Silicon Photonics

矽光子學

2 Basics of guided waves (A)

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Outline

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- **2.2 REFLECTION COEFFICIENTS**
- **2.3 PHASE OF A PROPAGATING WAVE AND ITS WAVEVECTOR**
- **2.4 MODES OF A PLANAR WAVEGUIDE**

2.1 THE RAY OPTICS APPROACH TO DESCRIBING PLANAR WAVEGUIDES

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

RAY OPTICS

- Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

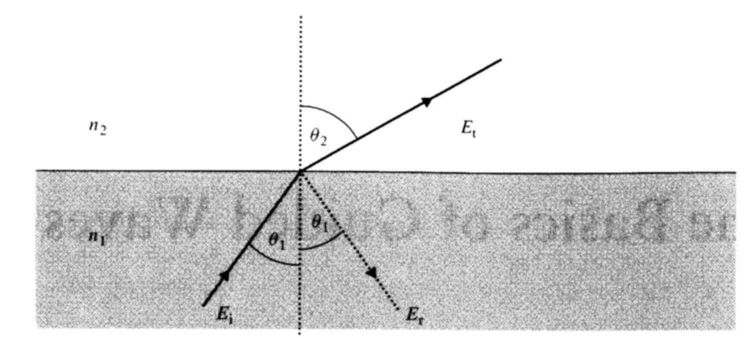
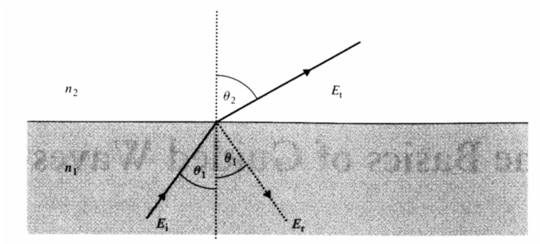


Figure 2.1 Light rays refracted and reflected at the interface of two media

Critical Angle

- At the critical angle ⇨ $\theta_2 = 90°$

$$n_1 \sin\theta_1 = n_2 \qquad \theta_c = \sin^{-1}\frac{n_2}{n_1}$$



Total Internal Reflection

- $\theta_i > \theta_c$ ⇨ no light is transmitted and total internal reflection (TIR) occurs.

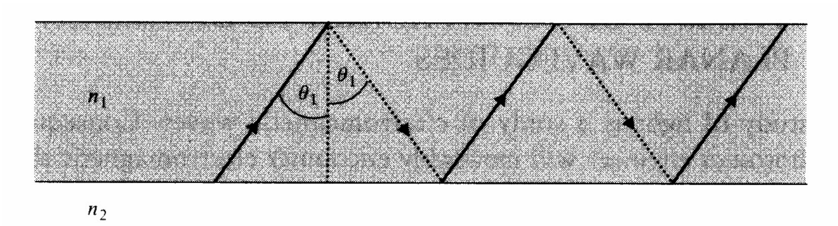


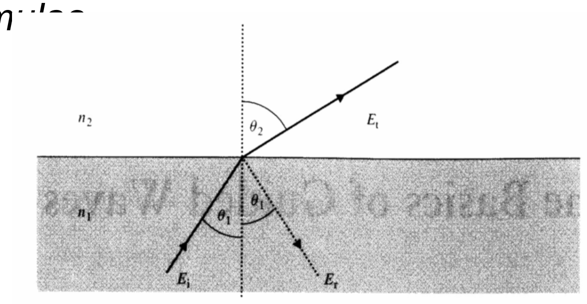
Figure 2.2 Total internal reflection at two interfaces, demonstrating the concept of a waveguide

2.2 REFLECTION COEFFICIENTS

REFLECTION COEFFICIENTS

- *Fresnel Form*

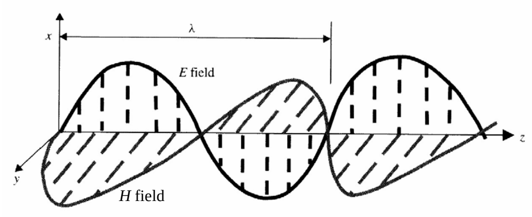
$$E_r = rE_i$$



  - r is a complex reflection coefficient
  - The reflection coefficient is a function of both the **angle of incidence** and the **polarization** of light
  - '**Partial**' reflection and '**partial**' transmission

