Silicon Photonics
矽光子學
2 Basics of guided waves (B)

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2.5 A TASTE OF ELECTROMAGNETIC THEORY

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

Maxwell's equations

- Maxwell's equations
  - Electric field **E** (V/m)
  - Magnetic field **H** (A/m)
  - Charge density ρ (C/m^3)
  - Current density *J* (A/cm^2)

$$\nabla \cdot \mathbf{D} = \rho \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Del Operator ∇

- **i**, **j** and **k** are unit vectors in the *x, y* and *z* directions respectively

$$\nabla = \left(\frac{\partial \mathbf{i}}{\partial x}, \frac{\partial \mathbf{j}}{\partial y}, \frac{\partial \mathbf{k}}{\partial z} \right)$$

- Note: **i** should not be confused with current, nor **j** with √-1 , nor **k** with a propagation constant

Field, Flux Density and Current

- Electric field **E** and magnetic field **H** are related to the electric flux density **D** and magnetic flux density **B** (assuming a lossless medium) by

$$\mathbf{D} = \varepsilon_{\mathrm{m}} \mathbf{E}$$

$$\mathbf{B} = \mu_{\mathrm{m}} \mathbf{H}$$

  - ε_{m} is the permittivity of the medium and μ_{m} is the permeability of the medium
- *Ohm's law*

$$\mathbf{J} = \sigma \mathbf{E}$$

Fields

$$\nabla \times \nabla \times \mathbf{E} = -\mu_{\mathrm{m}} \frac{\partial}{\partial t} (\nabla \times \mathbf{H})$$

$$\nabla \times \nabla \times \mathbf{E} = -\frac{\partial}{\partial t} (\nabla \times \mathbf{B})$$

$$\nabla \times \nabla \times \mathbf{E} = -\mu_{\mathrm{m}} \left[\frac{\partial \mathbf{J}}{\partial t} + \frac{\partial^2 \mathbf{D}}{\partial t^2} \right]$$

$$\nabla \times \nabla \times \mathbf{E} = -\mu_{\mathrm{m}} \left[\sigma \frac{\partial \mathbf{E}}{\partial t} + \varepsilon_{\mathrm{m}} \frac{\partial^2 \mathbf{E}}{\partial t^2} \right]$$

Wave Equation

- Vector identity

$$\nabla \times \nabla \times \mathbf{E} = \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

- No Charge and Current source
  - ρ = 0, J = 0
  - Laplacian Operator $\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$
- *Wave equation*

$$\nabla^2 \mathbf{E} = \mu_{\mathrm{m}} \varepsilon_{\mathrm{m}} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Wave Equation

- Velocity of wave

$$v^2 = \frac{1}{\mu_{\mathrm{m}} \varepsilon_{\mathrm{m}}}$$

- General wave equation

$$\nabla^2 \mathbf{E} = \frac{1}{v^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad \nabla^2 \mathbf{H} = \frac{1}{v^2} \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

2.6 SIMPLIFYING AND SOLVING THE WAVE EQUATION

Wave Equation in cartesian coordinates

- E field

$$\mathbf{E} = E_x \mathbf{i} + E_y \mathbf{j} + E_z \mathbf{k}$$

- General wave equation for E field

$$\nabla^2 \mathbf{E} = \mu_{\mathrm{m}} \varepsilon_{\mathrm{m}} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla^2 \mathbf{E} = \frac{\partial^2 \mathbf{E}}{\partial x^2} + \frac{\partial^2 \mathbf{E}}{\partial y^2} + \frac{\partial^2 \mathbf{E}}{\partial z^2}$$

$$= \left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} \right) \mathbf{i} + \left(\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} \right) \mathbf{j} + \left(\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} \right) \mathbf{k}$$

Asymmetrical Planar Waveguide

- Assume TE polarization with E field exists only in the *x* direction

$$\nabla^2 \mathbf{E} = \left(\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} \right) \mathbf{i}$$

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \mu_{\mathrm{m}} \varepsilon_{\mathrm{m}} \frac{\partial^2 E_x}{\partial t^2}$$

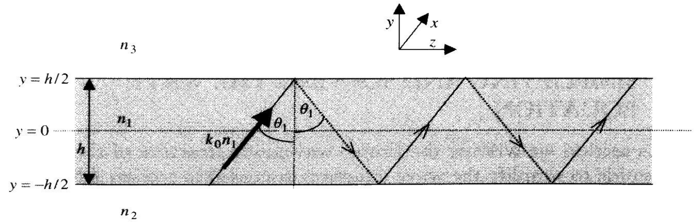


Figure 2.8 Propagation in an asymmetrical planar waveguide

Wave Equation

- General Solution for a Single Frequency

$$E_x = E_x(y)e^{-j\beta z}e^{j\omega t}$$

- Check

$$\frac{\partial E_x}{\partial z} = -j\beta E_x \qquad \frac{\partial^2 E_x}{\partial z^2} = -\beta^2 E_x$$

$$\Rightarrow$$

$$\frac{\partial E_x}{\partial t} = j\omega E_x \qquad \frac{\partial^2 E_x}{\partial t^2} = -\omega^2 E_x$$

