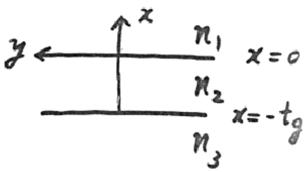
3. Theory of Optical Waveguides

- Physical-optic approach
- Particular waveguides of interest
  - Planar
  - Rectangular

Maxwell's equation (TE)



$$\nabla^2 E_y = \frac{n_i^2}{c^2}\frac{\partial^2 E_y}{\partial t^2}, \quad i=1,2,3$$

$$\Rightarrow \frac{d^2 E_y}{dx^2} + (k^2 n_i^2 - \beta^2) E_y = 0$$

$$\because E_y(x,y,z) = \mathcal{E}_y(x)\, e^{j(\omega t - \beta z)}$$

$$\mathcal{E}_y(x) = \begin{cases} A e^{-qx} & 0 \le x < \infty \\ B\cos(hx) + C\sin(hx) & -t_g \le x < 0 \\ D e^{p(x+t_g)} & -\infty < x < -t_g \end{cases}$$

where

$$q = \sqrt{\beta^2 - n_1^2 k^2}$$

$$h = \sqrt{n_2^2 k^2 - \beta^2} \qquad k = \frac{\omega}{c}$$

$$p = \sqrt{\beta^2 - n_3^2 k^2}$$

Boundary conditions:

$\left.\begin{array}{l} \mathcal{E}_y \\ \frac{\partial \mathcal{E}_y}{\partial x} \end{array}\right\}$ continuous at $n_1 - n_2$ boundary $(x=0)$

$\mathcal{E}_y$ " $n_2 - n_3$ " $(x = -t_g)$

then

$$\mathcal{E}_y = \begin{cases} C' e^{-qx} & 0 \le x < \infty \\ C'\left[\cos(hx) - \frac{q}{h}\sin(hx)\right] & -t_g \le x < 0 \\ C'\left[\cos(ht_g) + \left(\frac{q}{h}\right)\sin(ht_g)\right] e^{p(x+t_g)} & -\infty < x \le -t_g \end{cases}$$

$\frac{\partial \mathcal{E}_y}{\partial x}$ continuous at $n_2 - n_3$ boundary $(x = -t_g)$

$$-h\sin(-ht_g) - h(q/h)\cos(-ht_g)$$
$$= p\,[\cos(ht_g) + (q/h)\sin(ht_g)]$$

$$\Rightarrow \tan(ht_g) = \frac{p+q}{h\left(1-\frac{pq}{h^2}\right)} = \text{function of } \beta$$

Discrete values of $\beta \Rightarrow \beta_m$

C' : normalization costant

It is convenient to normalize so that $\mathcal{E}_y(x)$ represents

"power flow of one Watt per unit width in y direction"

Thus, when $E_y = A\,\mathcal{E}_y(x)$

then power flow $= |A|^2$ W/m

In this case, the normalization condition is

$$-\frac{1}{2}\int_{-\infty}^{\infty} E_y H_x^* \, dx = \frac{\beta_m}{2\omega\mu}\int_{-\infty}^{\infty}\left[\mathcal{E}_y^{m}(x)\right]^2 dx = 1$$

$$\left[\begin{array}{l} W = \oint_S \vec{P}\cdot d\vec{S} = \oint_S (\vec{E}\times\vec{H})\cdot dS \\ \vec{\nabla}\times\vec{E} = -j\omega\mu\vec{H} \Rightarrow_{64} \beta E_y = -j\omega\mu H_x \quad (\because E_z = 0) \end{array}\right]$$

$$C_m' = 2\sqrt{\frac{\omega\mu}{|\beta_m|\, t_g'}}$$

$$t_g' = \left(t_g + \frac{1}{q_m} + \frac{1}{p_m}\right)\left[1 + \left(\frac{q_m}{h_m^2}\right)^2\right]$$

For orthogonal modes

$$\int_{-\infty}^{\infty} \mathcal{E}_y^{(l)}\,\mathcal{E}_y^{(m)}\,dx = \frac{2\omega\mu}{\beta_m}\,\delta_{lm}$$

For the TM modes

$H_y \rightarrow E_x$ and E_z

$$\begin{cases} E_x = \dfrac{i}{\omega\epsilon}\dfrac{\partial H_y}{\partial z} \\ E_z = -\dfrac{i}{\omega\epsilon}\dfrac{\partial H_y}{\partial x} \end{cases} \qquad \left(\because \vec{\nabla}\times\vec{H} = j\omega\epsilon\vec{E}\right), \quad H_z = H_x = 0$$

H_y, $\frac{\partial H_y}{\partial x}$ continuous at $x = 0$

H_y " $x = -t_g$

$$H_y(x)=\begin{cases} -C'\dfrac{h}{\bar q}e^{-qx} & 0\le x<\infty \\ +C'\left[-\dfrac{h}{\bar q}\cos(hx)+\sin(hx)\right] & -t_g\le x<0 \\ -C'\left[\dfrac{h}{\bar q}\cos(ht_g)+\sin(ht_g)\right]e^{p(x+t_g)} & -\infty<x<-t_g \end{cases}$$

where $\bar q = \dfrac{n_2^2}{n_1^2}q$

$\dfrac{\partial H_y}{\partial x}$ continuous at $x=-t_g$

$$\tan(ht_g)=\frac{h(\bar p+\bar q)}{h^2-\bar p\,\bar q}, \qquad \bar p=\frac{n_2^2}{n_3^2}p$$

Normalization constant

$$C_m' = 2\sqrt{\frac{\omega\epsilon_0}{\beta_m t_g'}}$$

where

$$t_g' = \frac{\bar q^2+h^2}{\bar q^2}\left(\frac{t_g}{n_2^2}+\frac{q^2+h^2}{\bar q^2+h^2}\,\frac{1}{n_1^2 q}+\frac{p^2+q^2}{\bar p^2+h^2}\,\frac{1}{n_3^2 p}\right)$$

3.1.2 Symmetric Waveguide

Symmetric $n_1 = n_3$

Example : Multilayer GaAlAs OIC's

Cutoff Condition

At cutoff the field becomes oscillatory in Regions 1 and 3, the magnitude of β is given by

$$\beta = kn_1 = kn_3 \qquad \left(\frac{d^2E_y}{dx^2}=0\right)$$

or

$$p=\sqrt{\beta^2-k^2n_3^2}=0$$

$$q=\sqrt{\beta^2-k^2n_1^2}=0$$

$$h=k\sqrt{n_2^2-n_1^2}=k\sqrt{n_2^2-n_3^2}$$

$$\Rightarrow \tan(ht_g)=\frac{p+q}{h\left(1-\frac{pq}{h^2}\right)}=0$$

or

$$ht_g = m_s\pi, \quad m_s=0,1,2,\cdots$$

or $\Delta n = n_2 - n_1 > \dfrac{m_s^2\lambda_0^2}{4t_g^2(n_2+n_1)}$

- The lowest mode $m_s = 0$
  $\therefore \Delta n > 0$ always $(\because n_2 > n_1)$
  $\Rightarrow$ Exhibit no cutoff in principle!
- Small $\Delta n \Rightarrow$ poor confinement
  (or large λ_0/t_g)

- Special cases
  - $n_2 \simeq n_1$

$$\Delta n = n_2 - n_1 > \frac{m_s^2 \lambda_0^2}{8 t_g^2 n_2}$$

  - $n_2 >> n_1$

$$\Delta n > \frac{m_s^2 \lambda_0^2}{4 t_g^2 n_2}$$

air
glass
air

$Ga_{1-x}Al_xAs$
$Ga_{1-y}Al_yAs$
$Ga_{1-x}Al_xAs$
GaAs sub.