TEM waves

- *Transverse Electromagnetic Waves,* or TEM waves
  - The **electric** and **magnetic** fields of an electromagnetic wave are always **orthogonal** to one another, and both are **orthogonal** to the **direction of propagation**
  - ***Polarization*** *is the direction of the electric field associated with the propagating wave*



TE & TM Conditions

- Plane of incidence (formed by the wave normal & the normal to the interface)
- TE condition ⇨ E field ⊥ Plane of incidence
- TM condition ⇨ H field ⊥ Plane of incidence

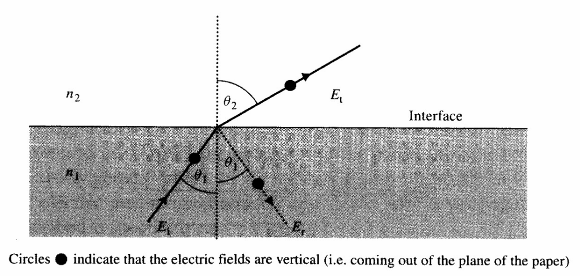


Figure 2.3 Orientation of electric fields for TE incidence at the interface between two media

Fresnel Formulae (I)

- TE polarization

$$r_{TE} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

- TM polarization

$$r_{TM} = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

Fresnel Formulae (II)

- Using Snell's law
- TE polarization

$$r_{TE} = \frac{n_1 \cos\theta_1 - \sqrt{n_2^2 - n_1^2 \sin^2\theta_1}}{n_1 \cos\theta_1 + \sqrt{n_2^2 - n_1^2 \sin^2\theta_1}}$$

- TM polarization

$$r_{TM} = \frac{n_2^2 \cos\theta_1 - n_1\sqrt{n_2^2 - n_1^2 \sin^2\theta_1}}{n_2^2 \cos\theta_1 + n_1\sqrt{n_2^2 - n_1^2 \sin^2\theta_1}}$$

Total Internal Reflection

- $\theta_i < \theta_c$
  - Only partial reflection occurs and the **reflection coefficient** is **real**.
- $\theta_i > \theta_c$
  - Total internal reflection occurs and the **reflection coefficient** is **complex**.

$$|r| = 1 \qquad r = \exp(j\phi)$$

Phase Shift due to Total Internal Reflection

$$\phi_{TE} = 2\tan^{-1}\frac{\sqrt{\sin^2\theta_1 - \left(\frac{n_2}{n_1}\right)^2}}{\cos\theta_1}$$

$$\phi_{TM} = 2\tan^{-1}\frac{\sqrt{\frac{n_1^2}{n_2^2}\sin^2\theta_1 - 1}}{\frac{n_2}{n_1}\cos\theta_1}$$

Reflected Power

- Poynting Vector
  - vector product of the electric and magnetic vectors

$$S = \frac{1}{Z}E^2 = \sqrt{\frac{\varepsilon_m}{\mu_m}}E^2$$

- Reflected Power

$$R = \frac{S_r}{S_i} = \frac{E_r^2}{E_i^2}$$

2.3 PHASE OF A PROPAGATING WAVE AND ITS WAVEVECTOR

PHASE OF A PROPAGATING WAVE

- the electric and magnetic fields associated with a propagating wave be described respectively as

$$E = E_0 \exp[j(kz \pm \omega t)]$$

$$H = H_0 \exp[j(kz \pm \omega t)]$$

- Phase

$$\phi = kz \pm \omega t$$

PHASE OF A PROPAGATING WAVE

- Angular Frequency ω and Frequency f

$$\left|\frac{\partial \phi}{\partial t}\right| = \omega = 2\pi f$$

- *Wavevector* k (*propagation constant*)