- Wave Equation for a Single Frequency

$$\frac{\partial^2 E_x}{\partial y^2} = \left(\beta^2 - \omega^2 \mu_\mathrm{m} \varepsilon_\mathrm{m}\right)E_x$$

Wave Equation

- Substitution with

$$k_0 = 2\pi / \lambda_0 = \omega / c$$

$$1/v^2 = \mu_\mathrm{m}\varepsilon_\mathrm{m} \qquad c/n = v$$

- Wave Equation for a Single Frequency

$$\frac{\partial^2 E_x}{\partial y^2} = \left(\beta^2 - k_0^2 n_i^2\right)E_x$$

- Or

$$\frac{\partial^2 E_x}{\partial y^2} + k_{yi}^2 E_x = 0 \qquad k_{yi}^2 = k_0^2 n_i^2 - \beta^2$$

  – i = 1, 2 or 3.

Wave Equation

- General Solution

$$E_x = E_x(y)e^{-j\beta z}e^{j\omega t}$$

- For Asymmetrical Planar Waveguide

$$E_x(y) = E_u \exp\left[-k_{yu}\left(y - \frac{h}{2}\right)\right] \qquad y \geq (h/2)$$

$$E_x(y) = E_c \exp\left(-jk_{yc}y\right) \qquad -(h/2) \leq y \leq (h/2)$$

$$E_x(y) = E_l \exp\left[k_{yl}\left(y + \frac{h}{2}\right)\right] \qquad y \leq -(h/2)$$

Boundary Conditions

- Electric field (*E*), and its derivative ($\partial E / \partial y$) are continuous at the boundary *y* = ±(*h*/2)
- At *y* = *h*/2

$$E_u = E_c \exp\left(-jk_{yc}\frac{h}{2}\right)$$

- At *y* = -*h*/2

$$E_l = E_c \exp\left[-jk_{yc}\left(-\frac{h}{2}\right)\right]$$

Boundary Conditions

- Exponential function represents both sinusoidal and cosinusoidal functions
  - Cosinusoidal (Cos) ⇨ Even Mode
  - Sinusoidal (Sin) ⇨ Odd Mode
- For Even Mode,
  - *E* fields are continuous at *y* = ±(*h*/2)

$$E_u = E_c \cos\left(k_{yc}\frac{h}{2}+\phi\right)$$

$$E_l = E_c \cos\left(-k_{yc}\frac{h}{2}+\phi\right)$$

Boundary Conditions

- And, the derivatives of the *E* fields ($\partial E/\partial y$) are continuous at *y* = ±(*h*/2)

$$E_u = \frac{k_{yc}}{k_{yu}}E_c \sin\left(k_{yc}\frac{h}{2}+\phi\right)$$

$$E_l = \frac{k_{yc}}{k_{yu}}E_c \sin\left(k_{yc}\frac{h}{2}-\phi\right)$$

- For core-upper cladding interface

$$E_u = E_c \cos\left(k_{yc}\frac{h}{2}+\phi\right) = \frac{k_{yc}}{k_{yu}}E_c \sin\left(k_{yc}\frac{h}{2}+\phi\right)$$

Boundary Conditions

– or

$$\tan^{-1}\left(\frac{k_{yu}}{k_{yc}}\right) = k_{yc}\frac{h}{2}+\phi+m\pi$$

- For core-lower cladding interface

$$E_l = E_c \cos\left(-k_{yc}\frac{h}{2}+\phi\right) = \frac{k_{yc}}{k_{yl}}E_c \sin\left(k_{yc}\frac{h}{2}-\phi\right)$$

– or

$$\tan^{-1}\left(\frac{k_{yl}}{k_{yc}}\right) = k_{yc}\frac{h}{2}-\phi+m\pi$$

Solution to Asymmetrical Planar Waveguide

- We can obtain

$$\tan^{-1}\left(\frac{k_{yl}}{k_{yc}}\right)+\tan^{-1}\left(\frac{k_{yu}}{k_{yc}}\right) = k_{yc}h+\phi+m\pi$$

- Compare

$$\left[k_0 n_1 h\cos\theta_1 - m\pi\right] = \tan^{-1}\left[\frac{\sqrt{\sin^2\theta_1-(n_2/n_1)^2}}{\cos\theta_1}\right] + \tan^{-1}\left[\frac{\sqrt{\sin^2\theta_1-(n_3/n_1)^2}}{\cos\theta_1}\right]$$

