\right)^{\alpha}} \qquad r \le a$$

$$n(r) = n_1\sqrt{1-2\Delta} \qquad r \ge a$$

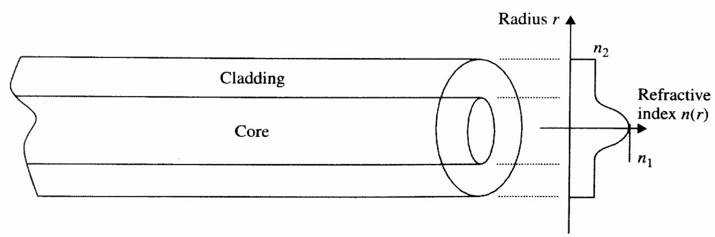


Figure 3.5 Structure of the graded-index optical fiber

Modes of a Graded-index Fiber

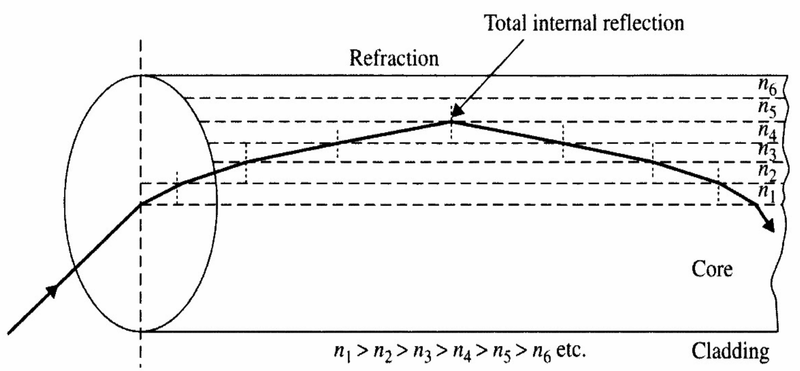


Figure 3.6 Total internal reflection in a graded-index fiber. Source: *Optical Fiber Communications, Principles and Practice,* 2nd edn, J M Senior, Pearson Education Limited. Reproduced by permission of Pearson Education

Modes of a Graded-index Fiber

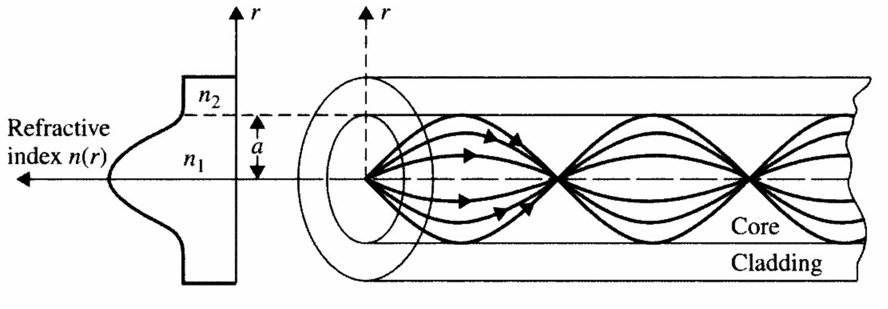


Figure 3.7 Mode trajectories in graded-index fiber. Source: *Optical Fiber Commu-nications, Principles and Practice,* 2nd edn, J M Senior, Pearson Education Limited. Reproduced by permission of Pearson Education

3.3 NUMERICAL APERTURE AND ACCEPTANCE ANGLE

NUMERICAL APERTURE AND ACCEPTANCE ANGLE

- Critical angle θ_c, Acceptance angle θ_a.

$$n_a \sin\theta_a = n_1 \sin(90^\circ - \theta_c) = n_1 \cos\theta_c = n_1\sqrt{1-\sin^2\theta_c}$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

