# The Role of Store Brand Positioning for Appropriating Supply Chain Profit under Shelf Space Allocation

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**Abstract**

We consider a retailer's decision of developing a store brand (SB) version of a national brand (NB) and the role that its positioning strategy plays in appropriating the supply chain profit. Since the business of the retailer can be regarded as selling to NB manufacturers the shelf space at its disposal, we formulate a game-theoretical model of a single-retailer, single-manufacturer supply chain, where the retailer can decide whether to launch its own SB product and sells scarce shelf-space to a competing NB in a consumer good category. As a result, the most likely equilibrium outcome is that the available selling amount of each brand is constrained by the shelf-space available for its products and both brands coexist in the category. In this paper, we conceptualize the SB positioning that involves both product quality and product features. Our analysis shows that when the NB cross-price effect is not too large, the retailer should position its SB's quality closer to the NB, more emphasize its SB's differences in features facing a weaker NB, and less emphasize its SB's differences in features facing a stronger NB. Our results stress the importance of SB positioning under the shelf-space allocation, in order to maximize the retailer's value appropriation across the supply chain.

*Keywords:* Private label positioning; shelf space management; supply chain management; marketing-operations interface; competitive strategy; game theory

## 1 Introduction

Large wholesalers or retailers including Costco, Kroger, and Wal-Mart have introduced store brand (i.e., private label) products in a large number of product categories. Since 2007, store brands have grown at a rate of 5% annually and the dollar sales for the store brands have reached to 92.7 billion across all the channels in 2011, see Private Label Manufacturer Association (2012) for

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details. Generally speaking, there are at least two different forms of competition for a consumer good category in a retail supply chain involving store brands:

1. Space competition: Shelf-space scarcity intensifies brand competition. Shelf-space size and allocation may result in competitive exclusion, where the retailer takes advantage of its power and harms the NB producer.

2. Position competition: Quality and feature differences between the store brand (SB) and national brand (NB), allowing price discrimination between consumers who are willing to pay more for a NB product that is advertised and consumers who are not.

From an operations perspective, the quantity that each brand can be placed on the retailer's shelf-space during the selling period does lead to another form of competition. Shelf-space is one of the retailer's most important assets. Obviously, it is a limited resource (see, e.g., Zhou et al., 2012). Due to spatial consideration of the retailer's shelf-space, the retailer needs to forecast the expected sales of the product category and decide the total shelf-space available on the basis of category goals at the beginning of the selling period (Kurtuluş and Toktay, 2011). The total shelf-space decision then forms as an upper limit for the category product selling amount. The next decision in the supply chain is how to allocate the limited space to each brand. Intuitively, the space allocated to each brand not only influences the sales of each brand but also plays a pivotal role in each party's profit. However, the first form of competition – shelf-space level and allocation – has received less attention. Indeed, White (2010) describes how NB manufacturers are forced to pay retailers tens of millions of dollars just for shelf-space, the retailers can up the asking prices when they like, and even they can refuse to place NB products at eye level.

From a marketing perspective, the second form is a well known fact that the SB, usually regarded as "lower-cost and lower-quality", increases the retailer's bargaining power in the determination of the NB wholesale price (see Martin, 2008). In other words, in a supply chain a retailer can use SBs as a strategic weapon of increasing its influence on the channel. This often causes channel conflict, as the retailer's SB product competes with the manufacturer's NB. On the other side, to avoid competition from SB products NB producers often reduce their wholesale price that renders the shelf placement for the SB unattractive to the retailer. It seems that such strategic wholesale price reductions can be used as a means for the NB manufacturers to expel the SB. This pricing standpoint has attracted the attention of both operations and marketing scholars in the literature (see, e.g., Groznik and Heese, 2010). However, the relationship between these two forms of competition has received comparatively little attention in most analytical works.

Motivated by these issues, we pose the following research question: How should the retailer assign its shelf-space to a category of consumer goods and the supply chain allocate space level to each brand when the category involves the SB? Namely, how does the supply chain "strategically" manage the channel conflict between the NB and SB by utilizing the shelf-space. To answer the questions, we consider a two-echelon supply chain where a retailer sells its own SB product along with a competing NB in a consumer good category. In our model, the retailer first determines the category shelf-space. Next, the manufacturer pays *space fee* to buy the space for placing its NB products and chooses the NB wholesale price. The retailer finally sets the retail price of each brand. Notice that in the proposed model, the NB manufacturer seems being able to be a monopolist in the product category by paying the expense to place its products on the total shelf-space available for the category. Of course, such allocation policy may not be optimal for the NB manufacturer since the decisions to the category shelf-space level and the retail price of each brand are on the retailer's hand. This setting is satisfied by most real world situations (see, e.g., Kurtuluş and Toktay, 2011; White, 2010). We characterize the equilibrium category shelf-space level and allocation, wholesale price, retail prices, and sales volumes for both brands. The space competition in our stylized model leads to the following managerial insights:

1. It is well known that the NB would like to be the monopolist in its product category. However, we find that with the shelf-space consideration both NB and SB always coexist and compete in the product category. This result is supported by the empirical observations made by Private Label Manufacturer Association (2012). Therefore, there is a need to better manage such channel conflict.

2. We show that the NB retail price is increasing but the SB retail price is decreasing in the NB's underlying market share. Also, both NB retail and wholesale prices are increasing in the space fee, but the NB shelf-space and the SB retail price are decreasing in the space fee. As the SB market share is sufficiently large (i.e., the NB market share is low enough), the retailer is willing to make competition intensively between two brands by raising the SB retail price and cutting the NB retail price at the same time.

3. We find that both NB retail and wholesale prices as well as its allocated shelf-space are increasing in the category market size. Consistent with intuition, when the product category including both NB and SB is popular, each party in the supply chain would like to raise the corresponding NB prices.

According to the above findings, we conclude that due to the existence of space competition, the wholesale price reductions may not be an effective strategy for the NB producers to impede the SB existence, which is supported by some empirical observations (The Korea Herald, 2009; White, 2010). In other words, the retailer can directly impact the NB producers' bargaining power through the exercise of its limited shelf-space in vertical strategic interaction. As mentioned in the beginning, the retailer can also indirectly impact the NB producers' bargaining power through the positioning of its SB products in horizontal strategic interaction. The product differentiation between the NB and SB can reflect *quality* differences or just differences in *features* (Choi and Coughlan, 2006).

Quality differentiation (i.e., vertical differentiation) is exhibited by the perception that the SB is of the lower quality than the corresponding NB. A premium-priced NB may lose its quality differentiation from the SB if the retailers are eventually able to match the NB's technology and perception. With Chen et al. (2011), the perceived quality of the SB is assumed to be not higher than that of the NB, which is very common in practice (Moorthy, 1988). Following Amrouche and Zaccour (2007), the SB quality is characterized in terms of the cross-price substitution. In contrast, feature differentiation (i.e., horizontal differentiation) refers to the degree to which products have different forms, sizes, colors, labeling, flavors, or packaging. Different with a "quality" characteristic, a "feature" characteristic of a product is one for which more is not always better, and can include characteristics where variety is valued by the consumer. That is, the degree of feature differentiation determines the substitution level between the NB and SB. We thus assume that for there being no significantly quality-differentiated between the NB and SB, the more substitutable the two brands the less feature-differentiated between them. In other words, we only consider the feature differentiation for very high quality SB. This is because for the low quality SB, it is less likely to substitute the NB but the reverse is not true so there is not too much need to differentiate itself in terms of product features. Through a comprehensive numerical investigation, the position competition in our stylized model leads to the following main managerial insights:

4. The quality differentiation literature such as Moorthy (1988) suggests that a SB should differentiate itself by offering a lower quality product than that of the NB. However, we find that for the NB cross-price effect being not too large, the retailer's value appropriation across the supply chain (i.e., capture more the entire chain profit) can be improved by raising the SB quality close to the NB quality. In fact, this is so called "me-too" strategy in the literature (Corstjens and Lal, 2000). So, the direction of impact of quality differentiation on retailers' bargaining power can be changed by considering the effect of shelf space.

5. In addition to quality differentiation, we find that the retailer should reduce the feature differentiation between the NB and SB if encountering a strong NB (i.e., first-tier brand with high market share and high brand equity); otherwise (i.e., facing a second-tier brand with low market share and low brand equity), increase the feature differentiation. The former is supported by the empirical literature including Sayman et al. (2002) that the SB should imitate the NB in terms of feature differentiation. But, our result points out that this positioning strategy may not work when the NB has low market share and low brand equity. Hence, the direction of impact of feature differentiation on retailers' bargaining power can be

more complicated than those in the literature without considering the effect of shelf space.

These two findings also suggest that beyond the low-price strategy, NB manufacturers should diversify product lines and offer more distinct functions to discover the needs of consumers. For instance of the recent story, the NB providers of soft drinks such as Coca-Cola Co. stepped up advertising to steal sales from the SB competitors including Cott even though Wal-Mart's decision is to reduce shelf-space for the soft drinks (Surridge, 2008; The Toronto Star, 2008). In sum, our study advances the operations, supply chain and marketing literatures by offering the SB positioning strategy with endogenously determining the category shelf-space level and allocation for both NB and SB products.

The rest of this paper is organized as follows. Section 2 surveys the related literature while emphasizing the position of our work. In Section 3 we formulate our retail supply chain model. Section 4 develops the retailer's best response functions and Section 5 develops the manufacturer's best response functions, respectively. A pure-strategy equilibrium in shelf-space level and allocation is characterized in Section 6. Finally, Section 7 extends our analysis numerically to explore the issue regarding how a SB should be positioned against a NB. The last section summarizes our findings. The appendix extends our analysis to the case in which the retailer can decide the NB space level. The proofs and technical details are provided in the online supplementary material if they are not reported in the main body of the paper.

## 2 Related Literature

There are two primary streams of research that relate to our analysis: the operations literature on allocation of shelf-space or capacity and the marketing literature on channel conflict between manufacturers and retailers (in particular for the competition between NBs and SBs).

Earlier research in the first stream can be traced back to the studies that consider how to determine the optimal product selection and shelf-space allocation (see, e.g., Anderson and Amato, 1974; Hansen and Heinsbroek, 1979; Corstjen and Doyle, 1981). In particular, Corstjens and Doyle (1981) develop an optimization model that examines the shelf-space allocation for the retailer subject to both store capacity and product availability constraints, assuming that the space positively affects the retailer's profit. Following this direction, many operations studies including Irion et al. (2012) develop more complicated optimization models and then proposes some heuristic algorithms to better allocate the shelf-space based on the descending order of sales profit. However, these operations studies mainly focus on the retailer side and do not consider the strategic interactions between retailers and manufacturers, see Hübner and Kuhn (2012) for a comprehensive review. Martin-Herran et al. (2006) develop a game-theoretical model to investigate the impact of the manufacturers' wholesale prices on the retailer's shelf-space allocation. They suggest that manufacturers' decisions on wholesale price play a critical role in the shelf-space allocation. Their numerical analysis reveals an opposite relation between the shelf-space elasticity and the wholesale prices. However, they do not consider the existence of retailer's SB products as we do in this paper.