$$\frac{\partial \phi}{\partial z} = k \qquad k = \frac{2\pi}{\lambda}$$

Wavevector

- *In free space*

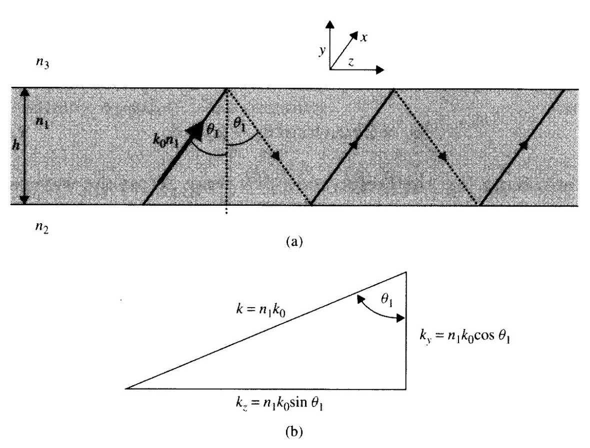
$$k_0 = \frac{2\pi}{\lambda_0}$$

- *In a medium with the refractive index n*

$$k = \frac{2\pi n}{\lambda_0} = nk_0$$

2.4 MODES OF A PLANAR WAVEGUIDE

PLANAR WAVEGUIDE



$$k_z = n_1 k_0 \sin\theta_1$$

$$k_y = n_1 k_0 \cos\theta_1$$

Figure 2.4 (a) Propagation in a planar waveguide. (b) The relationship between propagation constants in the *y*, *z* and wavenormal directions. Adapted with permis-sion from Artech House Publishing, Norwood, MA, USA, www.artechhouse.com

Standing Wave

- There will potentially be a *standing wave* across the waveguide in the *y* direction
- For a roundtrip in the *y* direction

$$\phi_h = 2k_y h = 2k_0 n_1 h \cos\theta_1$$

- The total phase including the effect of the reflection at the upper interface and lower interface

$$\phi_t = 2k_0 n_1 h \cos\theta_1 - \phi_u - \phi_l$$

Mode of Propagation

- For consistency, this total phase shift must be a multiple of 2π

$$2k_0 n_1 h \cos\theta_1 - \phi_u - \phi_l = 2m\pi$$

- There will be a series of discrete angles corresponding to integral values of *m*
- *Each solution* is referred to as a *mode of propagation*

Mode of Propagation

- The first *TE* mode, or *fundamental mode,* will be described as TE_0
- Higher-order modes are correspondingly described using the appropriate value of *m*
- *The limiting conditions of m correspond to the propagation angle, θ_1, becoming less than the critical angle at either the upper or lower waveguide interface.*

2.4.1 The Symmetrical Planar Waveguide

$n_2 = n_3$

Symmetrical Planar Waveguide

- **Symmetrical Planar Waveguide**

$$n_2 = n_3 \qquad \phi_u = \phi_l$$

- **TE Polarization**

$$2k_0 n_1 h \cos\theta_1 - 4\tan^{-1}\frac{\sqrt{\sin^2\theta_1 - (n_2/n_1)^2}}{\cos\theta_1} = 2m\pi$$

Symmetrical Planar Waveguide

- TE Polarization

$$\tan\left[\frac{k_0 n_1 h\cos\theta_1 - m\pi}{2}\right] = \frac{\sqrt{\sin^2\theta_1 - (n_2/n_1)^2}}{\cos\theta_1}$$

- TM Polarization

$$\tan\left[\frac{k_0 n_1 h\cos\theta_1 - m\pi}{2}\right] = \frac{\sqrt{(n_1/n_2)^2\sin^2\theta_1 - 1}}{(n_2/n_1)\cos\theta_1}$$

Number of Modes

- Minimum value of $\theta_1 = \theta_c$, where
- $m_{\max}$

$$k_0 n_1 h\cos\theta_1 - m_{\max}\pi = 0$$

– or

$$m_{\max} = \frac{k_0 n_1 h\cos\theta_c}{\pi}$$

- Allowed Modes are m = 0,1,2,…$[m_{\max}]_{\text{int}}$
- The Number of Modes = $[m_{\max}]_{\text{int}}+1$
- The lowest order mode (the fundamental mode, m = 0) is always allowed (never cut-off).

2.4.2 The Asymmetrical Planar Waveguide

Asymmetrical Planar Waveguide

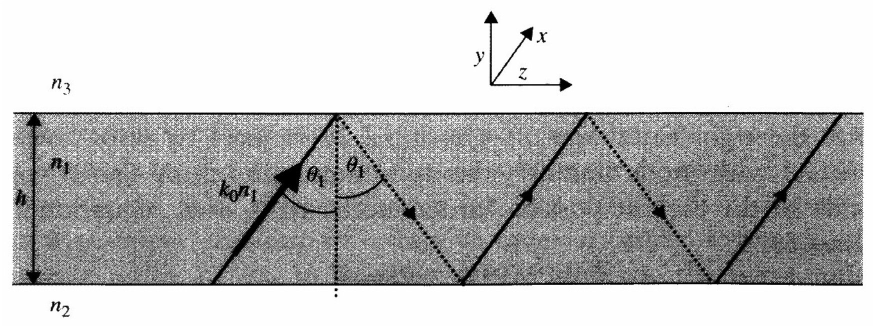


Figure 2.5 Propagation in an asymmetrical planar waveguide

Asymmetrical Planar Waveguide

- TE Polarization

$$\left[k_0 n_1 h \cos\theta_1 - m\pi\right] = \tan^{-1}\left[\frac{\sqrt{\sin^2\theta_1 - (n_2/n_1)^2}}{\cos\theta_1}\right] + \tan^{-1}\left[\frac{\sqrt{\sin^2\theta_1 - (n_3/n_1)^2}}{\cos\theta_1}\right]$$

- There is not always a solution for m = 0
- Especially when the waveguide is too thin (h is too small) or the refractive index difference between the core and the claddings is too small.