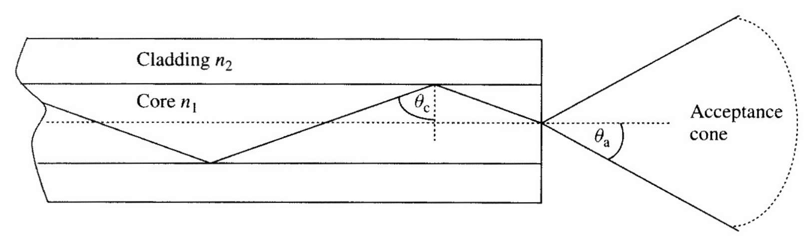


Figure 3.8 Acceptance angle of an optical fiber

NUMERICAL APERTURE AND ACCEPTANCE ANGLE

- Numerical Aperture

$$n_a \sin\theta_a = n_1\sqrt{1-n_2^2/n_1^2} = \sqrt{n_1^2 - n_2^2}$$

$$NA = n_a \sin\theta_a = \sqrt{n_1^2 - n_2^2} \qquad NA = n_1\sqrt{2\Delta}$$

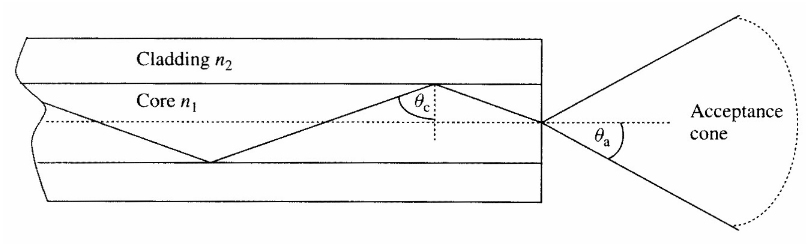


Figure 3.8 Acceptance angle of an optical fiber

NUMERICAL APERTURE AND ACCEPTANCE ANGLE

- For skew rays $NA = n_a \sin\theta_a \cos\gamma = \sqrt{n_1^2 - n_2^2}$

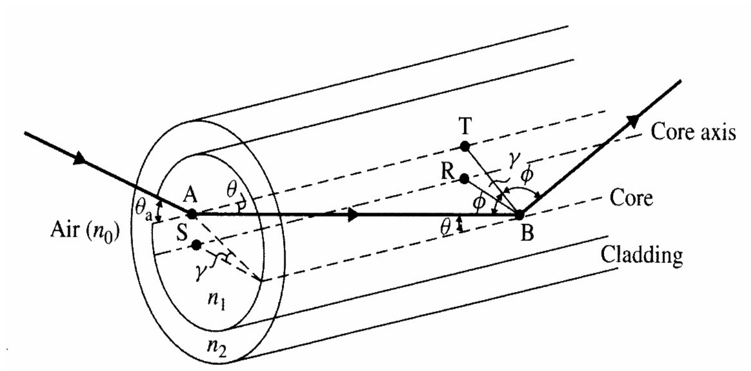
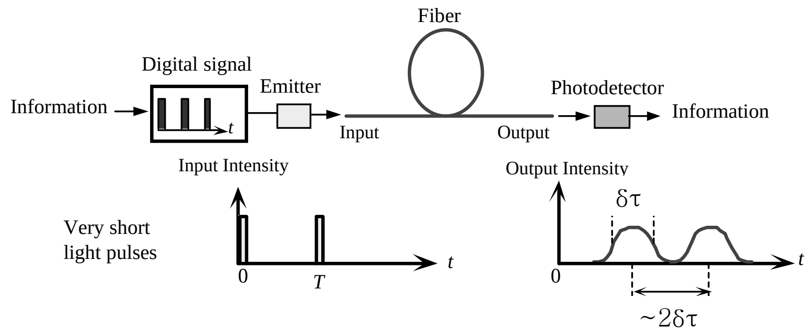


Figure 3.9 The acceptance angle of skew rays. Source: *Optical Fiber Communications, Principles and Practice,* 2nd edn, J M Senior, Pearson Education Limited. Reproduced by permission of Pearson Education

3.4 DISPERSION IN OPTICAL FIBERS

Dispersion

- Dispersion in optical fibers means that parts of the signal propagate through the fiber at slightly different velocities, resulting in distortion of the signal.