Amrouche and Zaccour (2007) investigate the pricing and shelf-space allocation equilibrium strategies in the SB versus NB context under a Stackelberg model (i.e., two-stage sequential move game) where the NB manufacturer acts as the leader. Their numerical outcomes suggest that the allocation of the shelf-space depends on the SB quality, measured by the demand characteristics such as the degree of product substitutability. Amrouche and Zaccour (2009) extend their previous work by comparing the Nash model (i.e., simultaneous move game) with the Stackelberg model. Their simulation outcomes suggest that the Stackelberg model benefits the NB manufacturer facing a me-too SB. Their models are suitable for the case that space fees need not to be paid by manufacturers while they are powerful (i.e., the Stackelberg model where the manufacturer is the leader). This takes place for some larger retailers, such as Wal-Mart, being compensated for shelf space primarily with lower wholesale prices offered by the manufacturers (Schoenberger, 2000; Klein and Wright, 2007). But, their time line setting in the Stackelberg model seems not being supported by some real-world cases as White (2010) points out that the NB producers are notified by the threat from the retailer that the available shelf-space can be taken by the SB if they refuse to pay the designated shelf-space rental fee (i.e., space fee). And, their studies do not consider space fee that is one of the major incomes for the retailer. Kurtuluş and Toktay (2011) recently develop a three-stage sequential move game where the retailer acts as the leader to determine the category shelf-space, recognizing the importance of shelf-space cost. However, they do not consider the SB existence. We complement the operations and supply chain management literature by developing a competitive shelf-space model of the NB versus SB based on Kurtuluş and Toktay's settings on the time line and demand functions.

Another stream of literature mainly analyzes channel conflict between different parties (see, e.g., Groznik and Heese, 2010). Choi (1991) analytically capture the strategic interactions between manufacturers and retailers by considering different move sequences between the parties. Later, Choi (1996) extends Choi (1991) by considering a more complex channel structure (with various combinations of the retailer and manufacturer number). The results suggest that the channel leader will be better off by being a leader in the game, whereas the total channel profit is larger when there is no channel leadership (i.e., a Nash model). However, the result shows that the Nash model is unstable since each channel member has an incentive to become a leader. There is a substantial body of research in this stream investigating the competition between manufacturers and retailers; only a handful of them introduce SB into consideration. The SB existence makes the retailer possess a strategic weapon to restrain the power of NB manufacturers and hence, increases the degree of confliction in channel (Wilcox, 1998). Our paper complements the marketing stream by considering the shelf-space decisions on level and allocation.

In sum, we contribute to both streams by explicitly studying the effects of shelf-space level and allocation on channel conflict between the NB and SB. No paper in the literature, to our best knowledge, considers a series of endogenous decisions including shelf-space level and allocation, NB wholesale price, and retail price of each brand on channel conflict.

# 3 Model Description

We consider a two-echelon supply chain comprised of one manufacturer ($n$) and one retailer ($s$) that each produces one product in a given category and sells them to consumers through the retailer. We adopt the convention of using feminine pronouns for the retailer and masculine pronouns for the manufacturer in the supply chain literature. The manufacturer produces *national brand* (NB) products at the unit cost of $c_n$, and sell them to the retailer at the unit wholesale price of $w_n$ that maximizes his profit. Besides selling the NB product to the end customers, the retailer develops her private label, called the *store brand* (SB), which is produced at the unit cost of $c_s$ by another manufacturer not playing any strategic role in our framework. Without loss of generality, we assume $c_s \leq c_n$ since the manufacturer spends additional costs, such as market promotion and administration, on producing the NB compared with the production expense on the SB. Throughout this paper, we assume that both parties in the supply chain are risk-neutral and all information is common knowledge.

Denote by $p_n$ the NB retail price and by $p_s$ the SB retail price. The demand for each brand at the retailer depends on these two prices and is given by the linear functions:

$$D_i = \alpha\beta_i - p_i + \gamma_i p_j,$$

where $i \in \{n, s\}$, $\alpha > 0$, $\beta_i \in [0, 1]$, and $\gamma_i \in [0, 1)$. The parameter values of $\alpha$, $\beta_i$, and $\gamma_i$ depend on the product attributes of the NB and SB. Due to its analytical simplicity, the choice of linear demand is widely used in the marketing-operations interface models (Tang, 2010) and is often a good approximation to more general demand functions (Shapiro, 1986). This downward sloping linear inverse demand function can also be thought of as the result of utility-maximizing behavior by customers with quadratic, additively separable utility functions (Sing and Vives, 1984). Raju et al. (1995) justify this demand model and apply it to determine the conditions under which a retailer should launch its own SB against the NB.

The parameter $\alpha$ refers to a market base and can be interpreted as a parameter giving the total market potential of the given product category (if the retail prices for both brands are set to zero). The parameter $\beta_i$ can be interpreted as brand $i$'s underlying *market share* so that $\sum_i \beta_i = 1$; thus, $\alpha\beta_i$ is the demand for brand $i$ if both brands' retail prices are set to zero. For the simplicity of mathematical expressions, we use

$$A_i = \alpha\beta_i$$

in the subsequent analysis. We refer $A_i$ as the *equity* of brand $i$ following Amrouche and Zaccour (2007).

The cross-price competition between two brands are captured by parameters $\gamma_n$ and $\gamma_s$, either of which is at most equal to one (i.e., the direct price effect). This setting is rather standard in game-theoretical modeling and plausibly depicts the empirical fact that the demand's inflation degree caused by the rival brand's price is less than that by its own price (see, e.g., Cotterill and Putsis, 2000; Cotterill et al., 2000). Empirical studies (Cotterill and Putsis, 2000; Cotterill et al., 2000) and analytical ones (Blattberg and Wisniewski, 1989; Bronnenberg and Waithieu, 1996; Amrouche and Zaccour, 2007) both have demonstrated that the NB cross-price effect of $\gamma_s$ is higher than SB cross-price effect of $\gamma_n$ when the SB is of lower quality. The difference between $\gamma_s$ and $\gamma_n$ is thus a measure of the *quality differentiation* between the NB and SB (Choi and Coughlan, 2006). With most analytical work such as Amrouche and Zaccour (2007), we suppose that the SB is, at best, of the same quality as the NB, i.e., $\gamma_s \geq \gamma_n$.

Accordingly, for the top-notch (i.e., highest quality) SB with $\gamma_n = \gamma_s = \gamma$, the analytical literature including Choi (1991) suggests that this cross-price effect of $\gamma$ measures the degree of product substitution across the brands. This parameter is inversely related to the degree of product differentiation. That is, the smaller the difference, the more substitutable (i.e., less differentiated) the two brands, hence the more potential price competition. In other words, when the SB quality is sufficiently high the parameter $\gamma$ expresses the degree of *feature differentiation* ranging from zero when the products are independent to one when the products are perfect substitutes (Sing and Vives, 1984; Zhang, 2002; Choi and Coughlan, 2006). Therefore, our demand function not only captures the quality differentiation between the NB and SB but also the feature differentiation for very high quality SB against the NB.

The category shelf-space level, decided by the retailer, is denoted by $M$ ($\geq 0$). Once the manufacturer decides on the NB shelf-space of $Q_n$, the SB shelf-space is $Q_s = M - Q_n$. For the retailer, the unit operational cost of shelf-space is $h$. Such space is valuable to manufacturers since in most cases it provides their only opportunity to build contact with ultimate consumers. Therefore, the business of the retailer may also be regarded as selling to manufacturers the space at her disposal. To induce the retailer to sell her space for carrying his NB product, the manufacturer must offer a price of $k$ for a unit of space which exceeds the opportunity cost of the space (Cairns, 1962; Aydin and Hausman, 2009). After knowing each brand's corresponding shelf-space level, the pricing decisions are made by the retailer subject to the space constraint $\sum_i D_i \leq M$. This shelf-space setting can be interpreted as that if a brand is given a shelf-space over its potential demand, it can be sold up to the potential demand; otherwise, it can be sold only with the level of its shelf-space, which is less than the demand. See Kurtuluş and Toktay (2011) for a discussion of the implications of this kind of shelf-space setting involving two competing brands.

Our problem is modeled as a three-stage sequential move game with the retailer as the leader and the manufacturer as the follower, similar to the model setting in Kurtuluş and Toktay (2011). The sequence of events is as follows and depicted in Figure 1. The retailer first announces the total shelf-space available for a product category. The manufacturer reacts by choosing the shelf-space allocation as well as his NB wholesale price. Next, the retailer chooses the retail prices for both brands ($p_n$ for the NB and $p_s$ for the SB) and then the demands for both brands are realized. In our setting, the manufacturer is able to determine the shelf-space allocation of his products by offering the retail shelf-space price. The manufacturer thus can use the shelf-space allocation as a means to influence the subsequent pricing decisions. But, note that the retailer can also use the category shelf-space level as a means to influence the manufacturer's decisions. As a result, the supply chain faces not only the "position" competition between two brands, but also the "space" competition that results in demand restrictions. This sequence of decisions is commonly employed in the literature and practice (see, e.g., Cairns, 1962; White, 2010; Kurtuluş and Toktay, 2011). Another alternative model with different setting in the sequence of events is discussed in the appendix.

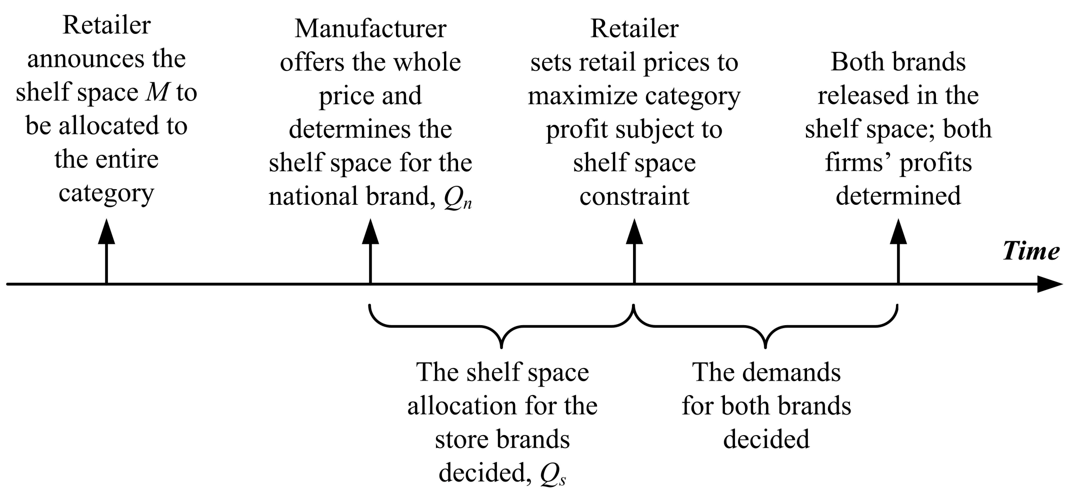


Figure 1: The sequence of events.