$y > x$

3.1.3 Asymmetric Waveguide

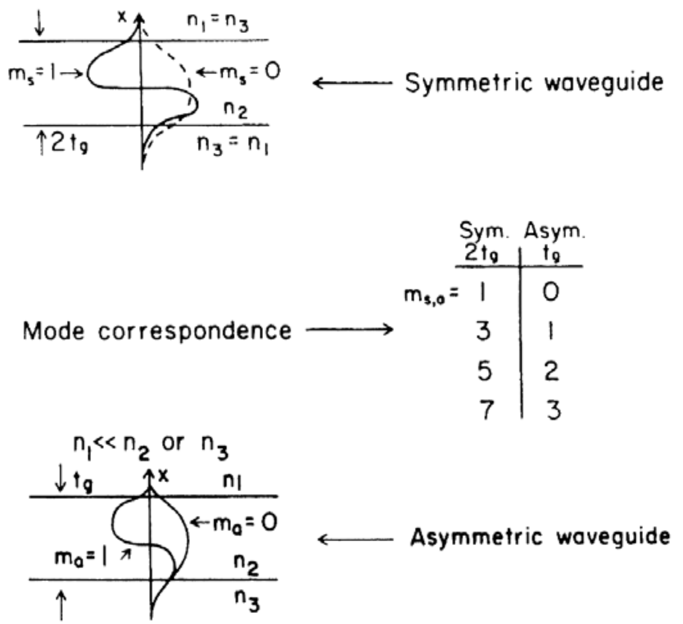
Asymmetric $n_2 > n_3 >> n_1$

Examples :

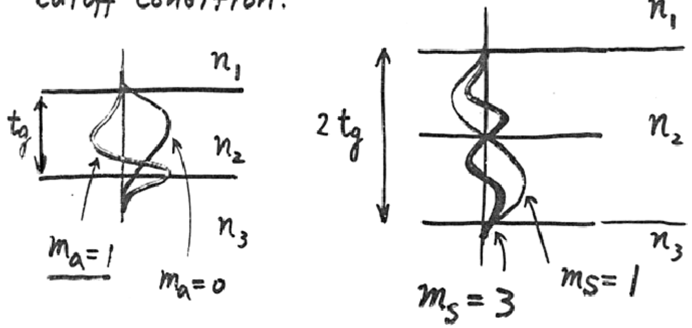
air
Ta_2O_5
SiO_2
Si substrate

Air
$Ti:LiNbO_3$
$LiNbO_3$

Fig. 3.2.



Approximate closed form expression for the cutoff condition.



$\because$ $n_3 \gg n_1$ The field near $n_1 - n_2$ boundary goes not zero very fast, which is similar to that of odd modes of a waveguide of thickness $2t_g$.

Symm ($2t_g$)	Asymm (t_g)
m_s	m_a
1	0
3	1
5	2
7	3
$m_s = 2m_a + 1$	m_a

$$\Delta n = n_2 - n_3 > \frac{(2m_a+1)^2 \lambda_0^2}{4(2t_g)^2 (n_2 + n_3)} = \frac{(2m_a+1)^2 \lambda_0^2}{32\, t_g^2\, n_0}$$

$n_2 \approx n_3$

- The cutoff conditions offer a convenient way to estimate the number of modes supported by a particular waveguide.

- Exact case

  TE waves

  Solve $\tan(h t_g) = \dfrac{h(p+q)}{h^2 - pq}$ (3.1.7)

  numerically

For TM waves

$$\tan(h t_g) = \frac{h(\bar{p} + \bar{q})}{h^2 - \bar{p}\bar{q}}$$

where $\bar{q} = \dfrac{n_2^2}{n_1^2} q$

$\bar{p} = \dfrac{n_2^2}{n_3^2} p \approx p$

The previous cutoff condition is also holds for TM waves as long as

$n_2 \approx n_3$

Thus, the asymmetric waveguide has a possible cutoff for all modes, particularly useful as an optical switch.

Unlike the symmetric waveguide for which the TE_0 mode can not be cut off.

Planar waveguides with Graded Index Profile

$\left\{ \begin{array}{l} \text{Diffusion} \\ \text{Ion Implantation} \end{array} \right. \Rightarrow$ Graded Index Profile $n(x) \neq$ constant

- Exact Solution
  - Parabolic
  - $\text{sech}^2(x)$
  - exponential
- Approximate Solution
  - Ray
  - WKB

TE modes are governed by

$$\frac{d^2 E_y}{dx^2} - (\beta^2 - k^2 n^2(x)) E_y = 0$$

Comparison $\left[\psi'' + \frac{2m}{\hbar^2} (E - V(x)) \psi = 0 \right]$

Planar Waveguides	Q.M. Potential Well
$k^2 n^2(x)$	$-\frac{2mV(x)}{\hbar^2}$
$-\beta_m^2$	$\frac{2mE}{\hbar^2}$

Parabolic Index Profile (Harmonic Oscillator)

$$n^2(x) = n_f^2 \left(1 - \frac{x^2}{x_0^2}\right)$$

For small x

$$n(x) \approx n_f \left(1 - \frac{1}{2} \frac{x^2}{x_0^2}\right)$$

Solutions

$$E_y(x) = H_m\left(\frac{\sqrt{2}\,x}{W}\right) e^{-\frac{x^2}{W^2}}$$

where

$$H_m(x) \equiv (-1)^m e^{x^2} \frac{d^m}{dx^m} e^{-x^2}$$

$$= \text{Hermite Polynomial}$$

$$W \equiv \frac{\lambda x_0}{\pi n_f} = \text{beam radius}$$

$$\beta_m^2 = k^2 n_f^2 - \frac{(2m+1)}{x_0} k n_f$$

$$N_m^2 = \text{effective index} = n_f^2 - \frac{(m+\frac{1}{2})\lambda n_f}{\pi x_0}$$

Note:

$H_0(x) = 1$ $\quad$ $H_2(x) = 4x^2 - 2$

$H_1(x) = 2x$ $\quad$ $H_3(x) = 8x^3 - 12x$

The Sech^2(x) Profile

$$n^2(x) = n_s^2 + 2 n_s \Delta n \,\mathrm{sech}^2\left(\frac{2x}{t}\right)$$

For small Δn

$$n(x) \approx n_s + \Delta n \,\mathrm{sech}^2\left(\frac{2x}{t}\right)$$

t : guiding layer thickness

Normalized thickness

$$V = k t \sqrt{2 n_s \Delta n}$$

The maximum number of guided modes

$$s = \frac{1}{2}\left(\sqrt{1+V^2} - 1\right)$$

Propagation constant

$$\beta_m^2 = k^2 n_s^2 + 4(s-m)^2/t^2$$

Effective index

$$N_m^2 = n_s^2 + (s-m)^2\left(\frac{\lambda}{\pi t}\right)^2$$

Field distribution

$$E_y = U_m\left(\frac{2x}{t}\right)\mathrm{sech}^2\left(\frac{2x}{t}\right)$$

where U_m are hypergeometric functions

$$U_m = 1 - \frac{m(2s-m)\sinh^2(2x/t)}{2\cdot 1\cdot 1!} + \frac{m(m-2)(2s-m)(2s-m-2)\sinh^4\left(\frac{2x}{t}\right)}{4\cdot 3\cdot 2!} - \cdots \quad (m = \text{even})$$

$$U_m = \sinh\left(\frac{2x}{t}\right)\left[1 - \frac{(m-1)(2s-m-1)\sinh^2\left(\frac{2x}{t}\right)}{2\cdot 3\cdot 1!} + \frac{(m-1)(m-3)(2s-m-1)(2s-m-3)\sinh^4\left(\frac{2x}{t}\right)}{4\cdot 3\cdot 5\cdot 2!} \cdots\right. \quad (m = \text{odd})$$