Pulse Broadening

- For a given pulse broadening, $\delta\tau$. If pulses are not to overlap, then the maximum pulse broadening must be a maximum of half of the transmission period, T .

$$\delta\tau \leq \tfrac{1}{2}T$$

- *The bit rate B_T,* is $1/T$

$$B_T \leq \frac{1}{2\delta\tau}$$

- Some texts use a more conservative rule of thumb, making $B_T \leq$ ¼ $\delta\tau$
- For a gaussian pulse, with an r.m.s. width σ . A typical rule of thumb is $B_T \leq 0.2/\sigma$

3.4.1 Intermodal (Multimode) Dispersion

Intermodal (multimode) Dispersion

- The fastest and slowest modes possible in such a fiber will be the axial ray, and the ray propagating at the critical angle θ_c. ($\sin\theta_c = n_2/n_1$)

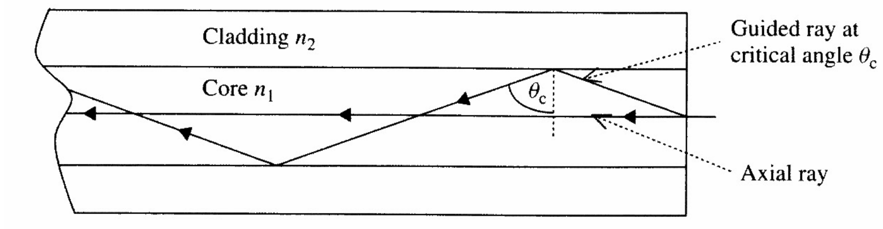
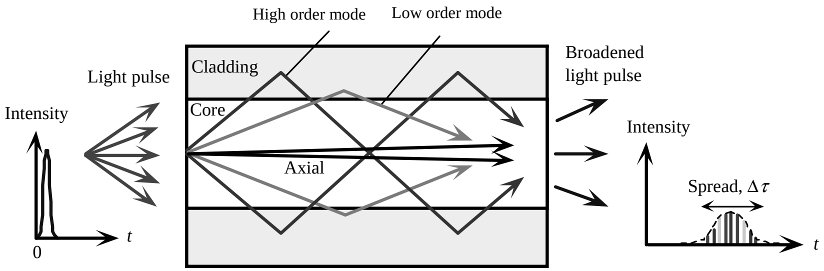


Figure 3.10 The origin of modal dispersion

Intermodal (Multimode) Dispersion

- Intermodal dispersion is a result of the different propagation times of different modes (different m) within a fiber.



Schematic illustration of light propagation in a slab dielectric waveguide. Light pulse entering the waveguide breaks up into various modes which then propagate at different group velocities down the guide. At the end of the guide, the modes combine to constitute the output light pulse which is broader than the input light pulse.

Intermodal (Multimode) Dispersion

- In a multimode fiber, generally

$$t_{\min} = \frac{\text{distance}}{\text{velocity}} = \frac{L}{c/n_1} = \frac{Ln_1}{c}$$

$$t_{\max} = \frac{L/\sin\theta_c}{c/n_1} = \frac{L}{c}\frac{n_1}{\sin\theta_c}$$

or

$$t_{\max} = \frac{L/\sin\theta_c}{c/n_1} = \frac{L}{c}\frac{n_1^2}{n_2}$$

Intermodal Dispersion in Step-Index Multi-mode Fiber

- Δ is the relative refractive index $(n_1 - n_2)/n_1$

$$\delta t_{si} = t_{\max} - t_{\min} = \frac{L}{c}\frac{n_1^2}{n_2} - \frac{Ln_1}{c} = \frac{L}{c}\frac{n_1^2}{n_2}\left(\frac{n_1 - n_2}{n_1}\right) = \frac{L}{c}\frac{n_1^2}{n_2}\Delta$$

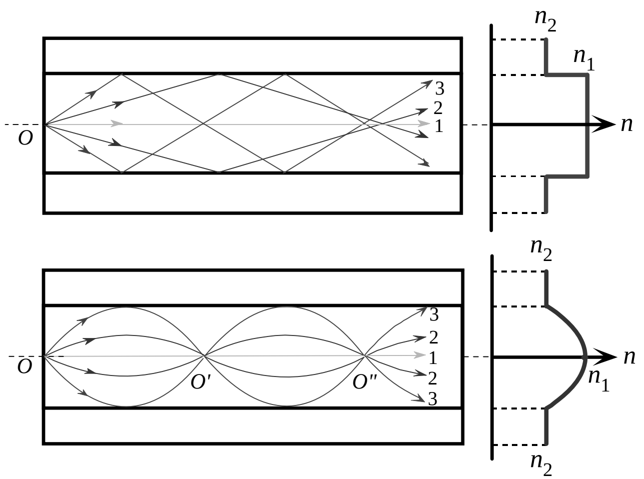
- *For $n_1 \approx n_2$*

$$\delta t_{si} = \frac{Ln_1\Delta}{c}$$

- If we let Δ = 0.01 and n_1 = 1.49. Hence $\delta\tau$ = 49.6 ns/km. The maximum bit rate becomes

$$B_T = \frac{1}{2\delta\tau} \approx 10 \text{ Mbit/s·km}$$

Intermodal Dispersion in Graded-Index Multi-mode Fiber



(a) Multimode step index fiber. Ray paths are different so that rays arrive at different times.