## 4 Retailer's Problem

We solve the game backwards by first considering the retailer's retail price decisions. Given the wholesale price and the shelf-space of the NB, the best responses of retail prices are identified to maximize the retailer's profit from both brands. We then find the optimal wholesale price and the shelf-space for the NB based on the best responses of the retailer in the next section. Finally, the equilibrium category shelf-space level and allocation are analyzed.

We begin with the decisions made by the retailer. Upon knowing the wholesale price of $w_n$ and the shelf-space of $Q_n$ for the NB, the retailer solves the following decision problem: given $w_n$ and $Q_n$,

$$\begin{aligned}\max_{p_n, p_s \geq 0} \Pi_s &= (p_n - w_n)^+ (D_n \wedge Q_n)^+ + (p_s - c_s)^+ (D_s \wedge Q_s)^+ + kQ_n - hM \\ &= (p_n - w_n) D_n + (p_s - c_s) [D_s \wedge (M - Q_n)]^+ + kQ_n - hM.\end{aligned} \tag{1}$$

Throughout this paper, the following notations are adopted: $x^+ = \max(0, x)$ and $x \wedge y = \min(x, y)$ for $x, y \in \mathbb{R}$. The retailer's profit consists of four parts. The first two terms in equation (1) represent the profits collected from the sales amount of the national and store brands, respectively. The non-negative constraints $(p_i - w_i)^+ \Leftrightarrow p_i \geq w_i$ for each $i$ ensure the profitability of each brand. The third term is the income of selling to the manufacturer the space and the last term represents the shelf-space operational cost. The available selling amount of each brand is constrained by its allocated shelf-space for its products. Note that we do not need having any constraint on the NB demand, i.e., $(D_n \wedge Q_n)^+ = D_n$, because the shelf-space constraint on the demand can be satisfied as the manufacturer chooses the wholesale price given the retailer's best responses. With the definition, $Q_s = M - Q_n$.

Assuming that retail prices are not so high as to make the available selling amount of both brands non-positive, we can omit the superscript of the 'positive sign' in the expression for the available selling amount of each brand. To solve the retailer's problem, we need to consider two different scenarios: (I) $D_s < Q_s$, the available SB selling amount is unrestricted by her shelf-space, and (II) $D_s = Q_s$, the available SB selling amount is restricted by her shelf-space. In the foregoing analysis, we use $F$ as shorthand for unconstrained (the product demand less than its allocated shelf-space) and $B$ for constrained (the product demand equal to its allocated shelf-space). The objective in (1) is jointly concave in prices and the capacity constraint $[D_s \wedge (M - Q_n)]^+ \Leftrightarrow A_s + Q_n - p_s - \gamma_s p_n \leq M$ is linear, so we can use standard optimization methods to find the optimal retail prices and summarize:

**Lemma 1.** *Given $w_n$ and $Q_n$, then the best responses of retail prices for both brands are:*

**(a)** *For $D_s < Q_s$:*

$$p_n^F(w_n) = \frac{2A_n + (\gamma_n + \gamma_s) A_s + (\gamma_n - \gamma_s) c_s}{4 - (\gamma_n + \gamma_s)^2} + \frac{2 - \gamma_n (\gamma_n + \gamma_s)}{4 - (\gamma_n + \gamma_s)^2} w_n, \tag{2}$$

$$p_s^F(w_n) = \frac{2A_s + (\gamma_n + \gamma_s) A_n + [2 - \gamma_s (\gamma_n + \gamma_s)] c_s}{4 - (\gamma_n + \gamma_s)^2} + \frac{\gamma_s - \gamma_n}{4 - (\gamma_n + \gamma_s)^2} w_n; \tag{3}$$

**(b)** *For $D_s = Q_s$:*

$$p_n^B(w_n, Q_n) = \frac{A_n + \gamma_n A_s}{2(1-\gamma_n\gamma_s)} - \frac{\gamma_n - \gamma_s}{2(1-\gamma_n\gamma_s)}(M - Q_n) + \frac{1}{2}w_n, \tag{4}$$

$$p_s^B(w_n, Q_n) = \frac{(2-\gamma_n\gamma_s)A_s + \gamma_s A_n}{2(1-\gamma_n\gamma_s)} - \frac{2-\gamma_s(\gamma_n+\gamma_s)}{2(1-\gamma_n\gamma_s)}(M - Q_n) + \frac{\gamma_s}{2}w_n. \tag{5}$$

We next consider the manufacturer's problem of choosing his NB wholesale price and shelf-space level.

# 5 Manufacturer's Problem

Upon knowing the retailer's pricing strategy, the NB manufacturer solves the following decision problem: given $p_n$ and $p_s$,

$$\begin{aligned}\max_{w_n, Q_n \geq 0} \Pi_n &= (w_n - c_n)^+ (D_n \wedge Q_n) - kQ_n \\ &= (w_n - c_n)(D_n \wedge Q_n) - kQ_n.\end{aligned}$$

The retail prices, $p_n$ and $p_s$, made by the retailer are the best response functions of $w_n$ and depend on whether the available selling amount of the SB is unconstrained by her shelf-space ($D_s < Q_s$) or not ($D_s = Q_s$), as given by (2) and (3) or by (4) and (5), respectively. For simplicity we assume that

$$w_n \geq c_n + k$$

in the following analysis; thus, we can omit the 'positive sign' in the expression of the marginal profit for the manufacturer.

Since each party may have either an unconstrained or a constrained shelf-space for its own product, there are four possible shelf-space configurations: $(B, F)$, $(F, F)$, $(B, B)$, and $(F, B)$ where in each two-tuple the first entry for the NB and the second entry for the SB. Either 'duopolistic' or 'monopolistic' outcome may take place in each configuration, depending on the manufacturer's profit. We can obtain the following result by knowing the optimal decisions for each shelf-space configuration.

**Lemma 2.** *Given the category shelf-space of M, then*

**(a)** *the $(F, F)$ configuration is impossible to occur since the $(B, F)$ configuration dominates the $(F, F)$ configuration;*

**(b)** *the duopolistic $(F, B)$ configuration is impossible to occur since the duopolistic $(B, B)$ configuration dominates the duopolistic $(F, B)$ configuration.*

We thus know that the monopoly outcome is not able to take place in the $(F, B)$ configuration and the $(F, F)$ configuration is not a possible choice for the manufacturer's decision. Since the manufacturer makes the space decision for the NB with considering the retailer's best responses of retail pricing, through the choices of $w_n$ and $Q_n$ he can induce the retailer to choose retail prices corresponding to his preferred space configuration. Formally,

**Lemma 3.** *Given the category shelf-space of $M$, then the manufacturer can dictate the retailer to follow his preferred shelf-space configuration.*

It is found in Lemma 2 that given the category shelf-space of $M$ there are four possible shelf-space configurations for the NB manufacturer to choose: $(B, F)$, duopolistic $(B, B)$, monopolistic $(B, B)$, and monopolistic $(F, B)$ configurations. Accordingly, we want to know the conditions under which the manufacturer prefers one configuration to another. The following lemma identifies these conditions.

**Lemma 4.** *Consider the pair of shelf-space configurations, $(F_l, L_l)$, listed in Table 1 where $l = 1, ..., 6$. For each pair, there exists a unique threshold of $\widehat{M_l}$ such that the NB manufacturer*

**(a)** *prefers configuration $F_l$ if the category shelf-space of $M < \widehat{M_l}$;*

**(b)** *prefers configuration $L_l$ otherwise.*

| Case | $F_l$ | $L_l$ | $\widehat{M_l}$ |
|---|---|---|---|
| (i) | Duopolistic $(B, B)$ | $(B, F)$ | $\widehat{M_1}$ |
| (ii) | Monopolistic $(B, B)$ | $(B, F)$ | $\widehat{M_2}$ |
| (iii) | Monopolistic $(B, B)$ | Duopolistic $(B, B)$ | $\widehat{M_3}$ |
| (iv) | Monopolistic $(F, B)$ | $(B, F)$ | $\widehat{M_4}$ |
| (v) | Monopolistic $(B, B)$ | Monopolistic $(F, B)$ | $\widehat{M_5}$ |
| (vi) | Monopolistic $(F, B)$ | Duopolistic $(B, B)$ | $\widehat{M_6}$ |

Table 1: Summary of threshold for each pair of space configuration.

Based on parts (ii), (iii), and (iv) of this lemma, we can see that for some shelf-space of $M_1 > 0$ such that $(B, F)$ is the preferred configuration, then for any $M > M_1$, $(B, F)$ is still preferred. Thus, the lowest $M \in [0, \infty)$ for configuration $(B, F)$ preferred is defined as $\overline{M}_3$. Continuing the same logic, based on parts (iii) and (vi), we can define the threshold, $\overline{M}_2$, as the lowest shelf-space at which the duopolistic $(B, B)$ configuration is preferred. Similarly, based on part (v), $\overline{M}_1$ can be directly defined, which concludes the proof of the following proposition to characterize the optimal shelf-space configuration for a given category shelf-space of $M$.

**Proposition 1.** *There exist three thresholds of $\overline{M}_t$ for $t = 1, 2, 3$ where $0 < \overline{M}_1 \leq \overline{M}_2 \leq \overline{M}_3 < \infty$, such that the optimal shelf-space configuration for the NB manufacturer is: given the category shelf-space of $M$,*

**(a)** *the monopolistic $(B, B)$ configuration for $M < \overline{M}_1$;*

**(b)** *the monopolistic $(F, B)$ configuration for $\overline{M}_1 \leq M < \overline{M}_2$;*

**(c)** *the duopolistic $(B, B)$ configuration for $\overline{M}_2 \leq M < \overline{M}_3$;*

**(d)** *the $(B, F)$ configuration for $M \geq \overline{M}_3$.*

The proposition shows the NB manufacturer's preferred shelf-space configuration for the different levels of category shelf-space, $M$. To help understand the behavior described in Proposition 1, it is useful to graphically compare optimal shelf-space configurations under different sizes of the given shelf-space. Figure 2 does this for a typical scenario. Propositions 1(a) and 1(b) indicate that when the space $M$ is sufficiently low ($M < \overline{M}_2$), the NB is the only product sold on the category shelf-space, i.e., the NB enjoys a monopoly position. Proposition 1(c) shows that when $M$ is moderate ($\overline{M}_2 \leq M < \overline{M}_3$), both brands compete for the limited category shelf-space. In Proposition 1(d), we see that when $M$ is large enough ($M \geq \overline{M}_3$), the manufacturer only takes a necessary space for the NB and leaves all the remaining space to the SB. In other words, except for very low shelf-space as given in most scenarios we expect to see that both brands play a duopoly on the category shelf-space.

We have characterized the shelf-space configuration the manufacturer prefers given the level of the category shelf-space. Before moving to equilibrium analysis, we are interested in knowing what the optimal level of the given category shelf-space is in the manufacturer's viewpoint. The following proposition shows that the optimal level of the given category shelf-space for the manufacturer falls into the range of $M \leq \overline{M}_1$, i.e., the range for the monopolistic $(B, B)$ configuration and the boundary between the monopolistic $(B, B)$ and $(F, B)$ configurations. The manufacturer would like the NB to be the only product on the category shelf-space and is happy with a limited space less than the NB demand. In other words, the NB manufacturer prefers a monopoly position instead of selling more to face a competition from the SB. It explains why the national brands of canned vegetables pay costly shelf-space fee to keep competing brands off the shelf and the large manufacturers of automotive air fresheners and tortilla also pay a lot of space fees to take all the shelf-space eventually (FTC, 2001: 31). This result provides a basis for comparison with the equilibrium shelf-space configuration for the retail supply chain.

