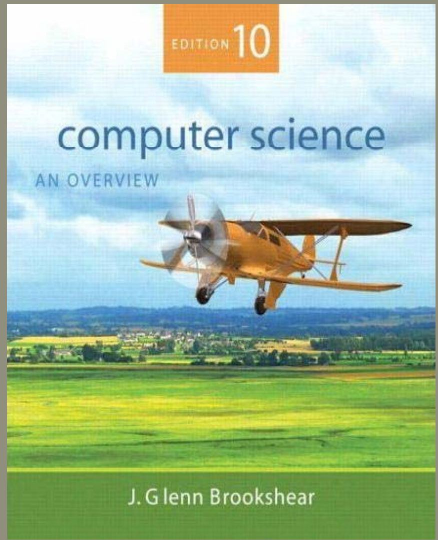
Chapter 12

Theory of Computation



Chapter 12: Theory of Computation

- 12.1 Functions and Their Computation
- 12.2 Turing Machines
- 12.3 Universal Programming Languages
- 12.4 A Noncomputable Function
- 12.5 Complexity of Problems
- 12.6 Public-Key Cryptography

Functions

- **Function:** A correspondence between a collection of possible input values and a collection of possible output values so that each possible input is assigned a single output

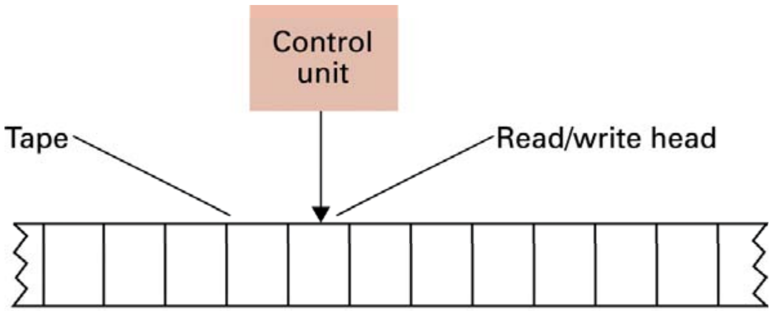
Functions (continued)

- **Computing a function**: Determining the output value associated with a given set of input values
- **Noncomputable function**: A function that cannot be computed by any algorithm

Figure 12.1 An attempt to display the function that converts measurements in yards into meters

Yards (input)	Meters (output)
1	0.9144
2	1.8288
3	2.7432
4	3.6576
5	4.5720
.	.
.	.
.	.

Figure 12.2 The components of a Turing machine



Turing Machine Operation

- Inputs at each step
  - State
  - Value at current tape position
- Actions at each step
  - Write a value at current tape position
  - Move read/write head
  - Change state

Figure 12.3 A Turing machine for incrementing a value

Current state	Current cell content	Value to write	Direction to move	New state to enter
START	*	*	Left	ADD
ADD	0	1	Right	RETURN
ADD	1	0	Left	CARRY
ADD	*	*	Right	HALT
CARRY	0	1	Right	RETURN
CARRY	1	0	Left	CARRY
CARRY	*	1	Left	OVERFLOW
OVERFLOW	*	*	Right	RETURN
RETURN	0	0	Right	RETURN
RETURN	1	1	Right	RETURN
RETURN	*	*	No move	HALT

Church-Turing Thesis

The functions that are computable by a Turing machine are exactly the functions that can be computed by any algorithmic means.

Universal Programming Language

A language with which a solution to any computable function can be expressed

- Examples: "Bare Bones" and most popular programming languages

The Bare Bones Language

- Bare Bones is a simple, yet universal language.
- Statements
  - **clear** ***name*****;**
  - **incr** ***name*****;**
  - **decr** ***name*****;**
  - **while** ***name*** **not 0 do; … end;**

Figure 12.4 A Bare Bones program for computing X x Y

```
clear Z;
while X not 0 do;
   clear W;
   while Y not 0 do;
      incr Z;
      incr W;
      decr Y;
   end;
   while W not 0 do;
      incr Y;
      decr W;
   end;
   decr X;
end;
```

Figure 12.5 "copy Today to Tomorrow" in Bare Bones

```
clear Aux;
clear Tomorrow;
while Today not 0 do;
   incr Aux;
   decr Today;
end;
while Aux not 0 do;
   incr Today;
   incr Tomorrow;
   decr Aux;
end;
```


The Halting Problem

- Given the encoded version of any program, return 1 if the program is self-terminating, or 0 if the program is not.

