Wholesale Price Rebate vs. Capacity Expansion: The Optimal Strategy for Seasonal Products in a Supply Chain

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Abstract

We consider a supply chain in which one manufacturer sells a seasonal product to the end market through a retailer. Faced with uncertain market demand and limited capacity, the manufacturer can maximize its profits by adopting one of two strategies, namely, wholesale price rebate or capacity expansion. In the former, the manufacturer provides the retailer with a discount for accepting early delivery in an earlier period. In the latter, the production capacity of the manufacturer in the second period can be raised so that production is delayed until in the period close to the selling season to avoid holding costs. Our research shows that the best strategy for the manufacturer is determined by three driving forces: the unit cost of holding inventory for the manufacturer, the unit cost of holding inventory for the retailer, and the unit cost of capacity expansion. When the single period capacity is low, adopting the capacity expansion strategy dominates as both parties can improve their profits compared to the wholesale price rebate strategy. When the single period capacity is high, on the other hand, the equilibrium outcome is the wholesale price rebate strategy.

Keywords: Supply chain management; seasonal product; capacity expansion; wholesale price rebate.

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1. Introduction

In a supply chain, manufacturers and retailers always seek to match supply with demand through production planning and demand management. For seasonal products, in particular, members of the supply chain need better strategies for coordinating supply and demand. For example, air conditioner manufacturers that face high demand during the summer may not fully respond to orders from their downstream partners because of a significantly long leadtime of production. Hence, manufacturers generally shift a proportion of their production to earlier periods and store produced goods in the warehouse to satisfy peak-season demand. This production movement indirectly heaps inventory holding pressure onto the manufacturers. A similar phenomenon also occurs within the fashion apparel industry in which retailers usually place a single order with their manufacturers because of offshore production, making replenishment difficult to arrange. Faced with these situations, manufacturers may adopt a variety of strategies to alleviate the pressure of product shortage and to foster supply chain efficiency.

To mitigate the risk of production shortage, manufacturers can initiate production at earlier periods and encourage retailers to hold these items. In pursuit of this goal, an incentive should be offered to the retailers to compensate for the costs of storing these items. One commonly used strategy is the wholesale price rebate. A notable example dates back to 1994 when Gree, China's largest air conditioner manufacturer, faced a situation where almost all the retailers ordered items close to the peak season. To tackle this issue, Gree came up with a new pricing policy, the off-peak season rebate. To wit, if the retailer was willing to take delivery of the air conditioners during the off-peak season, Gree provided a price discount. This strategy successfully filled the spare capacity of Gree's production in the off-peak season and reduced the possibility of product shortage in the peak season.

Another strategy manufacturers may adopt in response to peak season demand is to use a temporary workforce or equipment or to implement workforce overtime to increase capacity. Practical applications of this strategy are widely observed in both manufacturing and service industries, such as automobile and tourism sectors (Chopra and Meindl, 2009). These examples identify an intrinsic rationale through which the manufacturers or service providers adjust their capacity during the peak season to further avoid shortage loss. Although the capacity expansion is an additional expense for manufacturers, aggregating production close to the peak season not only reduces setup costs, but also increases the flexibility of matching demand orders.

In this paper, we consider a two-echelon supply chain in which a manufacturer sells a seasonal product to the end market through a retailer. On one hand, the retailer faces a fully competitive market wherein the retail price is exogenously determined and the only decision the retailer can make is the order quantity. The manufacturer, on the other hand, is able to allocate production across two time periods. To mitigate the pressure of holding inventories, the manufacturer should postpone the production to the period close to the selling season and set up the production line in an earlier period only if the order exceeds the capacity of a single period, which is called *basic strategy*. In addition, we further investigate two strategies of the manufacturer:

- *Wholesale price rebate strategy*: the manufacturer offers subsidies to the retailer for taking delivery in the earlier period.

- *Capacity expansion strategy*: the production capacity of the manufacturer is expanded in the second period.