For the lowest mode orders

$$u_0 = 1$$

$$u_1 = \sinh\left(\frac{2x}{t}\right)$$

$$u_2 = 1 - 2(s-1)\sinh^2\left(\frac{2x}{t}\right)$$

$$u_3 = \sinh\left(\frac{2x}{t}\right)\left[1 - \frac{2}{3}(s-2)\sinh^2\left(\frac{2x}{t}\right)\right]$$

The Exponential Profile

$$n^2(x) = n_s^2 + 2 n_s \Delta n \, e^{-2|x|/t}$$

For small Δn

$$n(x) \approx n_s + \Delta n \, e^{-2|x|/t}$$

Normalized layer thickness

$$V = kt\sqrt{2 n_s \Delta n}$$

Field distribution

$$E_y = J_p\left(V e^{-\frac{|x|}{t}}\right)$$ Bessel Function of the first kind

The order p_m is determined by matching the solutions at the boundary $x=0$ for given V.

$$\begin{cases} J_p'(V) = 0 & \text{even modes} \\ J_p(V) = 0 & \text{odd} \end{cases}$$

Each of the two solutions yields approximately

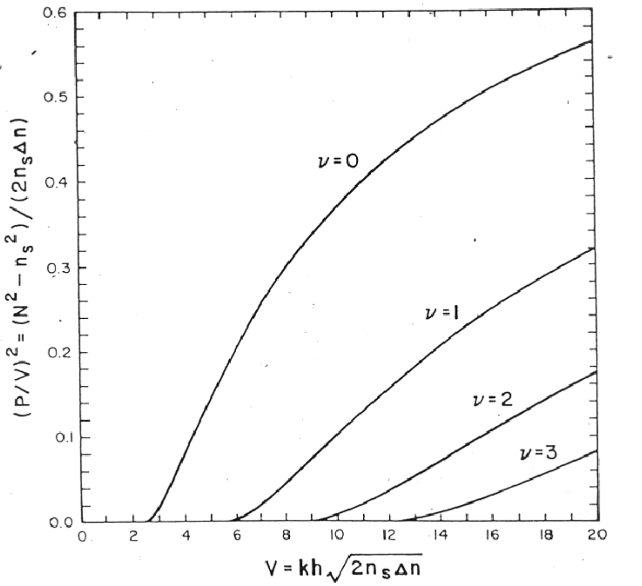
$\frac{V}{\pi}$ solutions for p_m

$$[\, x^2 y'' + x y' + (x^2 - p^2) y = 0 \,],\ J_p(x)$$

$$\beta_m^2 = k^2 n_3^2 + \frac{p_m^2}{t^2} \quad \text{(Propagation const.)}$$

$$N_m^2 = n_3^2 + \frac{p_m^2}{(kt)^2} \quad \text{(Effective index)}$$

$$\left(\frac{p_m}{V}\right)^2 = \frac{N_m^2 - n_3^2}{2 n_3 \Delta n} \approx \frac{N_m - n_3}{\Delta n}$$



Normalized ω-β diagram for planar guides with an exponential profile.

The Ray-Approximation Method

The index distribution of diffused waveguides is written in the form

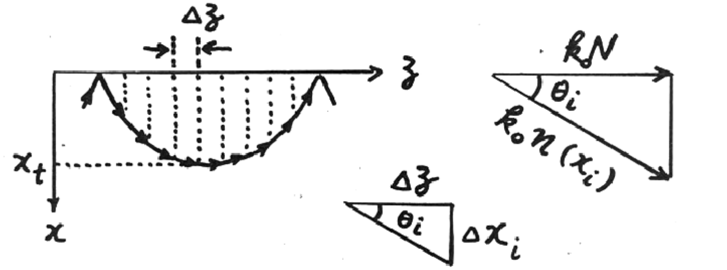
$$n(x) = n_s + \Delta n \, f\left(\frac{x}{d}\right)$$

n_s : substrate index

$f(\frac{x}{d})$: decreases monotonically with x

$$0 \le f\left(\frac{x}{d}\right) \le 1, \quad \infty > x \ge 0$$

d : diffusion depth, at $x=0$, $n(0) = n_f$



$$\theta_i = \cos^{-1}\left(\frac{N}{n(x_i)}\right), \quad \Delta x_i = \Delta z \tan\theta_i$$

At turning point $\theta_i = 0$, (or $n(x_t) = N$)

x_t is the effective guide thickness

The incremental transverse phase shift across Δx_i is

$$\phi_i = k_0 n(x_i) \sin\theta_i \, \Delta x_i = k_0\sqrt{n^2(x_i) - N^2}\, \Delta x_i$$

Phase shift due to TIR

at $x = 0$

$$\varphi_0 = \tan^{-1}\left(\frac{\sqrt{n_f^2 \sin^2\varphi_f - n_c^2}}{n_f \cos\varphi_f}\right) \underset{n_f >> n_c}{\approx} \tan^{-1}(\tan\varphi_f) = \varphi_f$$

$$\approx \frac{\pi}{2} \qquad \begin{pmatrix} n_f \to n_2 \\ n_c \to n_1 \end{pmatrix}$$

at $x = x_t$

$$\varphi_t = \tan^{-1}\left(\frac{\sqrt{n_s^2\sin^2\varphi_s - n_s^2}}{n_s\cos\varphi_s}\right) \underset{n_2=n_s}{=} \tan^{-1}\left(\frac{\sqrt{|\sin^2\varphi_s - 1|}}{\cos\varphi_s}\right)$$

$$\varphi_s \to \frac{\pi}{2}$$

$$= \tan^{-1}\left(\frac{\sqrt{|\cos^2\theta_s - 1|}}{\sin\theta_s}\right) = \tan^{-1} 1 = \frac{\pi}{4}$$

$$\theta_s \to 0$$

For a guided mode

$$\sum_i \phi_i - 2\phi_o - 2\phi_t = 2m\pi$$

$$\sum_i \phi_i \approx 2k_o\int_0^{x_t}\sqrt{n^2(x) - N^2}\,dx$$

$$n^2(x) \approx n_s^2 + (n_f^2 - n_s^2)\,f\left(\frac{x}{d}\right)$$

Normalized diffusion depth

$$V_d = k_o d\sqrt{n_f^2 - n_s^2}$$

Normalized expression

$$2V_d\int_0^{\xi_t}\sqrt{f(\xi) - b}\,d\xi = \left(2m + \frac{3}{2}\right)\pi$$

where

$$\xi \equiv \frac{x}{d}\ ;\ \xi_t \equiv \frac{x_t}{d}\ ;\ b = f(\xi_t)$$

Example : Ti-diffused $LiNbO_3$ waveguide

$$f(\xi) = e^{-\xi^2}$$

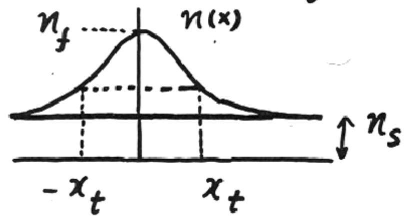
At cutoff $b = 0 \Rightarrow \xi_t = \infty$

$$\because \int_0^{\infty}\sqrt{e^{-\xi^2}}\,d\xi = \sqrt{\frac{\pi}{2}}$$

$$\therefore V_{dm} = \sqrt{2\pi}\left(m + \frac{3}{4}\right)$$

Strickly speaking, the phase shift $2\phi_o$ and $2\phi_t$ depend on the substrate birefringence.