Figure 12.6 Testing a program for self-termination

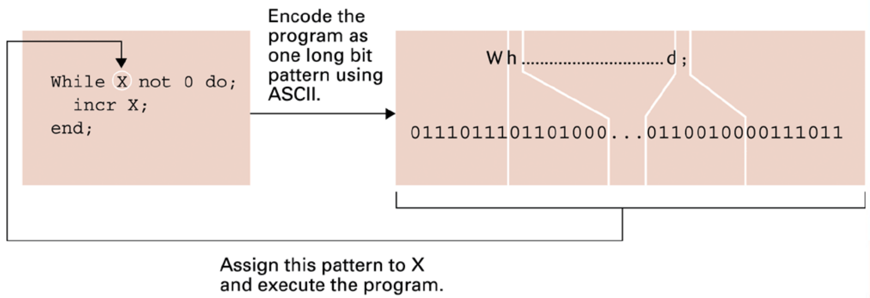
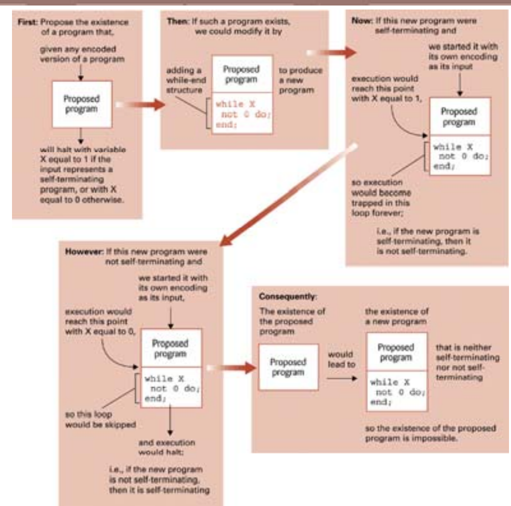


Figure 12.7 Proving the unsolvability of the halting program



Complexity of Problems

- **Time Complexity:** The number of instruction executions required
  - Unless otherwise noted, “complexity” means “time complexity.”
- A problem is in class O(f(n)) if it can be solved by an algorithm in Θ(f(n)).
- A problem is in class Θ(f(n)) if the best algorithm to solve it is in class Θ(f(n)).

Figure 12.8 A procedure MergeLists for merging two lists

```
procedure MergeLists (InputListA, InputListB, OutputList)

if (both input lists are empty) then (Stop, with OutputList empty)
if (InputListA is empty)
    then (Declare it to be exhausted)
    else (Declare its first entry to be its current entry)
if (InputListB is empty)
    then (Declare it to be exhausted)
    else (Declare its first entry to be its current entry)
while (neither input list is exhausted) do
        (Put the "smaller" current entry in OutputList;
         if (that current entry is the last entry in its corresponding input list)
             then (Declare that input list to be exhausted)
             else (Declare the next entry in that input list to be the list's current entry )
        )
Starting with the current entry in the input list that is not exhausted,
        copy the remaining entries to OutputList.
```


Figure 12.9 The merge sort algorithm implemented as a procedure MergeSort

```
procedure MergeSort (List)

if (List has more than one entry)
  then (Apply the procedure MergeSort to sort the first half of List;
        Apply the procedure MergeSort to sort the second half of List;
        Apply the procedure MergeLists to merge the first and second
            halves of List to produce a sorted version of List
       )
```


Figure 12.10 The hierarchy of problems generated by the merge sort algorithm

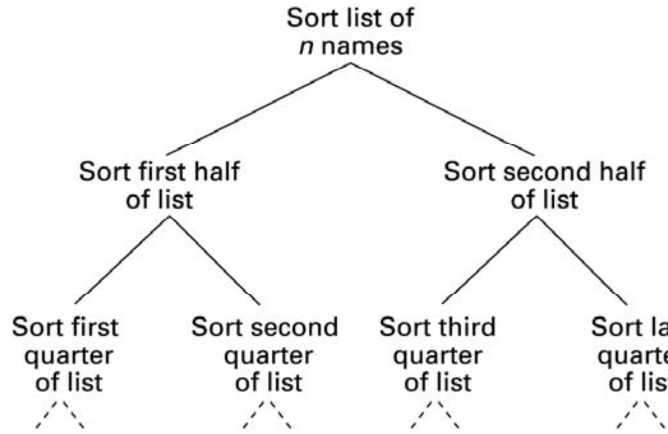
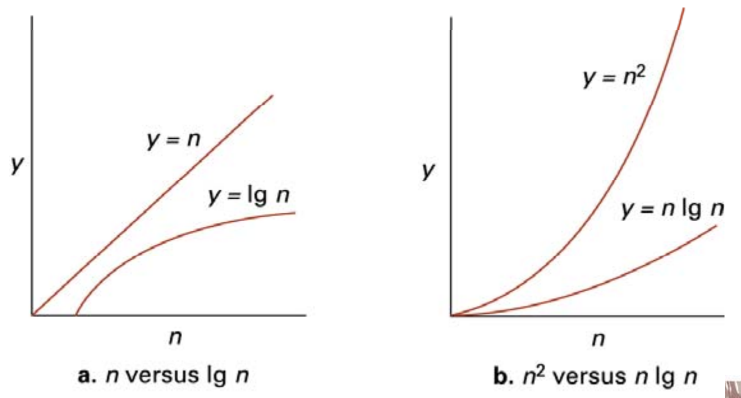