2.7 ANOTHER LOOK AT PROPAGATION CONSTANTS

2.8 MODE PROFILES

Propagation and Decay constant

- Equation $$\frac{\partial^2 E_x}{\partial y^2} = \left(\beta^2 - k_0^2 n_i^2\right) E_x$$
  - or $$\frac{\partial^2 E_x}{\partial y^2} + k_{yi}^2 E_x = 0 \qquad k_{yi}^2 = k_0^2 n_i^2 - \beta^2$$
  - Has a solution $$E_x = E_c e^{-k_y y} e^{-j\beta z} e^{j\omega t}$$
  - If $\beta > kn_{\mathrm{i}}$ ⇨ $k_{y\mathrm{i}}$ is imaginary ⇨ propagation along z direction
  - If $\beta < kn_{\mathrm{i}}$ ⇨ $k_{y\mathrm{i}}$ is real ⇨ decay along z direction

Mode Profile

- The field distribution, $E_x(y)$ or the intensity distribution, $|E_x(y)|^2$
- Example: n_1 = 3.5, n_2 = 1.5, n_3 = 1.0, λ_0= 1.3 μm, and h = 0.15 μm, m = 0 (SOI waveguide)

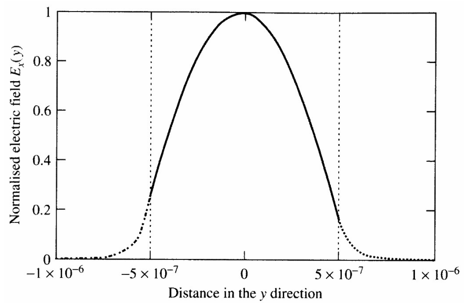


Figure 2.9 Electric field profile of the fundamental mode (*m* = 0)

Mode Profile

- Example: n_1 = 3.5, n_2 = 1.5, n_3 = 1.0, λ_0= 1.3 μm, and h = 0.15 μm, m = 2 (SOI waveguide)

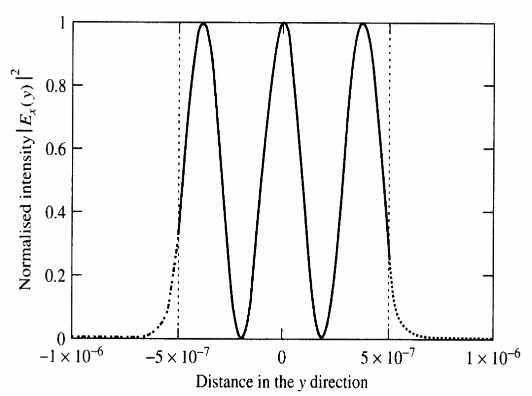


Figure 2.9 Electric field profile of the second even mode (m = 2)

2.9 CONFINEMENT FACTOR

Confinement Factor

- *Confinement Factor*

$$\Gamma = \frac{\int_{-h/2}^{h/2} E_x^2(y)\mathrm{d}y}{\int_{-\infty}^{\infty} E_x^2(y)\mathrm{d}y}$$

- *A function of*
  - *polarization*
  - *the refractive index difference between core and claddings*
  - *the thickness of the waveguide (relative to the wavelength)*
  - *the mode number*

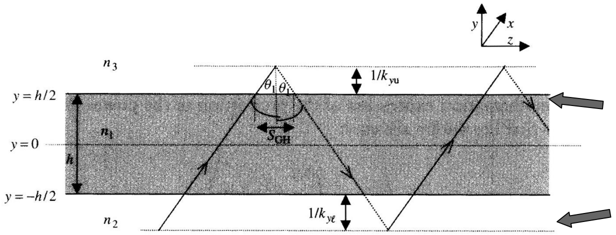
2.10 THE GOOS-HÄNCHEN SHIFT

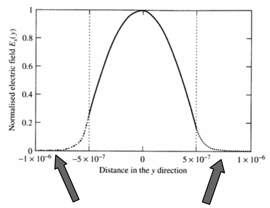
GOOS-HÄNCHEN SHIFT

- ***Goos-Hänchen Shift* (*S*$_{GH}$)**

$$\tan\theta_1 = \frac{S_{GH}/2}{1/k_{yu}} = S_{GH}k_{yu}/2$$

$$S_{GH} = \frac{2\tan\theta_1}{k_{yu}} = \frac{2}{k_{yu}}\frac{\beta}{k_{yc}}$$





Evanescent field

Figure 2.11 Propagation in an asymmetric planar waveguide, showing cladding penetration