We characterize the optimal decisions of the manufacturer and the retailer in each strategy and compare the profits of both parties to a situation where the basic strategy is adopted. To gain additional managerial insights, we assume the market demand follows a uniform distribution, and conditions are provided to obtain the manufacturer's preferred strategy. Our results reveal that when single period capacity is sufficiently high, the manufacturer tends to adopt the wholesale price rebate strategy and associated order quantity of the retailer is also larger under the strategy. This is due to the fact that under the wholesale price rebate strategy, the manufacturer raises the wholesale price charged to the retailer to a high level and the retailer gains more profits when the order quantity exceeds the single period capacity. This high wholesale price decision combined with a rebate to the retailer leads to a high order quantity accordingly. This effect diminishes as the single period capacity is low and in this case the equilibrium outcome moves to the capacity expansion strategy. Also, our numerical study shows that among the three strategies, both parties are better off by implementing the capacity expansion strategy compared to the basic strategy. On the other hand, the manufacturer can improve its profit under the wholesale price rebate strategy but the retailer is worse off even though a compensation is offered by the manufacturer under this strategy.

The remainder of this paper is organized as follows. Section 2 provides a survey of relevant literature. Sections 3 and 4 describe our models and derive analytical results, respectively. In Sections 5 and 6, we demonstrate the results when the demand follows a specific distribution and implement the numerical study. A discussion of the results is included in Section 7. All proofs are relegated to the Appendix.

2. Literature Review

We review the literature with regard to three aspects: seasonal products, supply chain contracts, and capacity management. Two distinct characteristics of the seasonal products, namely, cyclical demand and perishability, give rise to many interesting research topics in production planning. An earlier research that investigates seasonal products can be traced back to Chang and Fyffe (1971). Voros (1999) discusses the risks faced by manufacturers who produce seasonal goods and provides suggestions on minimizing risks by a learning effect on the uncertain demand. Chen and Xu (2001) conclude that downstream members in a supply chain, such as retailers, tend to issue demand orders for seasonal products as close to the selling season as possible.

In the traditional supply chain, seasonal products generally have a relatively low salvage value at the end of the selling season. One of the major factors affecting the number of unsold items is the retail price. Smith and Achabal (1998) conclude that clearance price at the end of the selling season and inventory management significantly influence a retailer's profits. Furthermore, Bitran and Mondschein (1997) consider the seasonal product as non-refundable. Thus, retailers lower retail prices to promote sales at the end of the selling season. In addition, Aviv and Pazgal (2008) consider the existence of strategic customers and adopt the Stackelberg game to obtain the optimal pricing strategy. DeYong and Cattani (2012) use a two-period newsvendor model for a case where an order quantity can be revised based on updated information.

Our research is also related to the topic of supply chain contracts in a decentralized supply chain. Bresnahan and Reiss (1985) investigate the relationship between the marginal revenue of a car dealer and the pricing model of a manufacturer. Lariviere and Porteus (2001) discuss the wholesale price decision for a manufacturer in a newsvendor setting. Padmanabhan and Png (1997) compare the profits of a manufacturer among different scenarios depending on whether the manufacturer allows the return of unsold items. Cachon and Zipkin (1999) study the effect of providing subsidies to the retailer, and Viswanathan and Wang (2003) study the effect of price elasticity on quantity discount. Marvel and Peck (1995) consider a scenario with uncertain demand where the manufacturer changes the return policy for the retailer. Cachon (2002) summarizes several contracts designed by a manufacturer to enhance profit and investigates whether the manufacturer can provide some incentives to the retailers so as to coordinate the supply chain. These widely used contracts include buyback contract (Emmons and Gilbert, 1998), revenue sharing contract (Dana and Spier, 2001, Giannoccaro and Pontrandolfo, 2004, and Cachon and Lariviere, 2005), sales rebate contract (Taylor, 2002) and quantity flexibility contract (Tsay, 1999, Tsay and Lovejoy, 1999, and Lian and Deshmukh, 2009).

A fair amount of research explores capacity management in a supply chain. Clearly, proper capacity planning and management can reduce costs, satisfy orders on time, lower inventory level, raise utilization of equipment, and alleviate the fluctuation of throughput and labor usage (Chase, 2006). For seasonal products with cyclical demands, the manufacturer has to determine the optimal capacity and inventory level to reduce shortage and inventory costs (Bradley and Arntzen, 1999). Metters (1997) develops a heuristic algorithm for a multiple-period production problem with stochastic seasonal demand and limited capacity. Metters (1998) also summarizes several principles for the manufacturer when producing seasonal products. Aviv and Federgruen (2001) investigate the trade-off between the investment of capacity and service level under the fluctuation of seasonal demand. Mathur and Shah (2008) study a case in which the manufacturer designs a contract with two-way penalties for coordinating supply and demand.