For symmetric waveguide



$2\phi_o$ is replaced by $2\phi_t$

$$2\left(\sum_i \phi_i - 2\phi_t\right) = 2m\pi$$

The normalized expression becomes

$$2V_d\int_0^{\xi_t}\sqrt{f(\xi) - b}\,d\xi = \left(m + \frac{1}{2}\right)\pi$$

When $f(\xi) = e^{-\xi^2}$ (Gaussian profile)

$$V_{dm} = \sqrt{\frac{\pi}{2}}\left(m + \frac{1}{2}\right)$$

Summary

- Ray-approximation is applicabe for arbitray index distribution.
- Does not provide electromagnetic field distribution.

The WKB Method

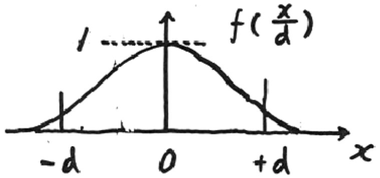
TE wave equation

$$\frac{d^2 E_y}{dx^2} + (k^2 n^2(x) - \beta^2) E_y = 0$$

$$\varphi \equiv k^2(n_f^2 - N^2), \quad N \equiv \frac{\beta}{k}$$

$$U(x) \equiv k^2(n_f^2 - n^2(x))$$

$$n(x) = n_s + (n_f - n_s) f\left(\frac{x}{d}\right)$$



$$\frac{d^2 E_y}{dx^2} + \underbrace{(\varphi - U(x))}_{k^2(x)} E_y = 0 \quad \text{(Schrödiger Eq.)}$$

The WKB method can provide approximate solutions for the wave equation as long as the index change is slow compared with the optical wavelength. Mathematically

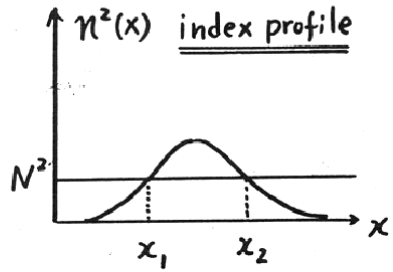
$$\lambda(x)\left|\frac{dk(x)}{dx}\right| \ll |k(x)|$$

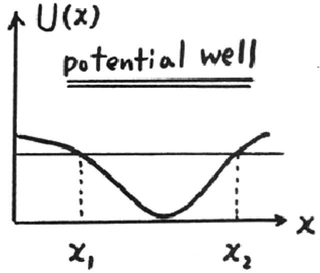
or

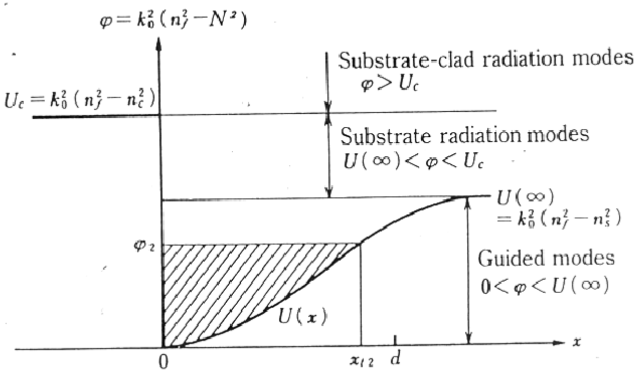
$$|k'(x)| \ll |k^2(x)|$$

turning points.

$$n(x_i) = N = \frac{\beta}{k} \; ; \quad i = 1, 2$$







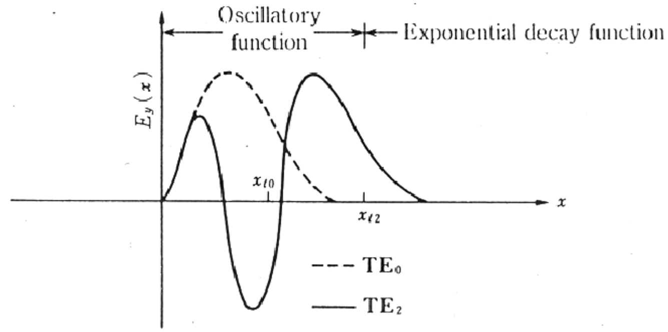
Electron-energy model and mode classification

$$
\begin{cases}
E_y = A\exp(\gamma_c x), \; x < 0 \\
E_y = B\dfrac{1}{\sqrt{k(x)}}\cos\left(\dfrac{\pi}{4} - K(x)\right), \; 0 < x < x_t \\
E_y = \dfrac{B}{2}\sqrt{\dfrac{2\pi K(x)}{3k(x)}}\,\{J_{1/3}(K(x)) + J_{-1/3}(K(x))\}, \; x \lesssim x_t \\
E_y = \dfrac{B}{2}\sqrt{\dfrac{2\pi \Gamma_s(x)}{3\gamma_s(x)}}\,\{I_{1/3}(\Gamma_s(x)) + I_{-1/3}(\Gamma_s(x))\}, \; x \gtrsim x_t \\
E_y = \dfrac{B}{2}\dfrac{1}{\sqrt{\gamma_s(x)}}\exp(-\Gamma_s(x)), \; x > x_t
\end{cases}
$$

where

$$k(x) = \sqrt{k_0^2 n^2(x) - \beta^2}, \quad \gamma_s(x) = \sqrt{\beta^2 - k_0^2 n_s^2(x)}$$

$$K(x) = \int_x^{x_t} k(x)\,dx, \quad \Gamma_s(x) = \int_{x_t}^{x} \gamma_s(x)\,dx$$



Electric field distributions of guided modes

The propagation constant β_m

$$\int_{x_1}^{x_2} \sqrt{k_0^2 n^2 - \beta_m^2} = \left(m + \tfrac{1}{2}\right)\pi$$

where x_1 and x_2 are turning points

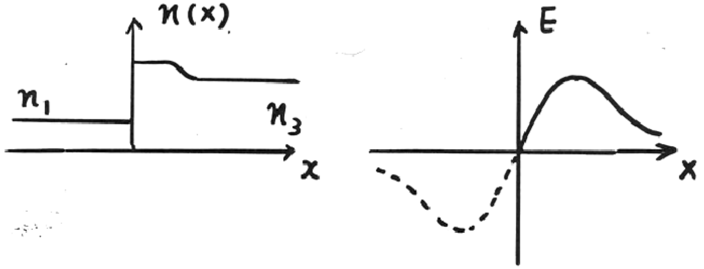
$$n(x_i) = N = \frac{\beta}{k_0}, \quad i = 1, 2$$

In terms of the effective index

$$\int_{x_1}^{x_2} \sqrt{n^2 - N^2} = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{2}$$

For the special case of the parobolic profile, the WKB predictions are known to agree exactly with the closed form solutions.