**Proposition 2.** *The manufacturer's profit is decreasing in $M$ for $M > \overline{M}_1$. So, the manufacturer prefers that the category shelf-space level is not larger than $\overline{M}_1$, i.e., $M \leq \overline{M}_1$.*

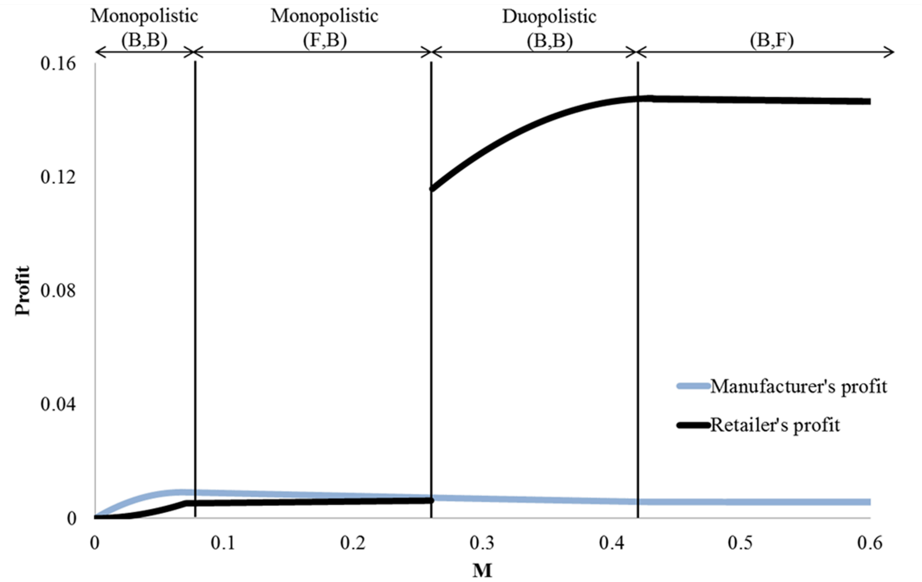


Figure 2: Manufacturer's and retailer's profits at different levels of the category shelf-space. Here, $A_n = 0.25$, $A_s = 0.75$, $\gamma_n = 0.06$, $\gamma_s = 0.09$, $c_n = 0.015$, $c_s = 0.01$, $h = 0.005$, and $k = 0.01$.

## 6 Equilibrium Analysis

After analyzing the retailer's problem, we now consider the supply chain equilibrium behavior in the category shelf-space level and allocation. We limit our analysis only to subgame-perfect Nash equilibrium in pure strategy that are described in the following proposition.

**Proposition 3.** *The equilibrium category shelf-space falls into the range of* $\left[\overline{M}_2, \overline{M}_3\right]$.

Together with Proposition 1, this equilibrium precisely describes the situation we observe in many category shelf-spaces: Both national and store brands compete in the product category and each brand's demand is constrained by the respective shelf-space (i.e., no brand has extra space more than its demand). Consistent with intuition, under the equilibrium there are no redundant spaces allocated by the retailer for the given product category. Notice that even if $\overline{M}_2 = \overline{M}_3 = M_d$, the optimal shelf-space for the supply chain is simply $M_d$ where the national and store brands still play a duopoly in the category shelf-space. Comparing with Proposition 2, we know that without the shelf-space consideration, the NB would like to be the monopolist in its product category. However, with the available shelf-space threat from the retailer both brands coexist in the category. Notice that it is possible to derive the exact equilibrium solution rather than the range here but there is a need to make other assumptions for each specific scenario.

Applying Proposition 1(c) and the optimal retail prices (see Lemma 1), we obtain the following comparative statics:

**Corollary 1.** *Let*

$$\sigma(\gamma_n) \equiv \frac{\gamma_n (3 - \gamma_n)}{1 + \gamma_n}.$$

*For* $M \in \left[\overline{M}_2, \overline{M}_3\right]$, *then*

**(a)** *the SB retail price is decreasing in* $M$*; and,*

**(b)** *the NB retail price is increasing in* $M$ *for* $\gamma_s \geq \sigma(\gamma_n)$ *but decreasing in* $M$ *otherwise.*

Under the category space range of $\left[\overline{M}_2, \overline{M}_3\right]$, the retailer raises the NB retail price as the category shelf-space level increases when the cross-price competition parameter $\gamma_s$ is significant, i.e., $\gamma_s \geq \sigma(\gamma_n)$. Note that the scenarios of $\gamma_s \geq \sigma(\gamma_n)$ are popular in practice that the NB retail price has a major impact on the SB retail demand but the SB retail price have little influence on the NB demand (see Cotterill and Putsis, 2000).

By considering various derivatives we can prove the following corollaries.

**Corollary 2.** *For* $M \in \left[\overline{M}_2, \overline{M}_3\right]$, *then both NB retail and wholesale prices are increasing in the space fee,* $k$, *but the SB retail price is decreasing in* $k$. *In addition, the shelf-space for the NB is decreasing in* $k$.

Notice that the space fee of $k$ is a means for the retailer to counteract the manufacturer's space decision. With a higher $k$, the NB manufacturer asks for smaller space and charges a higher wholesale price to maintain a certain margin. In the meantime, the retailer reactively increases the NB retail price. On the other hand, to supplement the demand gap the SB retail price is reduced so as to stimulate its demand. Furthermore, consistent with intuition a higher $k$ will induce the NB manufacturer to choose less shelf-space allocated to the NB, which enhances forceful competition from the SB.

In addition to the effect of the space fee the following corollary considers the effect of market share for both brands, $\beta_i$ for $i = n, s$ where $\beta_s = 1 - \beta_n$.

**Corollary 3.** *For* $M \in \left[\overline{M}_2, \overline{M}_3\right]$, *then*

**(a)** *the NB retail price is increasing in* $\beta_n$*;*

**(b)** *the SB retail price is decreasing in* $\beta_n$*;*

**(c)** *the NB wholesale price is increasing in* $\beta_n$*;*

**(d)** *the shelf-space for the NB is increasing in* $\beta_n$.

The intuition of Corollaries 3(c) and (d) is that when the NB market share keeps reduced, the NB manufacturer admittedly takes less shelf-space to avoid additional cost due to excess shelf-space and at the same time charges a lower wholesale price to induce a lower retail price for the NB product in order to compete with the SB product. From the retailer's perspective, Corollaries 3(b) shows that the retailer tends to raise the SB retail price when the NB market share is low. This is since the retailer uses its power of determining the total shelf-space level to foster the benefit from the NB product. To pursue more profits the retailer reduces the NB retail price to stimulate its demand, see Corollary 3(a). The above analysis uncovers this intrinsic rationale that when the SB market share is high, the retailer tends to make competition more intense between two brands by simultaneously increasing the SB retail price and decreasing the NB retail price for her own benefit.

Finally, we move our attention to the effect of market base, $\alpha$, on the pricing decisions of the manufacturer and the retailer as well as the resultant shelf-space for the NB products. The following corollary shows that the NB shelf-space can be increased through expanding the potential market size for the given product category. In addition, with the popularity of the product category including both NB and SB in the market, Corollaries 4(a) and (b) show that each party in the channel would raise the corresponding prices of the NB for a higher profit. Note that the relationship between the SB retail price and market base is not monotonic so that we do not pursue its analysis.

**Corollary 4.** *For $M \in \left[\overline{M}_2, \overline{M}_3\right]$, then*

**(a)** *the NB retail price is increasing in $\alpha$;*

**(b)** *the NB wholesale price is increasing in $\alpha$;*

**(c)** *the NB shelf-space is increasing in $\alpha$.*

Two possible extensions by relaxing our model appropriately can be addressed by our analytical results in Sections 5 and 6:

**Remark 1.** Our research focuses on the positioning strategy of the SB in the supply chain under shelf space allocation; thus in our model the unit price of space, $k$, is exogenously given. One may relax this assumption by allowing $k$ to be one of the endogenous decisions of the retailer before the shelf-space size and allocation decisions. In fact, our result reveals that the equilibrium outcome is both NB and SB coexisting on the category shelf-space, i.e., duopolistic (B,B) configuration, see Propositions 1 and 3. As a result, it is not difficult to show that the retailer's profit under equilibrium is concave in $k$ so the optimal decision of $k$ is able to be uniquely determined: $k^* = \frac{(\gamma_n+1)(M-A_n+M\gamma_n-A_s\gamma_n)}{2\gamma_n(\gamma_n\gamma_s-1)} - \frac{A_n-M+c_n\gamma_n+\gamma_s\gamma_n}{2\gamma_n}$.

**Remark 2.** One may consider a SB introduction fee of $f$ that is charged when the retailer initiates the SB in the market. Note from the proof of Proposition 3 that the retailer's profit function is increasing for $M < \overline{M}_2$, concave for $\overline{M}_2 \leq M < \overline{M}_3$, and then decreasing for $M \geq \overline{M}_3$. If the retailer does not introduce the SB, the optimal category shelf-space level is $\overline{M}_2$. Therefore, one can simply compare the profits of the retailer when $M = \overline{M}_2$ and $M = M^*$ that represents the equilibrium shelf-space level for $f$ being not included in the model. If the difference between these two profits exceeds $f$, then the equilibrium category shelf-space level remains the same and the SB is introduced; otherwise, the equilibrium shelf-space level would reduce to $\overline{M}_2$ and the NB is the only brand in the market. In other words, the NB can enjoy the monopoly position on the category shelf-space while the SB introduction fee is rather costly.

# 7 Sensitivity Analysis of Cross-Price Competition

As shown in Choi and Coughlan (2006) and Hall et al. (2010), a linear demand with asymmetric cross-price effects produces interdependent best responses in product pricing, resulting in very messy closed-form equilibrium quantities. These solutions are very complex and not amenable to immediate analytical interpretation as to the cross-price effect of $\gamma_n$ and $\gamma_n$ on the retailer's and supply chain profits. To understand the sensitivity of the above results about the cross-price effect to changes in market and operational parameters, we resort to an extensive numerical analysis to illustrate the effect of the store brand positioning on the retailer's ability to appropriate and create value across the supply chain. The management of this tension between value appropriation and creation is central to retail branding strategy (Ailawadi and Keller, 2004; MacDonald and Ryall, 2004). We examine 900 parameter instances consisting of every combination in Table 2, which were selected to provide a wide range of possible scenarios. In each case, we calculate the equilibrium quantities in terms of the total shelf-space size, space allocations, NB wholesale price, each brand's retail price, each party's profit, and the supply chain profit. Comparing the incidence of specific equilibria leads to findings below that are valid for a large range of the parameters considered. The results of this numerical analysis are presented in Tables 3 and 4, which report the summary outcomes across all the parameter instances excluding the ones with $\gamma_s < \gamma_n$.