3. Model Description

We consider a supply chain in which a manufacturer sells a product to end customers through a retailer. The time horizon is divided into two periods, and the demand is realized at the end of the second period. From the retailer's perspective, market demand is a random variable, D, that follows a probability distribution with support on $[0, R]$. We define $F(\cdot)$ as the cumulative density function (cdf) of D and $f(\cdot)$ as its probability density function (pdf). In addition, $\overline{F}(\cdot) := 1 - F(\cdot)$. We assume that the retail price, p, charged to the end customers is exogenously determined. That is, the retailer faces a newsvendor problem, and the only decision the retailer can make is the order quantity, q, that satisfies the end market demand.[1] In this paper, we assume that the leadtime of production is one period – the retailer who places the order at the beginning of period $i, i = 1, 2$ can receive the goods at the end of that period. In fact, we have witnessed numerous practical

[1]The newsvendor setting is commonly used in decentralized supply chain literature such as supply chain contracts (Cachon, 2002 and reference therein) and assembly systems (Bernstein et al., 2007). Adopting such setting helps facilitate the analysis of the retailer's optimal decisions, enabling us to mainly focus on the strategic moves between the manufacturer and the retailer and the optimal strategy in the supply chain.

examples where seasonal product retailers tend to place an order close to the peak season to avoid holding costs incurred if they receive the goods during the off-peak season. Therefore, unless the manufacturer provides some incentive, such as a wholesale price rebate, to the retailer in the first period, the retailer is better off ordering at the beginning of the second period.

The manufacturer can initiate the production in both periods to fulfill the quantity ordered by the retailer. Let Q_i be the number of products that the manufacturer determines to produce at period $i, i = 1, 2$. However, Q_i cannot exceed the manufacturer's production capacity, K, in each period. That is, the manufacturer can supply a maximum of $2K$ in both periods. For each period, the manufacturer incurs a unit production cost, $c(y) = a - by$, to produce y units where $a, b > 0$.[2] Thus, in a period, the total cost to produce y units is $c(y) \times y$. Note that we use the form of production cost to represent the economics of scale in terms of the manufacturer's production efficiency. For each unit of the product, the manufacturer charges a wholesale price, w, such that $c(y) \leq w \leq \bar{p} \leq p, \forall y$. Notice that $\bar{p}$ serves as the upper bound wholesale price that the manufacturer can set to ensure the retailer possesses a positive profit margin of selling the product. This positive profit margin provides incentive to the retailer for participating in the transaction with the manufacturer in the supply chain.[3] If the manufacturer chooses to produce during the first period, but the retailer does not take delivery of the manufactured products, the manufacturer incurs a unit holding cost, m. Hence, to minimize the inventory holding cost for the products produced in the first period, the manufacturer rationally uses up the capacity in the second period. When the manufacturer expects the order quantity to be greater than K, the production for the excess quantity is advanced in the first period.

The timing of events is as follows: At the beginning of the first period, the manufacturer, who is a Stackelberg leader, sets the wholesale price, w, and determines the production in both periods, Q_1 and Q_2, respectively. At the beginning of the second period, the retailer determines the order quantity, q. At the end of the second period, all the products are shipped to the retailer, and the end customer demand is realized. We assume that both the manufacturer and the retailer are

[2]Note that the unit production cost $c(y)$ represents the average cost of producing a unit considering both fixed and variable costs. Aggregating production in one period may take advantage of an order quantity discount for acquiring material and proportionates fixed costs to each unit produced in the same period such as a setup cost. Therefore, to capture this property, we assume the unit cost $c(y)$ decreases as the production quantity y increases.

[3]One may consider the upper bound $\bar{p}$ be a ratio of the retail price p (i.e., $\bar{p} = \alpha p$ where $\alpha \in (0, 1]$). The case where $\bar{p} = p$ (or equivalently, $\alpha = 1$) represents the fact that the manufacturer is able to set the wholesale price w equal to the retail price p.

risk-neutral, and that all the aforementioned parameters are known to each player in the supply chain. The manufacturer's objective is to determine the wholesale price, w, and the number of products to produce, $Q_i, i = 1, 2$, to maximize profit in two periods. The retailer chooses the order quantity, q, to maximize profit by satisfying the end customer demand. Let $S(q) := q - \int_0^q F(x)dx$ be the expected sales if the retailer chooses the order quantity, q.