Index Profile with Strong Asymmetry



Strongly asymmetric

$$n_3 >> n_1$$

Example :

$$n_3 = LiNbO_3 = 2.214$$

$$n_1 = air = 1$$

To obtain the first order approximation

$$\begin{cases} E = 0, & x < 0 \\ E = \text{symmetric-profile modal field}, & x > 0 \end{cases}$$

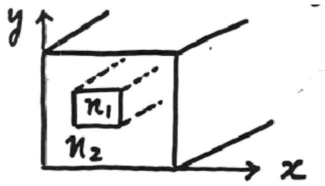
Simple relation

$$2 m_a + 1 = m_s$$

3.2 Rectangular Waveguide

- 1-dim confinement
  Spectrum analyzer
- 2-dim confinement
  Can yield
  - A laser with reduced threshold current and single mode oscillation.
  - An electrooptic modulator with reduced drive power requirement.
  - Guide light from one point on the surface of an OIC to another.
  - Interconnect two circuit elements in a manner analogous to that of the metallic stripes used in an electrical integrated circuit.

3.2.1 Channel Waveguide



$\begin{cases} n_1 > n_2 = \text{constant} \\ n_1 > n_2(x, y) \ \text{inhomogeneous} \end{cases}$

Often called
- channel waveguides
- strip "
- 3-D "

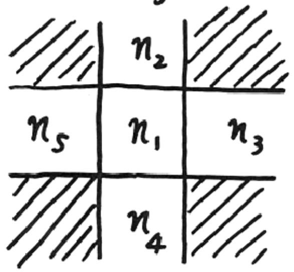
Review of Previous work

- Goell Uses cylindrical space harmonics to analyze guides with aspect ratios

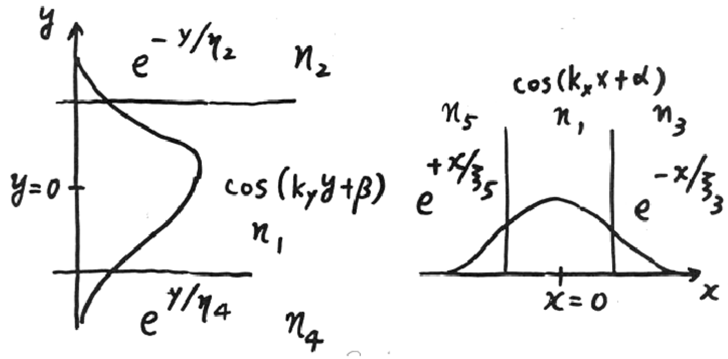
$$1 \leq \frac{\text{Width}}{\text{Height}} \leq 2$$

- Schlosser and Unger
  Use rectangular harmonics and numerical methods for large aspect ratio.

- Marcatili (approximate solution)
  Assume modes are well guided (well above cutoff)
  - field decays exponentially in regions 2 ~ 5 with most of the power being confined to region 1.



- The magnitudes of the fields in the shaded corner regions are small enough to be neglected.



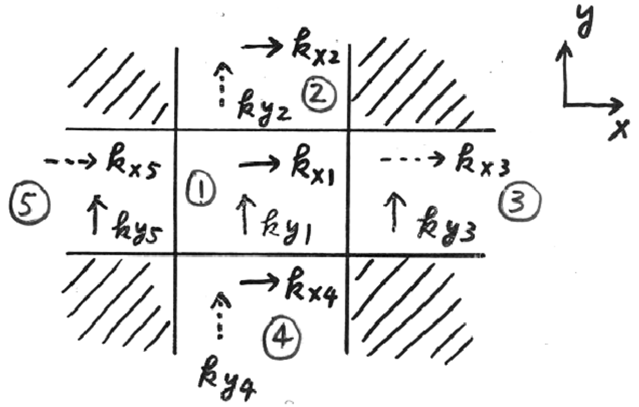
Modes

$$\begin{cases} E^x_{pq} \text{ modes} & E_x,\ H_y \\ E^y_{pq} \text{ modes} & E_y,\ H_x \end{cases}$$

Consider E^y_{11} mode

$$H_{x\nu} = e^{-ik_z z + i\omega t} \cdot \begin{cases} M_1 \cos(k_x x + \alpha)\cos(k_y y + \beta) \\ M_2 \cos(k_x x + \alpha)\, e^{-ik_{y2} y} \\ M_3 \cos(k_y y + \beta)\, e^{-ik_{x3} x} \\ M_4 \cos(k_x x + \alpha)\, e^{+ik_{y4} y} \\ M_5 \cos(k_y y + \beta)\, e^{+ik_{x5} x} \end{cases}$$

where $\nu = 1, 2, 3, 4$, and 5 for each region.



Wave guide theory

$$\vec{\nabla} \times \vec{E} = -j\omega\mu \vec{H}$$

$$\vec{\nabla} \times \vec{H} = j\omega\epsilon \vec{E}$$

Assume the propagation factor $e^{j(\omega t - k_z z)}$

(Note : $k_z = \beta$)

$$\frac{\partial E_z}{\partial y} + k_z E_y = -j\omega\mu H_x \quad (1) \qquad \frac{\partial H_z}{\partial y} + k_z H_y = j\omega\epsilon E_x \quad (4)$$

$$-k_z E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \quad (2) \qquad -k_z H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y \quad (5)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad (3) \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \quad (6)$$

Usually, H_x, H_y, E_x, and E_y are solved in terms of E_z and H_z as given by

$$\left.\begin{aligned} H_x &= \frac{1}{k^2 - k_z^2}\left(j\omega\epsilon \frac{\partial E_z}{\partial y} - jk_z \frac{\partial H_z}{\partial x}\right) \\ H_y &= -\frac{1}{k^2 - k_z^2}\left(j\omega\epsilon \frac{\partial E_z}{\partial x} + jk_z \frac{\partial H_x}{\partial y}\right) \\ E_x &= -\frac{1}{k^2 - k_z^2}\left(jk_z \frac{\partial E_z}{\partial x} + j\omega\mu \frac{\partial H_z}{\partial y}\right) \\ E_y &= \frac{1}{k^2 - k_z^2}\left(-jk_z \frac{\partial E_z}{\partial y} + j\omega\mu \frac{\partial H_z}{\partial x}\right) \end{aligned}\right\} \begin{matrix} \text{TE} \\ (E_z = 0) \\ \text{TM} \\ (H_z = 0) \end{matrix}$$

Usual

- Axial field components are solved.
- Transverse components are later derived from them.

New Feature

- Solve directly for the transverse components.

— Power is carried by the transverse field components. The axial fieds are negligible.

— The method involves solution of the full-wave equation, regions where ϵ varies continuously are solved without special treatment.