In the first observation below, we examine how the value appropriating outcome for the retailer, $\Pi_s / (\Pi_s + \Pi_n)$, is affected by the increase in the SB quality. The relevant results from our numerical study are represented in Table 3. In the table, the grey slots refer to the high-quality SB positioning strategy and the other ones are associated with the low-quality strategy. Because the retailer can capture over half profits of the supply chain when the NB cross-price effect of $\gamma_s$ is not larger than 0.5, an increase in the SB cross-price effect of $\gamma_n$ (0.1 to 0.3, 0.3 to 0.5) allows us to characterize the retailer's SB positioning strategy. This result of increasing the SB cross-price effect, however, does not carry over for the case when the NB cross-price effect is high ($\gamma_s > 0.5$).

| Parameter | Values |
|---|---|
| $\alpha$ | 1 |
| $\beta_n$ | $1-\beta_s$ |
| $\beta_s$ | $\{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$ |
| $\gamma_n$ | $\{0.1, 0.3, 0.5, 0.7, 0.9\}$ |
| $\gamma_s$ | $\{0.1, 0.3, 0.5, 0.7, 0.9\}$ |
| $c_n$ | 0.0015 |
| $c_s$ | 0.01 |
| $k$ | $\{0.005, 0.01\}$ |
| $h$ | $\{0.005, 0.0075\}$ |

Table 2: Parameter values used in numerical experiments.

**Observation 1.** [**Quality Differentiation**] *Given that the NB quality is fixed and its cross-price effect is not too large ($\gamma_s \leq 0.5$), an increase in the SB quality increases the retailer's value appropriation across the supply chain.*

| | Mean value of $\Pi_s/(\Pi_s+\Pi_n)$ (%) | | | | |
|---|---|---|---|---|---|
| $\gamma_s/\gamma_n$ | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 |
| 0.1 | 71.6% | 66.6% | 57.8% | 53.3% | 48.9% |
| 0.3 | - | 69.5% | 62.4% | 57.4% | 50.3% |
| 0.5 | - | - | 67.9% | 62.5% | 51.7% |
| 0.7 | - | - | - | 58.6% | 44.7% |
| 0.9 | - | - | - | - | 38.5% |

Table 3: The effect of quality differentiation on value appropriation.

The above observation suggests that in most scenarios the high-quality SB is preferred by the retailer, i.e., the NB and SB are not significantly quality differentiated. This implies the adoption of me-too strategy from the retailer's perspective on the SB positioning. Accordingly, we can make an assumption of $\gamma_s = \gamma_n = \gamma$ in the rest of analysis. This assumption implies the following comparative statics in equilibrium due to the fact that $\sigma(\gamma_n) = 1$:

**Proposition 4.** *When there is no quality differentiation between the NB and SB (i.e., $\gamma_s = \gamma_n = \gamma$), the equilibrium wholesale price, shelf-space and retail price for the NB are all decreasing in the category shelf-space of $M$.*

Several analytical models including Mills (1995) show that the SB gives retailers negotiating leverage over NB manufacturers when there is no space limit. We confirm that this claim is still true under the shelf-space consideration. In sum, the negotiating leverage provided by a high quality SB can make it easier for a retailer to capture more supply chain profits, i.e., strengthening its value appropriation capability.

Consider the effect of feature differentiation between the two brands, another dimension of the SB positioning strategy, by fixing $\gamma_s = \gamma_n = \gamma$. In Table 4, we look at how a change in $\gamma$ (which captures the degree of feature differentiation between the brands) affects the retailer's ability to appropriate the supply chain profit under different NB market shares of $\beta_n$. Note that in the table, the grey slots refer to the better SB positioning strategies in terms of $\Pi_s/(\Pi_s + \Pi_n)$. We find that the retailer will make the most profits by differentiating the SB from the NB if the NB market share (or brand equity) is not large ($\beta_n < 0.5$); otherwise, the retailer prefers the positioning strategy for the SB to imitate the NB in terms of product features. Formally,

**Observation 2.** [**Feature Differentiation**] *Given that the SB quality is sufficiently high ($\gamma_s = \gamma_n = \gamma$), in terms of the retailer's value appropriation across the supply chain (i.e., $\Pi_s/(\Pi_s + \Pi_n)$),*

**(a)** *a SB is better off positioning closer to the NB if the NB market share is high ($\beta_n \geq 0.5$);*

**(b)** *a SB is worse off positioning closer to the NB if the NB market share is low ($\beta_n < 0.5$).*

| | Mean value of $\Pi_s/(\Pi_s + \Pi_n)$ (%) | | | | |
|---|---|---|---|---|---|
| $\gamma$ | $\beta_n = 0.1$ | $\beta_n = 0.3$ | $\beta_n = 0.5$ | $\beta_n = 0.7$ | $\beta_n = 0.9$ |
| 0.1 | 99.5% | 92.3% | 71.3% | 46.6% | 35.3% |
| 0.3 | 99.5% | 85.6% | 67.8% | 50.0% | 39.8% |
| 0.5 | 91.5% | 82.7% | 69.6% | 54.3% | 45.3% |
| 0.7 | 70.9% | 64.7% | 89.6% | 53.2% | 48.7% |
| 0.9 | 39% | 38.8% | 38.5% | 38.3% | 38.1% |

Table 4: The effect of feature differentiation on value appropriation.

## 8 Conclusion

Shelf-space allocation is a predominant aspect of most retail supply chains. Our research shows that it has significant implications regarding the SB positioning. Incentive conflicts between the NB and SB in managing the shelf-space have largely been modeled in exogenous settings. In this paper, we study a game-theoretical model of endogenously determining both category shelf-space size and allocation in a two-echelon supply chain, distilling the direct (space) and indirect (position) competitions between the NB and SB. In our model, the category shelf-space level is first decided by the retailer who owns the SB. Consequently, the space allocation between the NB and SB is not exogenously fixed, but an outcome of negotiation by all firms in the supply chain. As a result, with the shelf-space consideration the wholesale price reduction seems not to be an effective strategy for the NB manufacturer to exclude the SB on the category shelf-space.

Our findings provide systematic answers to questions regarding the shelf space allocation between the NB and SB and the SB positioning strategy under the space competition with the NB, and also lead to several hypotheses that could be tested empirically. One of our main conclusions is that both brands always coexist on the category shelf-space and the retailer should raise the SB quality for its value appropriation across the supply chain. We further find that the retailer should reduce the feature differentiation between the NB and SB if encountering a strong NB; otherwise, increase the feature differentiation. Overall, we show that the direction of impact of SB positioning in product quality and features on retailers' bargaining power can be more complicated than those in the literature without considering the effect of shelf space.

Our results come with several limitations. In our work we do not endogenously scale diseconomies in shelf-space as does Kurtuluş and Toktay (2011). Further, we take a restrictive view of shelf-space as the store space without any impact on potential demand. By relaxing these assumptions the impact of shelf-space on retail supply chains could definitely be increased further by the thought experiment. However, these extensions may make the problem unsolvable analytically. This should prove to be an interesting problem for future studies. Future research could also extend our framework to stochastic demand models.

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## Appendix: The Alternative Model

In this paper, we focus on an allocation scheme in which the NB producer decides the shelf-space allocation. While this type of allocation scheme is commonly used in the literature and practice (Kurtuluş and Toktay, 2011; White 2010), i.e., the NB producer can purchase the shelf-space from the retailer by paying space fee, it might be also interesting to consider alternative scheme to determine whether our results are sensitive to the choice of shelf-space allocation scheme. To that end, in this appendix we discuss another possible allocation scheme in retail supply chains in which the shelf-space allocation is decided by the retailer. By investigating it we are able to see whether the findings and insights from our (base) model still hold in this alternative setting.

The sequence of events in the alternative scheme is as follows. First, the retailer determines the total category shelf-space along with the space allocations to the NB and to the SB, respectively. Upon the retailer's decisions, the manufacturer settles the wholesale price at which the manufacturer sells NB products to the retailer. Finally, the retailer chooses the retail prices to the end customers. The analysis of this model is similar to the base model described in Section 3; the only variant is that in the alternative model the shelf-space allocation constraints for both brands are endogenously determined by the retailer prior to the wholesale price decision. To find the equilibrium pricing, space level, and space allocations for the manufacturer and retailer, similar to the base model we solve the problem backward in the main body of the paper.

Following the same procedures as in Section 5, we articulate four possible shelf-space configurations from the manufacturer's point of view: $(B, F)$, $(F, F)$, $(B, B)$, and $(F, B)$. In this alternative model, however, the shelf-space allocation determined by the retailer may influence the boundaries of the demand of each brand and thus the pricing decisions of both parties. We now face a question: Whether can the retailer and manufacturer settle for an agreement as in the base model when the shelf-space allocation decision is determined by the retailer? In Lemma 3, we show that the manufacturer can induce the discretionary retailer to choose the manufacturer's preferred space configuration. As a result, given the total shelf-space we prove in Proposition 1 that there exist three thresholds so that each equilibrium configuration can be derived accordingly. Nevertheless, in the current model the retailer (making both total shelf-space and allocation decisions) needs to take the manufacturer's move into account while the manufacturer (only setting the wholesale price) as well needs to evaluate the retailer's pricing behavior. From a modeling perspective, such multi-stage sequential move game makes the problem much more complex than the game considered in the base model. At the very least, the numerical solution is manageable although the closed-form solution is not available. We report a numerical study here that investigates the alternative model in the same parameter settings in Section 7 for the sake of comparison. Based on the outcome of numerical analysis, we make the following observations that show the robustness of our findings in the main body of the paper.

A.1 [**Equilibrium Outcome**] The equilibrium shelf-space allocation in the alternative model is either $(F, F)$ or $(F, B)$, depending on the cross-price competition. In other words, in the alternative model, the equilibrium total shelf-space and the shelf-space allocation to the NB are more than enough than those in the base model under the same parameter values. More specifically, the retailer can always force the NB to take excess space but the SB still has to share a fraction of the excess space, in particular, when both $\gamma_n$ and $\gamma_s$ are sufficiently large and close to unity (i.e., the NB and the SB are highly substitutable to each other).

A.2 [**Quality Differentiation**] We observe from Table 5 that the high quality SB is preferred by the retailer in all scenarios including that the NB cross-price effect is high. That is, given that the NB quality is fixed, an increase in the SB quality increases the retailer's value appropriation across the supply chain (i.e., me-too strategy). Observation 1 is true in the alternative model.

| | Mean value of $\Pi_s/(\Pi_s+\Pi_n)$ (%) | | | | |
|---|---|---|---|---|---|
| $\gamma_s/\gamma_n$ | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 |
| 0.1 | 84.1% | 82.4% | 80.1% | 77.8% | 74.5% |
| 0.3 | - | 85.9% | 84.1% | 81.1% | 77.3% |
| 0.5 | - | - | 88.6% | 86.4% | 82.7% |
| 0.7 | - | - | - | 92.1% | 89.5% |
| 0.9 | - | - | - | - | 97.1% |

Table 5: The effect of quality differentiation on value appropriation when the retailer determines the shelf-space allocation.