(b) Graded index fiber. Ray paths are different but so are the velocities along the paths so that all the rays arrive at the same time.

Intermodal Dispersion in Graded-Index Multi-mode Fiber

- For graded-index multimode fiber, the velocity of the ray is inversely proportional to the local refractive index, it can be shown that

$$\delta t_{gi} = \frac{Ln_1\Delta^2}{8c}$$

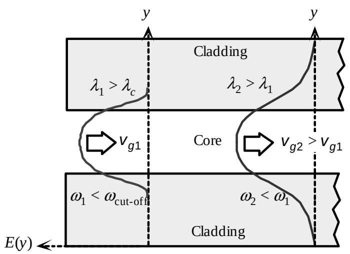
- If we let Δ = 0.01 and n_1 = 1.49. Hence $\delta\tau$ = 62 ps/km. The maximum bit rate becomes

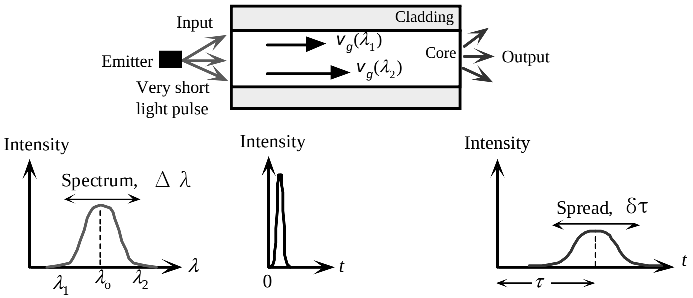
$$B_T = \frac{1}{2\delta\tau} \approx 8 \text{ Gbit/s·km}$$

3.4.2 Intramodal (Chromatic) Dispersion

Intramodal (Chromatic) Dispersion

- Different spectral components of the light source may have different propagation delays, and hence pulse broadening may occur, even if transmitted by a single mode.





Intramodal (Chromatic) Dispersion

- Phase velocity $v_p = \omega / \beta$
- Group velocity $v_g = \delta\omega / \delta\beta$
- Traveling time required for each frequency (wavelength) component $\tau = L / v_g$
- Hence

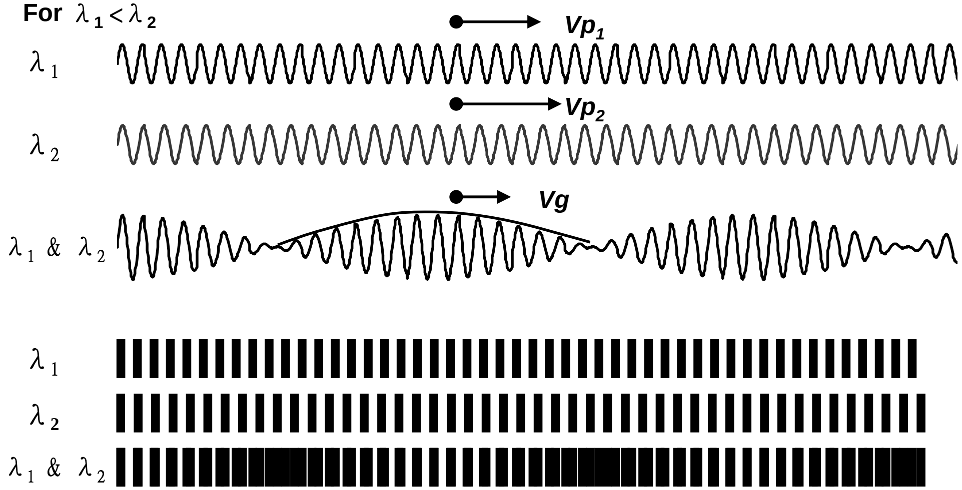
$$\frac{\partial \tau}{\partial \omega} = L \frac{\partial}{\partial \omega}\left(\frac{1}{v_g}\right)$$