Consider E^y_{pq} modes, E_y and H_x ; $H_y = 0$

(6) → (3.2.6) $$E_{z\nu} = \frac{j}{\omega\epsilon}\frac{\partial H_{x\nu}}{\partial y}$$

$$= \frac{j}{\omega\epsilon_0 n_\nu^2}\frac{\partial H_{x\nu}}{\partial y}$$

(2) → (3.2.4)

$$E_{x\nu} = -\frac{1}{k_z}\frac{\partial E_{z\nu}}{\partial x} = -\frac{j}{\omega\epsilon_0 n_\nu^2 k_z}\frac{\partial^2 H_{x\nu}}{\partial x\,\partial y}$$

(1) → (3.2.5)

$$E_{y\nu} = \frac{1}{k_z}\left[-j\omega\mu H_x - \frac{\partial E_z}{\partial y}\right] \qquad (3.2.4)$$

$$= \frac{1}{k_z}\left[-j\omega\mu H_x - \frac{j}{\omega\epsilon_0 n_\nu^2}\frac{\partial^2 H_{x\nu}}{\partial y^2}\right]$$

$$= \frac{-j\left[\omega^2\mu\epsilon_0 n_\nu^2 - k_{y\nu}^2\right]}{\omega\epsilon_0 n_\nu^2 k_z} H_{x\nu}$$

$$= \frac{-j\left[k^2 n_\nu^2 - k_{y\nu}^2\right]}{\omega\epsilon_0 n_\nu^2 k_z} H_{x\nu}$$

(4) → $$\frac{\partial H_z}{\partial y} = j\omega\epsilon E_x$$

Assume

$$\frac{\partial}{\partial y} \to -jk_y$$

$$\Rightarrow \quad -jk_y H_z = j\omega\epsilon E_x$$

For (3.2.3)

$$\because \vec{\nabla}\cdot\vec{H} = 0 \Rightarrow \frac{\partial H_x}{\partial x} + \underbrace{\frac{\partial H_y}{\partial y}}_{0} + \underbrace{\frac{\partial H_z}{\partial z}}_{-jk_z H_z} = 0$$

$$\therefore \quad H_z = -\frac{j}{k_z}\frac{\partial H_x}{\partial x}$$

Mathing the boundary conditions requires the assumption that

$$k_{x_1} = k_{x_2} = k_{x_4} = k_x$$

and

$$k_{y_1} = k_{y_3} = k_{y_5} = k_y$$

Also, $k_x^2 + k_y^2 + k_z^2 = k_1^2$

$$k_1 = k n_1 = \frac{2\pi}{\lambda_0} n_1$$

free space wavelength

Assuming

$$0 < \frac{n_1 - n_\nu}{n_1} \ll 1, \quad \nu = 2, 3, 4, 5$$

(Usual case in an OIC)

leads to the conditions

$$k_x \ll k_z \quad \text{and} \quad k_y \ll k_z$$

or $\lambda_x \gg \lambda_z$ and $\lambda_y \gg \lambda_z$

From (3.1.7)

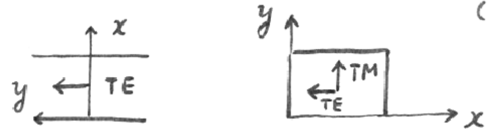
$$\tan(h t_g) = \frac{p+q}{h\left(1 - \frac{pq}{h^2}\right)}$$

$$p = \frac{1}{\xi_3}, \quad q = \frac{1}{\xi_5}, \quad h = k_x, \quad t_g = a$$

$$\therefore \tan(ha) = -\frac{k_x \xi_3 + k_x \xi_5}{1 - k_x^2 \xi_3 \xi_5}$$

$$= \tan\left\{-\left[(\tan^{-1} k_x \xi_3) + (\tan^{-1} k_x \xi_5)\right]\right\}$$

$$k_x a = p\pi - \tan^{-1} k_x \xi_3 - \tan^{-1} k_x \xi_5 \quad \text{(TE)} \quad (3.2.12)$$

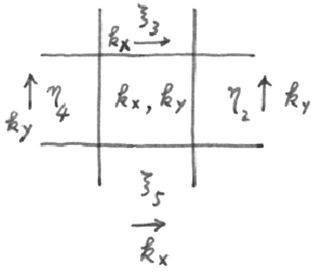


Similarly,

$$k_y b = q\pi - \tan^{-1} \frac{n_2^2}{n_1^2} k_y \eta_2 - \tan^{-1} \frac{n_4^2}{n_2^2} k_y \eta_4 \quad \text{(TM)} \quad (3.2.13)$$

$$\xi_3 = \frac{1}{q} = \frac{1}{\sqrt{k_z^2 - k^2 n_3^2}} = \frac{1}{\sqrt{k_1^2 - k_x^2 - \underset{\substack{\uparrow \\ \text{zero}}}{k_y^2} - \underbrace{k^2 n_3^2}_{k_3^2}}}$$

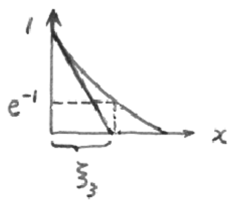
$$= \frac{1}{\sqrt{k_1^2 - k_x^2 - k_3^2}} = \frac{1}{\sqrt{\left(\frac{\pi}{A_3}\right)^2 - k_x^2}}$$



$$A_3 \equiv \frac{\pi}{\sqrt{k_1^2 - k_3^2}}$$

$$= \frac{\lambda_0}{2\sqrt{n_1^2 - n_3^2}}$$

= Characteristic depth



Similarly

$$\xi_5 = \frac{1}{p} = \frac{1}{\sqrt{k_z^2 - k^2 n_5^2}} = \frac{1}{\sqrt{\left(\frac{\pi}{A_5}\right)^2 - k_x^2}}$$

$$\eta_2 = \frac{1}{q} = \frac{1}{\sqrt{k_z^2 - k^2 n_2^2}} = \frac{1}{\sqrt{k_1^2 - \underset{\substack{\uparrow \\ \text{zero}}}{k_x^2} - k_y^2 - \underbrace{k^2 n_2^2}_{k_2^2}}}$$

$$= \frac{1}{\sqrt{k_1^2 - k_y^2 - k_2^2}} = \frac{1}{\sqrt{\left(\frac{\pi}{A_2}\right)^2 - k_y^2}}$$

$$A_2 = \frac{\pi}{\sqrt{k_1^2 - k_2^2}}$$

$$\eta_4 = \frac{1}{p} = \frac{1}{\sqrt{\left(\frac{\pi}{A_4}\right)^2 - k_y^2}}, \quad A_4 = \frac{1}{\sqrt{k_1^2 - k_4^2}}$$

The transcendental Eqs. (3.2.12) and (3.2.13) cannot be solved exactly in closed form.
Assume for well confined modes that most of the powder is in Region I.