A.3 [**Feature Differentiation**] Table 6 presents data on the effect of the degree of feature differentiation between the NB and SB on the retailer's value appropriation across the supply chain under different SB market shares. We find that the retailer will make the most profits by imitating the NB in terms of product features no matter of the NB market share. In other words, given the SB quality is sufficiently high, in terms of the retailer's value appropriation across the supply chain, a SB is always better off positioning closer to the NB in terms of feature differentiation (i.e., high $\gamma$). Thus, Observation 2(a) holds but 2(b) does not in the alternative model.

| | Mean value of $\Pi_s/(\Pi_s+\Pi_n)$ (%) | | | | |
|---|---|---|---|---|---|
| $\gamma$ | $\beta_n=0.1$ | $\beta_n=0.3$ | $\beta_n=0.5$ | $\beta_n=0.7$ | $\beta_n=0.9$ |
| 0.1 | 88.4% | 98.3% | 90.4% | 69.8% | 44.8% |
| 0.3 | 89.1% | 96.9% | 91.7% | 75.6% | 54.3% |
| 0.5 | 90.3% | 98.7% | 93.6% | 82.3% | 66.8% |
| 0.7 | 92.6% | 99% | 95% | 89.3% | 81.8% |
| 0.9 | 96.7% | 99.6% | 98.4% | 97% | 94% |

Table 6: The effect of feature differentiation on value appropriation when the retailer determines the shelf-space allocation.

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## Supplementary Material for "The Role of Store Brand Positioning for Appropriating Supply Chain Profit under Shelf Space Allocation"

**Proof of Lemma 1**

When the available SB selling amount is unconstrained by her shelf-space, by solving the first-order (optimality) conditions (FOCs) we obtain equations (2) and (3). When the available SB selling amount is constrained by her shelf-space, the retailer's problem becomes:

$$\max_{p_n, p_s, \lambda \geq 0} \Pi_s \quad = (p_n - w_n) D_n + (p_s - c_s)(M - Q_n) + kQ_n - hM - \lambda (M - Q_n - D_s)$$

where $\lambda$ is the Lagrangian multiplier for the equality constraint $D_s = Q_s \Leftrightarrow A_s + Q_n - p_s - \gamma_s p_n = M$. By the FOCs, the optimal retail prices for both brands are obtained in equations (4) and (5). So Lemma 1 is proved. ■

**Proof of Lemma 2**

$(B, F)$ *Configuration:* If the NB selling amount is shelf-space constrained but the SB is not, by substituting $Q_n = D_n$ and equations (2) to (3) then the manufacturer's problem becomes:

$$\max_{w_n, Q_n \geq 0} \Pi_n \quad = (w_n - c_n - k)\left[A_n - p_n^F(w_n) + \gamma_n p_s^F(w_n)\right].$$

Solving the FOCs yields the optimal wholesale price and the optimal shelf-space allocation for the NB:

$$w_n(B, F) = \frac{\Lambda_n(B, F)}{4(1 - \gamma_n \gamma_s)} \quad \text{and} \tag{A-1}$$

$$Q_n(B, F) = \frac{\Lambda_n(B, F)}{8 - 2(\gamma_n + \gamma_s)^2}, \tag{A-2}$$

where

$$\Lambda_n(B, F) \equiv [2 - \gamma_s(\gamma_n + \gamma_s)] A_n + (\gamma_n - \gamma_s) A_s + (\gamma_n + \gamma_s)(1 - \gamma_n \gamma_s) c_s + 2(1 - \gamma_n \gamma_s)(c_n + k).$$

Therefore, substituting (A-1) and (A-2) into $\Pi_n$ the associated profit of the NB manufacturer is given by

$$\Pi_n(B, F) = \frac{[\Lambda_n(B, F)]^2}{8(1 - \gamma_n \gamma_s)[4 - (\gamma_n + \gamma_s)^2]}. \tag{A-3}$$

$(F, F)$ *Configuration:* If the SB selling amount is also unconstrained, the corresponding retail prices for both brands are defined by (2) and (3). Because both prices are not functions of $Q_n$ and $D_n \wedge Q_n = D_n$ under the $(F, F)$ configuration, it is conceivable to see that $\Pi_n$ strictly decreases in $Q_n$ (due to $\partial \Pi_n / \partial Q_n = -k < 0$). It directly follows from the above fact that the manufacturer profit is decreasing in $Q_n$ and hence, he will set $Q_n$ for the minimum requirement, i.e., the NB demand of $D_n$. This decision rationale is exactly the $(B, F)$ configuration, thus leading to result (a).

$(B, B)$ *Configuration:* If both NB and SB selling amounts are shelf-space constrained, the retail prices are based on equations (4) and (5). With $Q_n = D_n$ the manufacturer's problem is given by:

$$\max_{w_n, Q_n \geq 0} \Pi_n = \quad (w_n - c_n - k) \left[ A_n - p_n^B (w_n, Q_n) + \gamma_n p_s^B (w_n, Q_n) \right].$$

Solving the above problem, we obtain the optimal wholesale price and the optimal shelf-space allocation for the NB as follows:

$$w_n^d(B, B) = \frac{A_n + \gamma_n A_s - (\gamma_n + \gamma_s)M + (1 - \gamma_n \gamma_s)(c_n + k)}{2(1 - \gamma_n \gamma_s)}, \tag{A-4}$$

$$Q_n^d(B, B) = \frac{A_n + \gamma_n A_s - (\gamma_n + \gamma_s)M - (1 - \gamma_n \gamma_s)(c_n + k)}{4 - 2(\gamma_n + \gamma_s)}. \tag{A-5}$$

Notice that the shelf-space allocation for the NB, $Q_n$, is bounded above by $M$. We consider two cases depending on whether $Q_n^d(B, B)$ in (A-5) is greater than $M$. First, if $Q_n^d(B, B)$ is less than $M$, i.e., $M \geq H$ where

$$H \equiv \frac{A_n + \gamma_n A_s - (1 - \gamma_n \gamma_s)(c_n + k)}{4 - (\gamma_n + \gamma_s)},$$

from (A-4) and (A-5) the manufacturer's profit is

$$\Pi_n^d(B, B) = \frac{\left[ A_n + \gamma_n A_s - (\gamma_n + \gamma_s)M - (1 - \gamma_n \gamma_s)(c_n + k) \right]^2}{4 \left[ 2 - (\gamma_n + \gamma_s) \right] (1 - \gamma_n \gamma_s)}. \tag{A-6}$$

We call this case the "duopolistic $(B, B)$ configuration" since both brands are sold on the category shelf-space.

On the other hand, if $M$ is sufficiently low, i.e., $M < H$, the manufacturer takes all the space and then becomes a monopolist in the product category. In the case, there are only NB products on the category shelf-space so the retailer's best response is to set the retail prices $p_n^B (w_n, Q_n)$ and $p_s^B (w_n, Q_n)$ leading to a zero demand for the SB: for $M < H$,

$$w_n^m(B, B) = \frac{A_n + \gamma_n A_s - 2M}{1 - \gamma_n \gamma_s}, \tag{A-7}$$

$$Q_n^m(B, B) = D_n^m(B, B) = M, \tag{A-8}$$

$$\Pi_n^m(B, B) = \frac{(A_n + \gamma_n A_s)M - (1 - \gamma_n \gamma_s)(c_n + k)M - 2M^2}{1 - \gamma_n \gamma_s}. \tag{A-9}$$

The above case is referred to the "monopolistic $(B, B)$ configuration".

$(F, B)$ *Configuration:* If the shelf-space allocation constraint for the SB is binding but the space constraint for the NB is unbinding, according to equations (4) and (5), we have the manufacturer's problem:

$$\max_{w_n \geq 0, Q_n > D_n \geq 0} \Pi_n = \quad (w_n - c_n) \left[ A_n - p_n^B (w_n, Q_n) + \gamma_n p_s^B (w_n, Q_n) \right] - kQ_n. \tag{A-10}$$

To solve the above problem, we need to first find the optimal wholesale price given the space allocation of $Q_n$. We then can obtain the optimal $Q_n$ that maximizes the manufacturer's profit. Given $Q_n$, the manufacturer chooses the wholesale price:

$$w_n(Q_n) = \frac{A_n + \gamma_n A_s + (\gamma_n + \gamma_s)Q_n - (\gamma_n + \gamma_s)M}{2(1 - \gamma_n\gamma_s)} + \frac{c_n}{2}. \tag{A-11}$$

Substituting (A-11) into (A-10) yields:

$$\Pi_n(F, B) = \frac{[A_n + \gamma_n A_s + \gamma_n + \gamma_s - (\gamma_n + \gamma_s)(M - Q_n) - (1 - \gamma_n\gamma_s)c_n]^2}{8(1 - \gamma_n\gamma_s)} - kQ_n$$

which is quadratic convex in $Q_n$. Under the $(F, B)$ configuration, the upper bound of $Q_n$ is the category shelf-space of $M$ and the lower bound of $Q_n$ is $D_n + \epsilon$ (because of unconstrained shelf-space for the NB) where $\epsilon$ is a small positive number. Due to the strict convexity of $\Pi_n(F, B)$, the optimal space allocation for the national brand is either $M$ or $D_n + \epsilon$.

Consider the former case, i.e.,

$$Q_n^m(F, B) = M. \tag{A-12}$$

From equation (A-11), the resultant optimal wholesale price is

$$w_n^m(F, B) = \frac{A_n + \gamma_n A_s}{2(1 - \gamma_n\gamma_s)} + \frac{c_n}{2}. \tag{A-13}$$

So, the NB manufacturer's profit becomes:

$$\Pi_n^m(F, B) = \frac{[A_n + \gamma_n A_s - (1 - \gamma_n\gamma_s)c_n]^2}{8(1 - \gamma_n\gamma_s)} - kM. \tag{A-14}$$

This case is referred to the "monopolistic $(F, B)$ configuration" as the manufacturer will take all the category shelf-space.

The latter case is termed as "duopolistic $(F, B)$ configuration" because of $Q_n = D_n + \epsilon$. The optimal wholesale price and the shelf-space allocation are:

$$w_n^d(F, B) = \frac{2(A_n + \gamma_n A_s) - 2(\gamma_n + \gamma_s)}{(1 - \gamma_n\gamma_s)\left[4 - (\gamma_n + \gamma_s)\right]} + \frac{\left[2 - (\gamma_n + \gamma_s)\right] c_n}{4 - (\gamma_n + \gamma_s)} + \epsilon,$$

$$Q_n^d(F, B) = \frac{A_n + \gamma_n A_s - (\gamma_n + \gamma_s)M - (1 - \gamma_n\gamma_s)c_n}{4 - (\gamma_n + \gamma_s)} + \epsilon.$$

Thus, the manufacturer's optimal profit is

$$\Pi_n^d(F, B) = \left[w_n^d(F, B) - c_n\right] D_n\left(w_n^d(F, B)\right) - kQ_n^d(F, B).$$

Notice that due to $\epsilon > 0$, we are able to show that $\Pi_n^d(F, B) \leq \Lambda_n(F, B)$ where

$$\Lambda_n(F, B) \equiv \left[w_n^d(F, B) - c_n\right] D_n\left(w_n^d(F, B)\right) - kD_n\left(w_n^d(F, B)\right).$$

Furthermore, it is not difficult to show that $\Lambda(F,B) \leq \Pi_n^d(B,B)$. As a result, with simple algebra, we obtain result (b). ■

**Proof of Lemma 3**

According to Lemmas 1 and 2, given the category shelf-space of $M$ the optimal configuration is one of the following cases: (1) the monopolistic $(F,B)$ configuration, (2) the monopolistic $(B,B)$ configuration, (3) the duopolistic $(B,B)$ configuration, and (4) the $(B,F)$ configuration. For the first two configurations, the manufacturer chooses the shelf-space allocation of $Q_n$ equal to the total category shelf-space, $M$. That is, the manufacturer takes all the shelf-space. Observe this, the retailer's best response is to set the retail prices of both brands such that the SB demand is zero since any positive demand does not add any profit from the SB but definitely reduce the benefit received from the NB. Thus, under cases (1) and (2) the manufacturer can dictate the retailer to follow his preferred shelf-space configurations.