2.4.3 Solving the Eigenvalue Equations for Symmetrical and Asymmetrical Waveguides

Solving the Eigenvalue Equations (I)

- Let n_1 = 1.5, n_2 = 1.49, n_3 = 1.40, λ_0= 1.3 μm, and h = 0.3 μm, m = 0 (Silica waveguide)

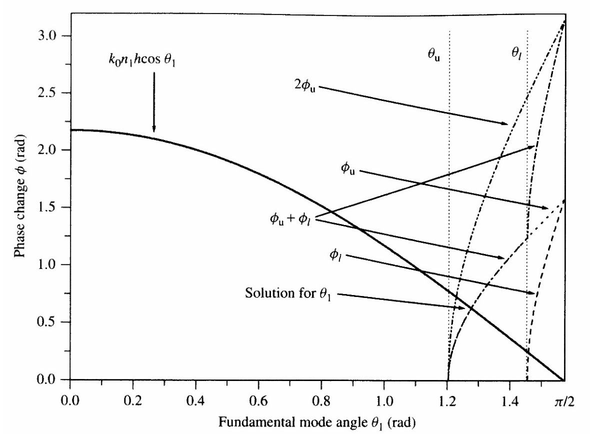


Figure 2.6 Solution of the eigenvalue equation for m = 0

Solving the Eigenvalue Equations (II)

- Let n_1 = 3.5, n_2 = 1.5, n_3 = 1.0, λ_0= 1.3 μm, and h = 0.15 μm, m = 0 (SOI waveguide)

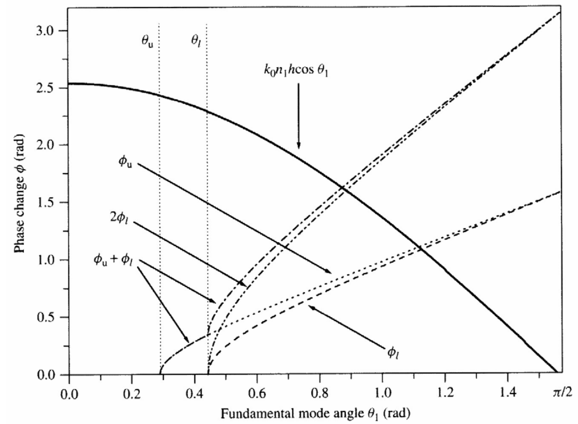


Figure 2.7 Solution of the eigenvalue equation for m = 0 (silicon-on-insulator)

2.4.4 Monomode Conditions

Monomode Conditions

- TE Polarization in a Symmetrical waveguide

$$\tan\left[\frac{k_0 n_1 h\cos\theta_1 - m\pi}{2}\right] = \left[\frac{\sqrt{\sin^2\theta_1 - (n_2/n_1)^2}}{\cos\theta_1}\right]$$

- Monomode condition

$$\tan\left[\frac{k_0 n_1 h\cos\theta_1 - m\pi}{2}\right] = 0$$

$$\cos\theta_c = \frac{\pi}{k_0 n_1 h} = \frac{\lambda_0}{2n_1 h} \quad \Rightarrow \quad \theta_c < \cos^{-1}\left(\frac{\lambda_0}{2n_1 h}\right)$$

2.4.5 Effective Index of a Mode

Effective Index of a Mode

- *The propagation constant in the z direction* k_z is often replaced in many texts by the variable β

$$k_z = n_1 k_0 \sin\theta_1 \qquad k_y = n_1 k_0 \cos\theta_1$$

- The effective index of the mode N (n_{eff})

$$N = n_1 \sin\theta_1 \qquad k_z = \beta = N k_0$$

Effective Index of a Mode

- Range of β

$$n_1 k_0 \geq \beta \geq n_1 k_0 \sin\theta_l \left(= k_0 n_2\right)$$

- Range of N

$$n_1 \geq N \geq n_2$$