For expositional reasons, we assume the following condition holds throughout the article:

$$2K < R < \frac{a}{2b}. \tag{1}$$

The condition rules out some rather trivial and uninteresting cases. The first inequality stands for the case where the total capacity of both periods will not cover the maximum end customer demand. In fact, we implicitly assume that this case holds because the paper mainly focuses on the manufacturer as pricing and production decisions are made to maximize profits under capacity constraint. The second inequality guarantees that the total production cost, $c(y) \times y$, always increases in the produced quantity y. Prior to presenting the settings of the three strategies, Table 1 summarizes the notations used in this paper.

Notation	Definition
D	market demand with a cumulative distribution, $F(\cdot)$, with support on $[0, R]$
p	retail price of the product, which is exogenously determined
$\bar{p}$	upper bound of wholesale price of the product
K	single-period capacity of the manufacturer
m	unit holding cost for the manufacturer
h	unit holding cost for the retailer
g	unit capacity expansion cost for the manufacturer
w	wholesale price of the product set by the manufacturer
q	order quantity by the retailer
$S(q)$	expected sales if the order quantity is q where $S(q) := q - \int_0^q F(x)dx$
Q_i	production amount by the manufacturer in period i where $i = 1, 2$
$c(y)$	unit production cost of manufacturing y units in each period and $c(y) = a - by$ where $a > 0$ and $b \geq 0$
Δk	incremental production capacity by the manufacturer in the second period

Table 1: Summary of notations.

3.1 Basic Strategy

We first consider the basic strategy. Note that as transparent information is assumed in our model, the manufacturer is able to expect the order quantity requested by the retailer at the beginning of the first period when the manufacturer makes the wholesale price decision. Therefore, if the quantity is less than or equal to the manufacturer's production capacity, K, in one period, the manufacturer only initiates production at the second period. If the order quantity is above the single period capacity, K, the manufacturer has to produce in both periods to satisfy the order quantity placed by the retailer. Based on the assumptions described above, the expected profits for the manufacturer and the retailer can be formulated in the following two cases: (i) $0 \leq q \leq K$ and (ii) $K < q \leq 2K$.

In case (i) (i.e., $0 \leq q \leq K$), the manufacturer produces all units of products in the second period to avoid the holding cost and to maximize the benefits from the economies of scale (i.e., $Q_1 = 0$ and $Q_2 = q$). The unit production costs for the manufacturer in the second period is $c(Q_2 = q) = a - bq$. Let π_m and π_r denote the expected profits for the manufacturer and the retailer. We have

$$\begin{aligned} \pi_m &= (w - c(q))q, \\ \pi_r &= pS(q) - wq. \end{aligned} \tag{2}$$

In case (ii) (i.e., $K < q \leq 2K$), the manufacturer fully utilizes capacity in the second period, and produces and stocks the remaining quantity, $q - K$, in the first period. The expected profits for the two parties are

$$\begin{aligned} \pi_m &= (w - m - c(q-K))(q-K) + (w - c(K))K, \\ \pi_r &= pS(q) - wq. \end{aligned} \tag{3}$$

The manufacturer's profit in Equation (3) includes the production costs, $c(q-K) \times (q-K)$ and $c(K) \times K$, in the two periods, respectively, and the holding cost, $m(q-K)$. We call this case as *basic strategy*, which serves as a benchmark for comparison with the two strategies, wholesale price rebate and capacity expansion, introduced in Sections 3.2 and 3.3.

We use backward induction by first characterizing the retailer's optimal order quantity, given the wholesale price in the basic strategy. Notice that the retailer's expected profit in the basic strategy is $\pi_r = pS(q) - wq$ defined in Equations (2) and (3) above, where $S(q) = q - \int_0^q F(x)dx$. As F is a continuous and strictly increasing function, based on the first-order condition, we find the retailer's order quantity, q, and the wholesale price, w, have one-to-one correspondence. That is, for a given wholesale price, w, the retailer will choose the corresponding order quantity, q, according to the following relationship.

$$q = F^{-1}(\frac{p-w}{p}). \tag{4}$$

Note also that the second derivative of π_r is negative ($\frac{\partial^2 \pi_r}{\partial q^2} = -pf(q) < 0$); hence, the solution of q given in Equation (4) is indeed optimal. Then, we substitute Equation (4) into the manufacturer's profit function derived in Equation (2) if $0 \leq q \leq K$, and in Equation (3) if $K < q \leq 2K$, where K is the single period capacity. Note that when $0 \leq q \leq K$, the manufacturer's profit function can be rewritten as $\pi_m = (p\overline{F}(q) - c(q))q$. Therefore, from the manufacturer's viewpoint, determining an optimal wholesale price to maximize expected profits in Equation (2) is equivalent to choosing an optimal q to maximize $\pi_m = (p\overline{F}(q) - c(q))q$. Defining q^* and w^* as the optimal order quantity of the retailer and the optimal wholesale price of the manufacturer, respectively, we have

$$q^* = \arg \max_{F^{-1}(\frac{p-\bar{p}}{p}) \leq q \leq K} \pi_m \quad \text{and} \quad w^*(q^*) = \min\{\bar{p}, p\overline{F}(q^*)\}.$$