- Thus

$$\frac{\partial \tau}{\partial \omega} = L \frac{\partial^2 \beta}{\partial \omega^2}$$

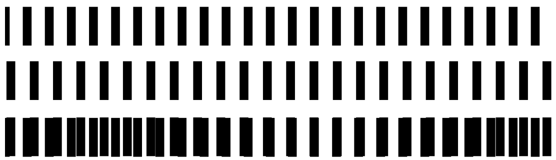
Phase Velocity & Group velocity

For $\lambda_1 < \lambda_2$

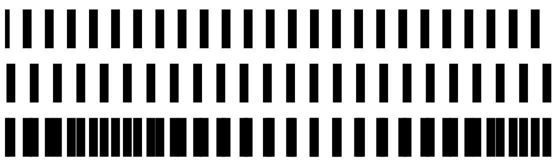


For $\lambda_1 < \lambda_2$

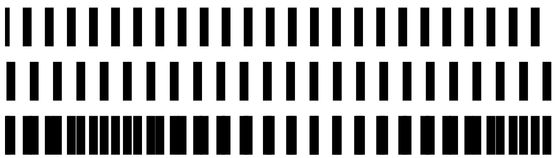
Zero dispersion $D = 0$, $Vp_1 = Vp_2$ **(Light Packet at the same speed** $Vg = Vp$**)**



Normal dispersion $D < 0$, $Vp_1 < Vp_2$ **(Slow Light Packet** $Vg < Vp$**)**



Anomalous dispersion $D > 0$, $Vp_1 > Vp_2$ **(Fast Light Packet** $Vg > Vp$**)**



Intramodal (Chromatic) Dispersion

- Delay time for a pulse with a specific frequency bandwidth $\Delta\omega$

$$\delta\tau_{ch} = \left|\frac{\partial\tau}{\partial\omega}\right|\Delta\omega = \left|\frac{\partial^2\beta}{\partial\omega^2}\right|L\Delta\omega$$

- Group velocity dispersion (GVD) parameter, D ps/(km·nm).

$$D = -\frac{2\pi c}{\lambda^2}\left(\frac{\partial^2\beta}{\partial\omega^2}\right)$$

- Thus

$$\delta\tau_{\text{ch}} = |D| L\Delta\lambda$$

Intramodal (Chromatic) Dispersion

- There are two main contributions to chromatic dispersion:
  - *Material dispersion.* The refractive index of any medium is a function of wavelength, and hence different wavelengths that see different refractive indices will propagate with different velocities, resulting in intramodal dispersion.
  - *Waveguide dispersion.* Even if the refractive index is constant, and material dispersion eliminated, the propagation constant β would vary with wavelength for any waveguide structure, resulting in intramodal dispersion.

Intramodal (Chromatic) Dispersion

- Total chromatic dispersion D is the sum of the material dispersion D_{M} and the waveguide dispersion D_{w}.
- There is zero dispersion close to 1.31 μm (λ_{ZD}) in glass optical fiber.

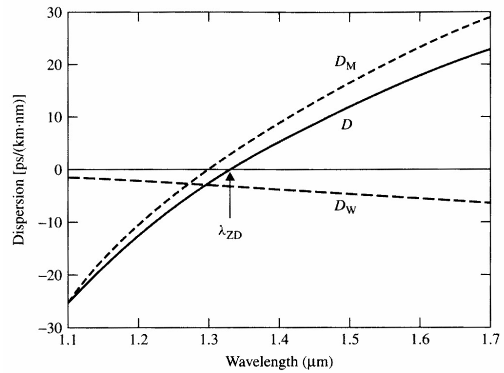


Figure 3.11 The variation in chromatic dispersion with wavelength.

Intramodal (Chromatic) Dispersion in Dispersion-modified Single-mode Optical Fiber

- The contribution of waveguide dispersion is dependent on fiber parameters such as refractive indices and core diameter.
- *Dispersion shifted fibers*: shift λ_{ZD} typically to 1.55 μm, where the optical loss of optical fiber is a minimum.
- *Dispersion flattened fibers*: the total chromatic dispersion is relatively small over the wavelength range 1.3-1.6 μm.