$\because$ small $\xi_3 \Rightarrow$ good confinement

$$\because \ \xi_3 = \frac{A_3}{\sqrt{\pi^2 - (A_3^2 k_x^2)}}, \quad \pi^2 \gg A_3^2 k_x^2$$

Similarly $\pi^2 \gg k_x^2 A_5^2$, $k_y^2 A_2^2$, and $k_y^2 A_4^2$

$$\xi_3 = \frac{A_3}{\pi}\left(1-\frac{A_3^2 k_x^2}{\pi^2}\right)^{-\frac{1}{2}} \approx \frac{A_3}{\pi}$$

$$\xi_5 \approx \frac{A_5}{\pi}$$

Note that $\tan^{-1}x \approx x$ if x is small

(3.2.12) gives

$$k_x a \approx p\pi - k_x \xi_3 - k_x \xi_5 \approx p\pi - k_x\left(\frac{A_3+A_5}{\pi}\right)$$

$$\therefore k_x \approx \frac{p\pi}{a}\Big/\left(1+\frac{A_3+A_5}{\pi a}\right) \qquad (3.2.18)$$

Similarly

$$k_y \approx \frac{q\pi}{b}\Big/\left(\frac{1+n_2^2 A_2+n_4^2 A_4}{\pi n_1^2 b}\right) \qquad (3.2.19)$$

Once k_x and k_y are solved, k_z , ξ_3, ξ_5, η_2, and η_4 can be obtained upon substitution.

$$k_z = \left[k_1^2 - \left(\frac{p\pi}{a}\right)^2\left(1+\frac{A_3+A_5}{\pi a}\right)^{-2} - \left(\frac{q\pi}{b}\right)^2\left(1+\frac{n_2^2 A_2+n_4^2 A_4}{\pi n_1^2 b}\right)^{-2}\right]^{1/2} \qquad (3.2.20)$$

$$\xi_{\substack{3\\5}} = \frac{A_{\substack{3\\5}}}{\pi}\left[1-\left(\frac{p A_{\substack{3\\5}}}{a}\,\frac{1}{1+\frac{A_3+A_5}{\pi a}}\right)^2\right]^{-\frac{1}{2}}$$

$$\eta_{\substack{2\\4}} = \frac{A_{\substack{2\\4}}}{\pi}\left[1-\left(\frac{q A_{\substack{2\\4}}}{b}\,\frac{1}{1+\frac{n_2^2 A_2+n_4^2 A_4}{\pi n_1^2 b}}\right)^2\right]^{-\frac{1}{2}}$$

For E^y_{pq} modes

E_y : the only significant component of electric field

$\left.\begin{matrix} E_x \\ E_z \end{matrix}\right\}$ negligibly small

For E^x_{pq} modes

E_x : the only significant component of electric field

$\left.\begin{matrix} E_y \\ E_z \end{matrix}\right\}$ negligibly small

To develop relationships for the E_{pq}^{x} modes

Change $E \leftrightarrow H$

$\mu_0 \leftrightarrow -\epsilon_0$

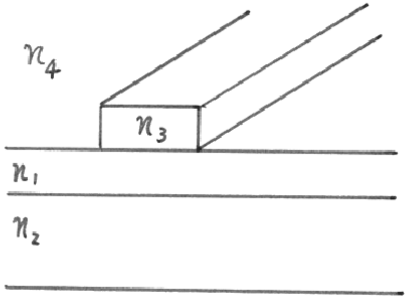
As long as

$$\frac{n_1 - n_\nu}{n_1} \ll 1, \quad \nu = 2, 3, 4, \text{ or } 5$$

k_z, ξ_3, ξ_5, η_2, η_4 are the same as those of E_{pq}^{x}

3.2.2 Strip-Loaded Waveguide

A special case of rectangular waveguide.

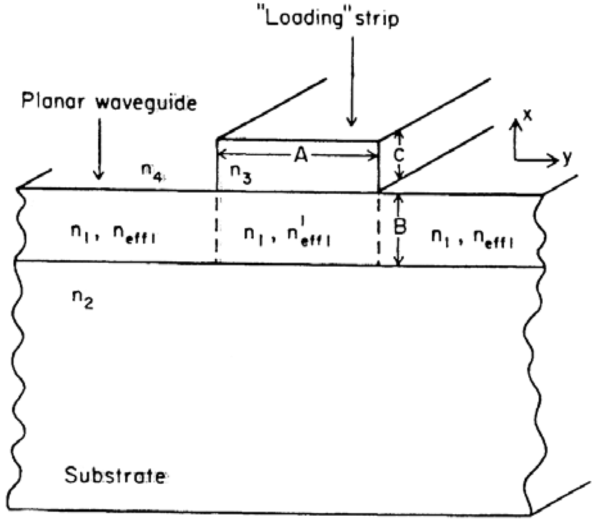


$n_1 > n_2 \geq n_3 > n_4$

$n_1 > n_3 \geq n_2 > n_4$

"strip-loaded waveguide" or "optical stripline"

Fig. 3.6.

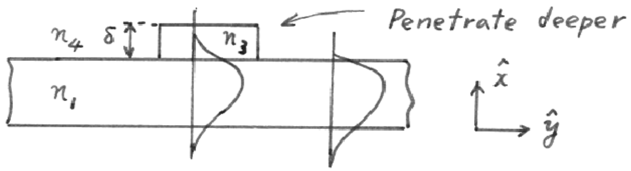


$n_1 > n_2 \geq n_3 > n_4$

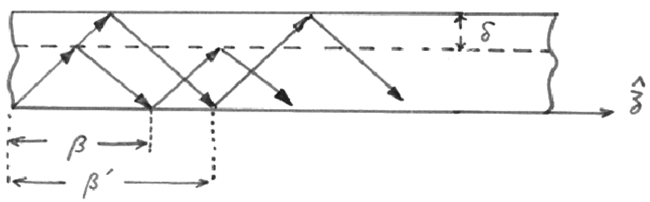
or $n_1 > n_3 \geq n_2 > n_4$

Vertical confinement $\quad n_1 > n_2, n_3$

Horizontal confinement $\quad n_3 > n_4$



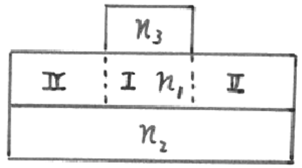
Side View



$$n_{eff1} = \frac{\beta}{k}$$

$$n'_{eff1} = \frac{\beta'}{k} > \frac{\beta}{k} > n_{eff1}$$

Ray optics approximation



Regions I and II have thickness ⇒ same h

$$\underbrace{k^2 n_1^2 - \beta'^2}_{\mathrm{I}} = h^2 = \underbrace{k^2 N_{eq}^2 - \beta^2}_{\mathrm{II}}$$

or

$$k^2 n_1^2 - k^2 n_{eff_1}'^2 = k^2 N_{eq}^2 - k^2 n_{eff_1}^2$$

$$\therefore \; N_{eq} = \sqrt{n_1^2 - n_{eff_1}'^2 + n_{eff}^2}$$

Marcatili's model

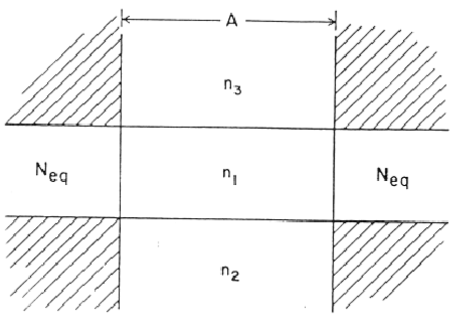
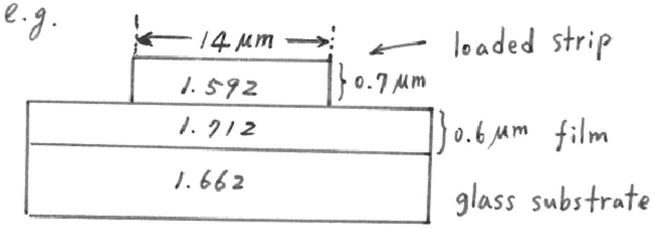


Fig. 3.7. Cross-sectional view of rectangular dielectric waveguide equivalent to the strip-loaded waveguide of Fig. 3.6

e.g.