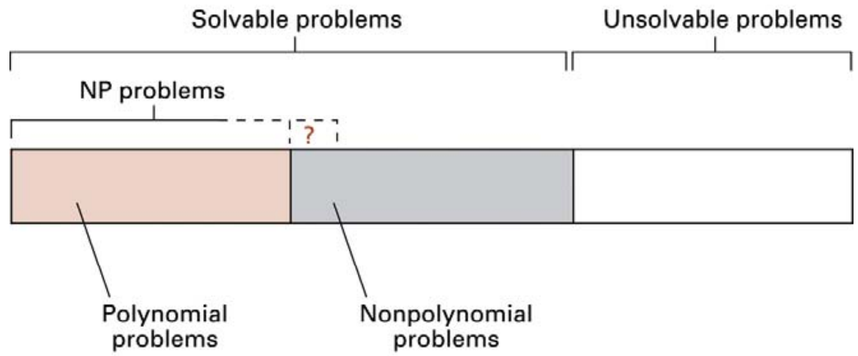
Figure 12.11 Graphs of the mathematical expression n, lg, n, n lg n, and n^2



P versus NP

- **Class P:** All problems in any class Θ(f(n)), where f(n) is a polynomial
- **Class NP:** All problems that can be solved by a nondeterministic algorithm in polynomial time
  - **Nondeterministic algorithm** = an "algorithm" whose steps may not be uniquely and completely determined by the process state
- Whether the class NP is bigger than class P is currently unknown.

Figure 12.12 A graphic summation of the problem classification



Public-Key Cryptography

- **Key:** A value used to encrypt or decrypt a message
  - **Public key**: Used to encrypt messages
  - **Private key**: Used to decrypt messages
- **RSA:** A popular public key cryptographic algorithm
  - Relies on the (presumed) intractability of the problem of factoring large numbers

Encrypting the Message 10111

- Encrypting keys: n = 91 and e = 5
- $10111_{two} = 23_{ten}$
- $23^e = 23^5 = 6{,}436{,}343$
- $6{,}436{,}343 \div 91$ has a remainder of 4
- $4_{ten} = 100_{two}$
- Therefore, encrypted version of 10111 is 100.

Decrypting the Message 100

- Decrypting keys: d = 29, n = 91
- $100_{two} = 4_{ten}$
- $4^d = 4^{29} = 288{,}230{,}376{,}151{,}711{,}744$
- $288{,}230{,}376{,}151{,}711{,}744 \div 91$ has a remainder of 23
- $23_{ten} = 10111_{two}$
- Therefore, decrypted version of 100 is 10111.

Figure 12.13 Public key cryptography

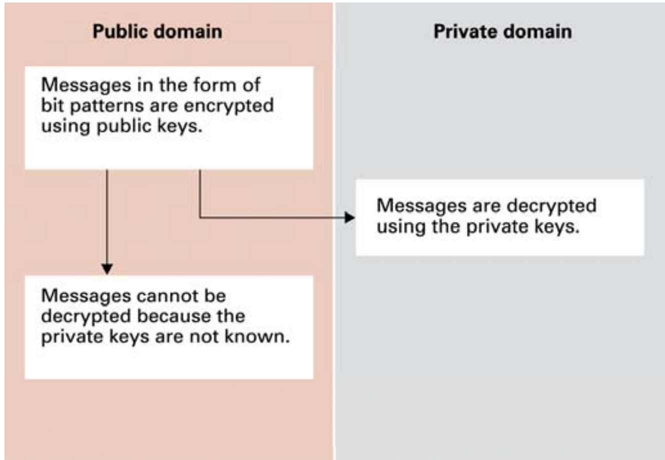


Figure 12.14 Establishing a RSA public key encryption system