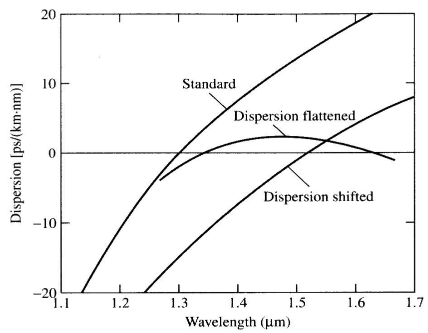


Figure 3.12 Dispersion-shifted and dispersion-flattened fibers.

Intramodal (Chromatic) Dispersion in Single-mode Optical Fiber

- For dispersion flattened single-mode optical fiber, operated near λ_{ZD}, e.g. $D = 1$ ps/(nm·km)

$$B_T = \frac{1}{2\delta\tau_{ch}} \approx 500 \text{ Gbit/s·km}$$

- Comparison

  SI MM fiber $B_T \approx$ 10 Mbit/s·km

  GI MM fiber $B_T \approx$ 8 Gbit/s·km

  SI SM fiber $B_T \approx$ 500 Gbit/s·km

表 2.3 多模步級折射率、單模步級折射率及斜射率光纖之一般特性的比較。

特性	多模步級折射率光纖	單模步級折射率光纖	斜射率光纖
$\Delta = (n_1 - n_2)/n$	0.02	0.003	0.015
核心直徑	100	8.3(MFD = 9.3 μm)	62.5
包層直徑	140	125	125
NA	0.3	0.1	0.26
帶寬×距離或色散	20 ~ 100 MHz km	< 3.5 ps $km^{-1}nm^{-1}$ 在 850 nm > 100 Gb s^{-1}km 常用	300 MHz km – 3 GHz km 在 1.3 μm 在 1.3 μm
光的衰減	4 ~ 6 dB km^{-1} 在 850 nm 0.7 ~ 1 dB km^{-1} 在 1.3 μm	1.8 dB km^{-1} 在 850 nm 0.34 dB km^{-1} 在 1.3 μm 0.2 dB km^{-1} 在 1.55 μm	3 dB km^{-1} at 850 nm 0.6 – 1 dB km^{-1} 在 1.3 μm 0.3 dB km^{-1} 在 1.55 μm
典型光源	發光二極體 (LED)	雷射 單模注入雷射	雷射
典型應用	短矩離或用戶區域網路通訊	長距離通訊	區域及廣面積網路，中距通訊

3.5 SINGLE-MODE FIBERS: MODE PROFILE, MODE-FIELD DIAMETER, AND SPOT SIZE

SINGLE-MODE FIBERS: MODE PROFILE, MODE-FIELD DIAMETER, AND SPOT SIZE

- Cylindrical coordinates (r, ϕ and z)
  - exp $j\beta z$
  - *exp* $jm\phi$
  - $J_m(r)$, $K_m(r)$

$$E_{LP} = E_{lm}(r,\varphi)\exp j(\omega t - \beta_{lm} z)$$

- Cartesian coordinates (x, y and z)
  - exp $j\beta z$
  - *exp* jmx *or indep. of* x
  - *cos* $k_{ym}y$, *exp* $-k_{ym}y$

$$E_m = E_m(y)\exp j(\omega t - \beta_m z)$$

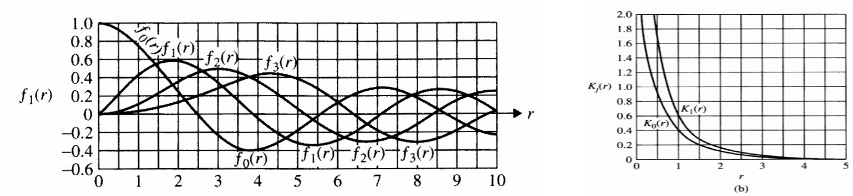


Figure 3.13 (a) Variation of the first four Bessel functions. (b) Variation of the first two modified Bessel functions.