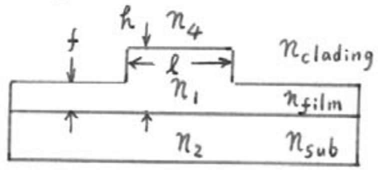


$\lambda = 0.6328\,\mu m$

Good agreement was found

- strip width ≈ 22λ
- strip thickness ≈ 1.1λ
- film thickness ≈ 0.95λ

Ridge Waveguides or Rib Waveguides



$n_1 = n_3$

$\lambda = 0.8\,\mu m$

Ti-indiffused waveguides in $LiNbO_3$

$n_f = 2.234$

$n_s = 2.214$

$n_c = 1$

$h = 1.8\,\mu m$

$f = 1\,\mu m$

$\ell = 2\,\mu m$

Normalized film thickness

$$V_h = kh\sqrt{n_f^2 - n_s^2} = 4.2$$
$$V_f = kf\sqrt{n_f^2 - n_s^2} = 2.3$$

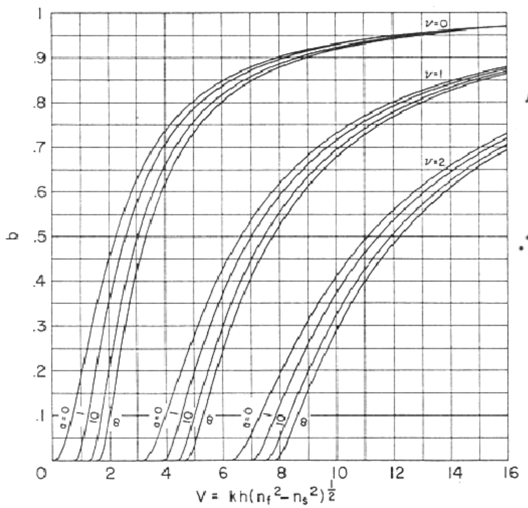
Asymmetry

$$a_E = \frac{n_s^2 - n_c^2}{n_f^2 - n_s^2} = 43.9$$

Normalized guide indices (TE-modes)

$$b_h = 0.65$$
$$b_f = 0.2$$

b-V curves of a planar slab waveguide

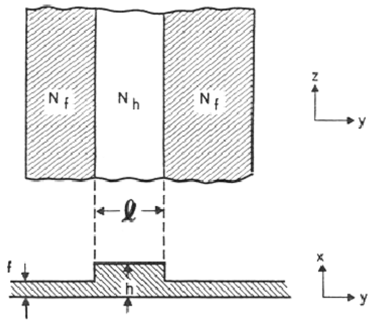


Note that

$$b_h = (N_h^2 - n_s^2)/(n_f^2 - n_s^2)$$
$$b_f = (N_f^2 - n_s^2)/(n_f^2 - n_s^2)$$

$\therefore$

$$N_h = 2.227$$
$$N_f = 2.218$$



Symm.

Asymm.

X-direction Asymmetric slab waveguide

Y-direction Symmetric " "

$$V_y = kl\sqrt{N_h^2 - N_f^2} = kl\sqrt{(n_f^2 - n_s^2)(b_h - b_f)} = 3.14$$

The corresponding $b = 0.64$

The effective index $N^2 = N_f^2 + b(N_h^2 - N_f^2)$

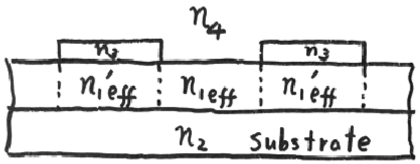
$$= 2.224$$

The number of guided modes

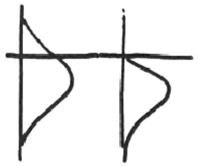
$$\nu_y = \frac{2l}{\lambda}\sqrt{N_h^2 - N_f^2} = 0.4\,\frac{l}{\lambda} = 1$$

Single mode!

Metallic-Strip Loaded Waveguides



$n'_{1eff} < n_{eff}$



Confinement poor good

For electrooptic modulators

Comparison

strip loaded waveguide v.s. burried-in waveguide

optical loss <

(Scattering due to side wall roughness is reduced.

e.g.

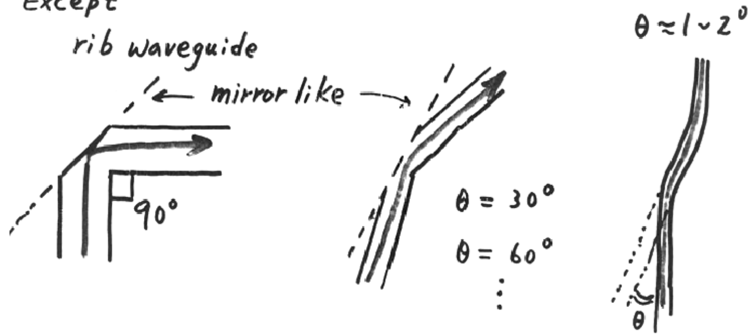
Blum	$\alpha = 1\ cm^{-1}$	GaAs strip loaded waveguide
Reinhart	$\alpha < 2\ cm^{-1}$	GaAs - GaAlAs rib waveguide

Limitations

Small effective index difference in the lateral direction is insufficient to limit radiation loss occuring from bends in waveguides.

Except

rib waveguide



Large angle bending is possible

No observation of excess radiation loss

Beam Propagation Method (BPM)

– Uniform in the propagation direction

Mode solver

– Nonuniform in the propagation direction

– Wave equation

$$\nabla^2 \psi = \frac{n^2}{c^2} \frac{\partial^2 \psi}{\partial t^2} \qquad (1)$$

ψ : electric/magnetic field

Assume $e^{-i\omega t}$ dependence

$$\nabla^2 \psi + k^2 n^2 \psi = 0 \qquad (2)$$

Assume

$$\psi(x, y, z) = \phi(x, y, z)\, e^{-j\beta z}$$

propagating beam

No $e^{+j\beta z}$ component

$\beta = n_{eff} k_0$

$\llcorner$ reference index, $\approx n_s$ or n_c

$$\frac{\partial^2 \psi}{\partial z^2} = \frac{\partial^2 \phi}{\partial z^2} e^{-j\beta z} - 2j\beta \frac{\partial \phi}{\partial z} e^{-j\beta z} - \beta^2 \phi e^{-j\beta z} \qquad (3)$$

Dividing by $e^{-j\beta z} \neq 0$, (2) becomes

$$\frac{\partial^2 \phi}{\partial z^2} - 2j\beta \frac{\partial \phi}{\partial z} + \underbrace{\nabla_\perp^2 \phi}_{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}} + \underbrace{(k_0^2 n^2 - \beta^2)}_{k_0^2 (n^2 - n_{eff}^2)} \phi = 0 \qquad (4)$$

(Wide-angle BPM)

Assume slowly varying along z, $\frac{\partial^2 \phi}{\partial z^2} \approx 0$, (4) becomes

$$2j\beta \frac{\partial \phi}{\partial z} = \nabla_\perp^2 \phi + k_0^2 (n^2 - n_{eff}^2) \phi$$

(Para-axial or Fresnel approximation)

Numerical Methods

(Ref. K. Kawano & T. Kitoh, Introduction to Optical Waveguide Analysis, Wiley, 2001)