Consider case (3) where the manufacturer prefers the monopolistic $(B,B)$ configuration. If the retailer deviates from the manufacturer's preferred configuration, she will choose $p_n^F$ and $p_s^F$ such that the SB demand is strictly less than the shelf-space allocated for the SB, $M - Q_n^d(B,B)$. However, based on equations (2), (3), and (A-5), we have

$$\begin{aligned} D_s(w_n^d(B,B)) &= \frac{\left[\begin{array}{c} [4-\gamma_n(\gamma_n+\gamma_s)]\,A_s + (3\gamma_s-\gamma_n)\gamma_n - 4(1-\gamma_s\gamma_n)c_s \\ -(\gamma_n+\gamma_s)^2 M + (1-\gamma_n\gamma_s)(\gamma_n+\gamma_s)(c_n+k) \end{array}\right]}{2\,[2+(\gamma_n+\gamma_s)]\,[2-(\gamma_n+\gamma_s)]} \\ &\geq M - Q_n^d(B,B) \end{aligned}$$

which shows the infeasibility of SB demand if the retailer chooses the unbinding retail prices. Therefore, the retailer will not deviate as long as the manufacturer prefers the monopolistic $(B,B)$ configuration. Finally, for the situation while the $(B,F)$ configuration is preferred by the manufacturer, it is intuitive that the retailer will not choose the retail prices of $p_n^B$ and $p_s^B$ that make the SB demand binding. The reason for that is due to the flexibility under the $(B,F)$ configuration which induces the retailer to gain more benefits from the SB. Thus, the retailer will follow the $(B,F)$ configuration if it is preferred by the manufacturer. This completes the proof. ■

**Proof of Lemma 4**

In the following, we prove the result based on each $(F_l, L_l)$ pair, $l = 1, \ldots 6$.

**(i) Duopolistic $(B,B)$ versus $(B,F)$**

The manufacturer's profits under the duopolistic $(B,B)$ configuration, $\Pi_n^d(B,B)$, and under the $(B,F)$ configuration, $\Pi_n(B,F)$, are given by (A-6) and (A-3). With simple algebra, we have

$\Pi_n^d(B,B) \geq \Pi_n(B,F)$ if

$$\begin{aligned}&[A_n+\gamma_n A_s-(\gamma_n+\gamma_s)M-(1-\gamma_n\gamma_s)(c_n+k)]^2\\ &\quad\geq \frac{[[2-\gamma_s(\gamma_n+\gamma_s)]A_n+(\gamma_n-\gamma_s)A_s+(\gamma_n+\gamma_s)(1-\gamma_n\gamma_s)c_s-2(1-\gamma_n\gamma_s)(c_n+k)]^2}{4+2(\gamma_n+\gamma_s)}\end{aligned}$$

Recall that the shelf-space allocated to the NB under both configurations are $Q_n^d(B,B)$ and $Q_n(B,F)$ based on (A-5) and (A-2), respectively. To ensure the nonnegativity of $Q_n^d(B,B)$ and $Q_n(B,F)$, the following conditions must hold:

$$A_n+\gamma_n A_s-(\gamma_n+\gamma_s)M-(1-\gamma_n\gamma_s)(c_n+k)\geq 0 \quad \text{and}$$

$$(2-\gamma_s(\gamma_n+\gamma_s))A_n+(\gamma_n-\gamma_s)A_s+(\gamma_n+\gamma_s)(1-\gamma_n\gamma_s)c_s-2(1-\gamma_n\gamma_s)(c_n+k)\geq 0.$$

With the above conditions, we can rewrite the condition for $\Pi_n^d(B,B) \geq \Pi_n(B,F)$ as $M \leq \widehat{M}_1$ where

$$\widehat{M}_1 \equiv \frac{G_1}{\gamma_n+\gamma_s} \quad \text{and}$$

$$\begin{aligned}G_1 &= A_n+\gamma_n A_s-(1-\gamma_n\gamma_s)(c_n+k)\\ &\quad -\frac{[2-\gamma_s(\gamma_n+\gamma_s)]A_n+(\gamma_n-\gamma_s)A_s+(\gamma_n+\gamma_s)(1-\gamma_n\gamma_s)c_s-2(1-\gamma_n\gamma_s)(c_n+k)}{\sqrt{4+2(\gamma_n+\gamma_s)}}.\end{aligned}$$

**(ii) Monopolistic $(B,B)$ versus $(B,F)$**

The manufacturer's profits under the two configurations are shown in (A-3) and (A-9). With simple algebra, we have $\Pi_n^m(B,B)-\Pi_n(B,F)\geq 0 \Leftrightarrow$

$$\begin{aligned}&M^2-\frac{A_n+\gamma_n A_s-(1-\gamma_n\gamma_s)(c_n+k)}{2}M\\ &\quad+\frac{[[2-\gamma_s(\gamma_n+\gamma_s)]A_n+(\gamma_n-\gamma_s)A_s+(\gamma_n+\gamma_s)(1-\gamma_n\gamma_s)c_s-2(1-\gamma_n\gamma_s)(c_n+k)]^2}{16\,[4-(\gamma_n+\gamma_s)^2]}\leq 0.\end{aligned}$$

Now, define

$$\Delta \equiv G_2^2-4G_3$$

where

$$\begin{aligned}G_2 &= \frac{A_n+\gamma_n A_s-(1-\gamma_n\gamma_s)(c_n+k)}{2} \qquad \text{and}\\ G_3 &= \frac{[[2-\gamma_s(\gamma_n+\gamma_s)]A_n+(\gamma_n-\gamma_s)A_s+(\gamma_n+\gamma_s)(1-\gamma_n\gamma_s)c_s-2(1-\gamma_n\gamma_s)(c_n+k)]^2}{16\,[4-(\gamma_n+\gamma_s)^2]}.\end{aligned}$$

There are two cases depending on the sign of $\Delta$ as discussed below.

**Case 1:** $\Delta < 0$

Note that $\Delta < 0$ implies $\Pi_n(B,F) \geq \Pi_n^m(B,B)$ for all $M \geq 0$; however, we need to consider the feasibility of $(B,F)$ configuration from the constraint of

$$G_4 \equiv D_s(B,F)+Q_n(B,F) \leq M$$

where $M = Q_s(B,F) + Q_n(B,F)$ and $D_s(B,F)$ is the SB demand under the $(B,F)$ configuration. Note from equations (2), (3), (A-1), and (A-2) that the space allocation for the NB and the demand for the SB under the $(B,F)$ configuration are given by (A-2) and

$$\begin{aligned} D_s(B,F) &= \frac{A_s + \gamma_s A_n - (1-\gamma_n\gamma_s)c_s}{4} \\ &+ \frac{2\left[A_s + \gamma_s A_n - (1-\gamma_n\gamma_s)c_s\right] - (\gamma_n + \gamma_s)\left[A_n + \gamma_n A_s - (1-\gamma_n\gamma_s)(c_n + k)\right]}{2\left[4 - (\gamma_n + \gamma_s)^2\right]}. \end{aligned}$$

So explicitly define

$$G_4 = \frac{A_s + \gamma_s A_n - (1-\gamma_n\gamma_s)c_s}{4} + \frac{A_s + \gamma_s A_n + A_n + \gamma_n A_s - (1-\gamma_n\gamma_s)(c_n + c_s + k)}{4 + 2(\gamma_n + \gamma_s)}.$$

Therefore, if $M < G_4$ the $(B,F)$ configuration is infeasible for the manufacturer. In sum, the manufacturer prefers the monopolistic $(B,B)$ configuration if $M < G_4$ and the $(B,F)$ configuration otherwise.

**Case 2:** $\Delta \geq 0$

In this case, we have $\Pi_n^m(B,B) = \Pi_n(B,F) \geq 0$ if

$$\frac{G_2 - \sqrt{G_2^2 - 4G_3}}{2} \leq M \leq \frac{G_2 + \sqrt{G_2^2 - 4G_3}}{2}.$$

The part of $M \geq \frac{G_2 - \sqrt{G_2^2 - 4G_3}}{2}$ is irrelevant in the analysis. That is because if the monopolistic $(B,B)$ configuration is preferred for some $M_1$, then the monopolistic $(B,B)$ configuration is still preferred for $M_2 < M_1$. Notice that the result is intuitive in the sense that from the definition of $(B,F)$ configuration, if the $(B,F)$ configuration is preferred for $M = M_2$, then at $M_1 > M_2$ the manufacturer will still choose the $(B,F)$ configuration and leave the extra shelf-space of $M_1 - M_2$ to the SB, doing which the manufacturer maintains the optimum of his profit function. Thus, we have the $(B,F)$ configuration being preferred if $M \geq \frac{G_2 + \sqrt{G_2^2 - 4G_3}}{2}$ and the monopolistic $(B,B)$ configuration otherwise.

Combining both cases, we conclude that the manufacturer chooses $(B,F)$ configuration when

$$M \geq \max\left[\frac{G_2 + \sqrt{G_2^2 - 4G_3}}{2}, G_4\right] \equiv \widehat{M}_2.$$

**(iii) Monopolistic $(B,B)$ versus Duopolistic $(B,B)$**

Let

$$\widehat{M}_3 \equiv H$$

for the ease of consistent expression where $H$ is defined in Section 5. As mentioned in Section 5, the manufacturer prefers the monopolistic $(B,B)$ configuration if $M < \widehat{M}_3$ and the duopolistic $(B,B)$ configuration otherwise.

**(iv) Monopolistic $(F,B)$ versus $(B,F)$**

From equations (A-3) and (A-14), we have $\Pi_n^m(F,B) \geq \Pi_n(B,F)$ if $M \leq G_5 - G_6$ where

$$\begin{aligned} G_5 &= \frac{[A_n + \gamma_n A_s (1-\gamma_n\gamma_s)c_n]^2}{8k(1-\gamma_n\gamma_s)} \quad \text{and} \\ G_6 &= \frac{\left[[2-\gamma_s(\gamma_n+\gamma_s)]\,A_n + (\gamma_n-\gamma_s)A_s + (\gamma_n+\gamma_s)(1-\gamma_n\gamma_s)c_s - 2(1-\gamma_n\gamma_s)(c_n+k)\right]^2}{8k(1-\gamma_n\gamma_s)\,[4-(\gamma_n+\gamma_s)^2]}. \end{aligned}$$

That is, let

$$\widehat{M_4} \equiv G_5 - G_6$$

the manufacturer prefers the $(F,B)$ configuration if $M \leq \widehat{M_4}$ and the monopolistic $(B,F)$ configuration otherwise.