MODE PROFILE

- **MODE PROFILE**
  - – $J_0(r)$, $K_0(r)$

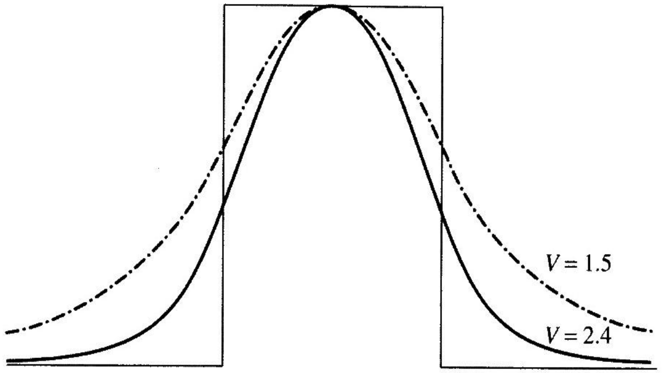


Figure 3.14 Field shape of the fundamental mode of a step-index fiber, for normalized frequencies of V = 1.5 and V = 2.4.

MODE FIELD DIAMETER

- **Mode Field Diameter = $2w_0$**
- **Spot size = w_0**

$$2w_0 = 2a\frac{V+1}{V}$$

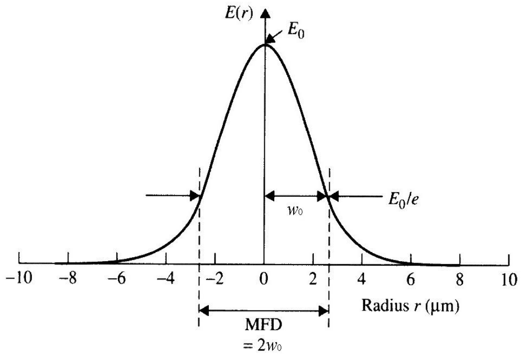


Figure 3.15 Approximation to the fundamental mode, showing the mode-field diameter (MFD) and the spot size w_0

3.6 NORMALIZED FREQUENCY, NORMALIZED PROPAGATION CONSTANT, AND CUTOFF WAVELENGTH

Normalized Frequency

- **Normalized Frequency: V *number***

$$V = \frac{2\pi}{\lambda_0} a\sqrt{n_1^2 - n_2^2} = \frac{2\pi}{\lambda_0} a n_1 \sqrt{2\Delta}$$

- ***Normalized propagation constant: b***

$$b = \frac{(\beta / k_0)^2 - n_2^2}{n_1^2 - n_2^2} = \frac{(\beta / k_0)^2 - n_2^2}{2n_1^2\Delta}$$

- ***a: fiber core radius***
- ***Δ: relative refractive index difference***
- ***λ_0 operating wavelength***

Dispersion (b-V) Diagram

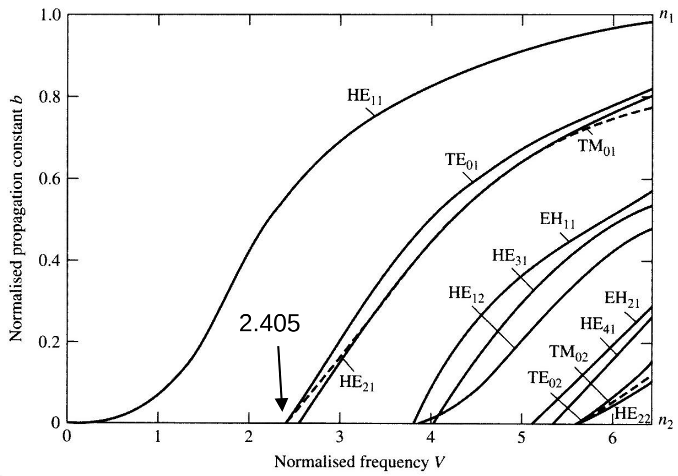


Figure 3.16 Propagation constant of the exact fiber modes plotted against normalized frequency.

Cut-off Condition for Single Mode Fiber

- Fitting curve for LP_{01}

$$b \approx \left(1.1428 - \frac{0.996}{V}\right)^2 \qquad (1.5 < V < 2.5)$$

- Cut-off Condition for Single Mode Fiber
- V_c = 2.405 for step-index fibers

$$\lambda_c = \frac{2\pi}{V_c} a n_1 \sqrt{2\Delta}$$

- $\lambda_c / \lambda_0 = V/V_c$.

$$\lambda_c = \frac{V\lambda_0}{2.405}$$