**(v) Monopolistic $(B,B)$ versus Monopolistic $(F,B)$**

The manufacturer' profits under the monopolistic $(B,B)$ and monopolistic $(F,B)$ configurations are given by equations (A-9) and (A-14), respectively. Since both $\gamma_n$ and $\gamma_s$ are between 0 and 1, we obtain that for $M \geq 0$,

$$\Pi_n^m(F,B) - \Pi_n^m(B,B) = \frac{2\left(M - \widehat{M_5}\right)^2}{1-\gamma_n\gamma_s} \geq 0$$

where

$$\widehat{M_5} \equiv \frac{A_n + \gamma_n A_s - (1-\gamma_n\gamma_s)c_n}{4}.$$

From equations (4), (5), and (A-12), if $M < \widehat{M_5}$ we then have $Q_n^m(F,B) = M < D_n^m(F,B) = \widehat{M_5}$, resulting in the infeasibility of the NB demand under the monopolistic $(F,B)$ configuration. As a result, we have that the manufacturer prefers the monopolistic $(B,B)$ configuration if $M < \widehat{M_5}$ and the monopolistic $(F,B)$ configuration otherwise.

**(vi) Monopolistic $(F,B)$ versus Duopolistic $(B,B)$**

The manufacturer's profits under the monopolistic $(B,B)$ and monopolistic $(F,B)$ configurations are given by equations (A-6) and (A-14), respectively, we have $\Pi_n^d(F,B) \geq \Pi_n^m(B,B)$ if $\frac{G_7-G_8}{4(\gamma_n+\gamma_s)^2} \leq M < \frac{G_7+G_8}{4(\gamma_n+\gamma_s)^2}$ where

$$\begin{aligned} G_7 &= 4(\gamma_n+\gamma_s)\,[A_n + \gamma_n A_s - (1-\gamma_n\gamma_s)c_n] - 4\,[4-(\gamma_n+\gamma_s)]\,(1-\gamma_n\gamma_s)k \quad \text{and} \\ G_8 &= \sqrt{4-2(\gamma_n+\gamma_s)}\,\left|2(\gamma_n+\gamma_s)\,[A_n+\gamma_n A_s - (1-\gamma_n\gamma_s)c_n] - 8(1-\gamma_n\gamma_s)k\right|. \end{aligned}$$

Combining the above results with the fact that the manufacturer chooses the duopolistic $(B,B)$ configuration if $M \geq \widehat{M_3}$ and the monopolistic $(B,B)$ configuration otherwise (see case (iii)), we can conclude that the duopolistic $(B,B)$ configuration will never be the optimal from the manufacturer's perspective when $M < \frac{G_7-G_8}{4(\gamma_n+\gamma_s)^2}$. We thus obtain that the monopolistic $(F,B)$ configuration is preferred when

$$M < \frac{G_7+G_8}{4(\gamma_n+\gamma_s)^2} \equiv \widehat{M_6}$$

and the duopolistic $(B, B)$ configuration is preferred otherwise. This completes the proof. ■

**Proof of Proposition 2**

Note that for $M \in \big(0, \overline{M}_1\big)$, the monopolistic $(B, B)$ configuration is optimal and we have from equation (A-9) that the manufacturer's profit $\Pi_n^m(B, B)$ is concave in $M$. For $M \in \big[\overline{M}_1, \overline{M}_2\big)$, the monopolistic $(F, B)$ configuration is optimal and we have from equation (A-14) that the manufacturer's profit is decreasing in $M$. For $M \in \big[\overline{M}_2, \overline{M}_3\big)$, the duopolistic $(B, B)$ configuration is optimal and it is not hard to observe from equation (A-6) that the manufacturer's profit decreases in $M$. Finally, when $M \in \big[\overline{M}_3, \infty\big)$, the $(B, F)$ configuration is optimal and observe from equation (A-3) that the manufacturer's profit is independent of $M$. Besides, it is not hard to show the continuities of $\Pi_n$ while the supply chain switches the configurations from the monopolistic $(B, B)$ to the monopolistic $(F, B)$, from the monopolistic $(F, B)$ to the duopolistic $(B, B)$, and from the duopolistic $(B, B)$ to the $(B, F)$. As a result, we conclude that the manufacturer prefers the category shelf-space of $M \leq \overline{M}_1$. ■

**Proof of Proposition 3**

The retailer's profit is given by equation (1). Substituting the retail prices (in equations (2), (3), (4), and (5)), the wholesale price, and the shelf-space allocation into the retailer's profit function and differentiating the retailer's profit function with respect to $M$, we obtain that for $M \in \big(0, \overline{M}_1\big)$, the monopolistic $(B, B)$ configuration is optimal and

$$\frac{\partial \Pi_s(p_n^B, p_s^B, w_n^m(B, B), Q_n^m(B, B))}{\partial M} = \frac{2M}{1 - \gamma_n \gamma_s} + (k - h) > 0.$$

For $M \in \big[\overline{M}_1, \overline{M}_2\big)$, the monopolistic $(F, B)$ configuration is optimal and

$$\frac{\partial \Pi_s(p_n^B, p_s^B, w_n^m(F, B), Q_n^m(F, B))}{\partial M} = k - h > 0.$$

For $M \in \big[\overline{M}_2, \overline{M}_3\big)$, the duopolistic $(B, B)$ configuration is optimal and

$$\frac{\partial^2 \Pi_s(p_n^B, p_s^B, w_n^d(B, B), Q_n^d(B, B))}{\partial M^2} = \frac{-32 + 16(\gamma_n + \gamma_s)}{(1 - \gamma_n \gamma_s)(4 - 2(\gamma_n + \gamma_s))^2} < 0.$$

For $M \in \big[\overline{M}_3, \infty\big)$, the $(B, F)$ configuration is optimal and

$$\frac{\partial \Pi_s(p_n^F, p_s^F, w_n(B, F), Q_n(B, F))}{\partial M} = -h < 0.$$

We therefore obtain that $\Pi_s$ is increasing for $M < \overline{M}_2$, concave for $\overline{M}_2 \leq M < \overline{M}_3$, and decreasing for $M \geq \overline{M}_3$. Furthermore, it is not hard to show the continuities of $\Pi_s$ while the supply chain switches the configurations from the monopolistic $(B, B)$ to the monopolistic $(F, B)$ and from the monopolistic $(F, B)$ to the duopolistic $(B, B)$. As a result, we conclude that the equilibrium category shelf-space for the retailer falls into the range of $\left[\overline{M}_2, \overline{M}_3\right]$. ■

**Proof of Corollary 1**

From Proposition 1, we know that when $\overline{M}_2 \leq M < \overline{M}_3$, the optimal configuration is the duopolistic $(B, B)$. To show the result, we substitute the optimal wholesale price and space allocation, given in equations (A-4) and (A-5), into equations (4) and (5), and differentiate both retail prices with respect to $M$, we obtain:

$$\begin{aligned}\frac{\partial p_n^B(w_n^d\left(B,B\right), Q_n^d\left(B,B\right))}{\partial M} &= \frac{\gamma_n^2 + \gamma_n\gamma_s + \gamma_s - 3\gamma_n}{\left(1-\gamma_n\gamma_s\right)\left[4 - 2(\gamma_n+\gamma_s)\right]},\\ \frac{\partial p_s^B(w_n^d\left(B,B\right), Q_n^d\left(B,B\right))}{\partial M} &= \frac{4 - (\gamma_n+\gamma_s)(1+\gamma_s)}{\left(1-\gamma_n\gamma_s\right)\left[4 - 2(\gamma_n+\gamma_s)\right]}.\end{aligned}$$

Note from the above, $p_s^B$ decreases in $M$ since both $\gamma_n, \gamma_s \in [0, 1)$, and thus, $4-(\gamma_n+\gamma_s)(1+\gamma_s) > 0$. Also, $p_n^B$ decreases in $M$ when $\gamma_s < \sigma(\gamma_n)$. Hence, for $\overline{M}_2 \leq M < \overline{M}_3$, the SB retail price decreases in $M$ under the duopolistic $(B, B)$ configuration. The NB retail price decreases in $M$ under the duopolistic $(B, B)$ configuration when $\gamma_s < \sigma(\gamma_n)$. ■

It is conceivable to see in Corollary 1 that the condition $\gamma_s \geq \sigma\left(\gamma_n\right)$ leads to $\gamma_s \geq \gamma_n$, meaning that the retailer tends to raise the NB retail price when $\gamma_s$ is relatively high. The intuition behind the observation is that with a higher $\gamma_s$, the NB retail price influences significantly the SB demand. For the retailer, raising the NB retail price can not only forces more space allocated to the SB, but also enlarge the profit margin of the NB. As a result, the directions of both retail prices will differ when $\gamma_s$ is high enough. The aforementioned effect, however, diminishes when $\gamma_s$ is relatively low as the retailer reduces both retail prices accordingly.

**Proof of Corollaries 2, 3, and 4**

Note from Proposition 3, the duopolistic $(B, B)$ configuration is the equilibrium and thus, the result directly follows by equations (4), (5), (A-4), and (A-5). This completes the proof. ■

Applying this proposition and the optimal decisions of the wholesale price and the NB shelf-space (see Section 4), we obtain the comparative statics:

**Corollary 5.** *For $\overline{M}_2 \leq M < \overline{M}_3$, the optimal NB wholesale price and the optimal NB shelf-space are both decreasing in $M$.*

**Proof:** From Proposition 1, the optimal configuration is the duopolistic $(B, B)$ when $\overline{M}_2 \leq M < \overline{M}_3$. From equations (A-4) and (A-5), we have that $\frac{\partial w_n^d(B,B)}{\partial M} = \frac{-(\gamma_n+\gamma_s)}{2(1-\gamma_n\gamma_s)}$ and $\frac{\partial Q_n^d(B,B)}{\partial M} = \frac{-(\gamma_n+\gamma_s)}{4-2(\gamma_n+\gamma_s)}$.

Since $0 < \gamma_n, \gamma_s < 1$, we conclude that both the optimal wholesale price and shelf-space allocation decrease in $M$. ■

The above corollary suggests that when $M$ is moderate, i.e., $\overline{M}_2 \leq M < \overline{M}_3$, both brands compete for the limited shelf-space, i.e., the space size is less than the demands for both brands. Intuitively, both brands play a conventional duopoly here so the optimal wholesale price is decreasing in $M$. With competition, the manufacturer is forced to reduce the space allocated to the NB since the retailer takes advantage of setting retail prices to influence the NB demand.

**Proof of Proposition 4**

Corollaries 1, 5, and Proposition 3 lead to the result. Proposition 4 is proved. ■