When $K < q \leq 2K$, the manufacturer's expected profit is represented in Equation (3). Similarly, when Equation (4) is substituted into the manufacturer's profit function, the manufacturer's profit function becomes

$$\pi_m = (w - m - c(q(w) - K))(q(w) - K) + (w - c(K))K. \tag{5}$$

Defining q^{**} and $w^{**}(q^{**})$ as the optimal order quantity of the retailer and the optimal wholesale price of the manufacturer when the retailer tends to order beyond the capacity, K. Hence, we obtain

$$q^{**} = \arg \max_{\max\{K, F^{-1}(\frac{p-\bar{p}}{p})\} < q \leq 2K} \pi_m \quad \text{and} \quad w^{**}(q^{**}) = \min\{\bar{p}, p\overline{F}(q^{**})\}.$$

3.2 Wholesale Price Rebate Strategy

We consider a case in which the manufacturer provides a wholesale price rebate to encourage the retailer to take products manufactured in the first period. This model corresponds to the situation when Gree, an example mentioned in the Introduction section, offered a price discount if its retailers ordered and took possession of the products in the off-peak period. In this strategy, the manufacturer offers a unit wholesale price rebate, r, for any quantity that the retailer accepts in the first period. Notice that both parties are aware that the manufacturer always fulfills the capacity in the second period and produces the remaining items in the first period. Therefore, such price rebate only applies to the quantity that exceeds the single period capacity, $q - K$. Although the retailer can benefit from a compensation in the first period, the retailer has to pay $h > 0$ to store each product unit.

Under the wholesale price rebate strategy, the retailer will make the order quantity decision at the beginning of the first period after the wholesale price and the price rebate are revealed. Clearly, if the order quantity is above the single period capacity, K, the retailer will be compensated by accepting the earlier delivery of products from the manufacturer. The timing of events under the wholesale price rebate strategy is as follows: At the beginning of the first period, the manufacturer sets the wholesale price and the price rebate, w and r, respectively. The retailer then determines the order quantity, q, after which the manufacturer allocates the production in each period, $Q_i, i = 1, 2$. At the end of the first period, Q_1 units of products are shipped to the retailer and for each unit the retailer is charged $w - r$. At the end of the second period, Q_2 units of products are delivered to the retailer who pays unit wholesale price, w, and the end customer demand is realized.

Similarly, we consider the expected profits under the wholesale price rebate strategy for the manufacturer and the retailer, denoted as $\pi_{r,m}$ and $\pi_{r,r}$, in two cases: $0 \leq q \leq K$ and $K < q \leq 2K$. In the first case, the manufacturer does not start producing until the second period, and thus, the rebate strategy is not initiated. The expected profit functions for the manufacturer and the retailer are identical to those in the basic strategy (see Equation (2)).

When $K < q \leq 2K$, K units of the products are manufactured at the second period ($Q_2 = K$) and the excess quantity is scheduled for production at the first period ($Q_1 = q - K$). According to the wholesale price rebate strategy, the manufacturer ships Q_1 units of the product to the retailer at the end of the first period and only charges $w - r$ for each unit. As a result, the expected profit for the manufacturer can be written as

$$\pi_{r,m} = (w - r - c(q - K))(q - K) + (w - c(K))K. \tag{6}$$

The manufacturer's objective is to determine the wholesale price, w, and the rebate, r, to maximize expected profit defined in Equation (6). Similarly, the retailer's profit under the wholesale price rebate strategy can be also obtained:

$$\pi_{r,r} = pS(q) - wq + (r - h)(q - K). \tag{7}$$

The retailer is compensated for paying the holding cost, h, by receiving a unit rebate, r, from the manufacturer. Given the wholesale price and the rebate, the retailer chooses the order quantity, q, to maximize expected profit in Equation (7).

3.3 Capacity Expansion Strategy

In this section, we consider an option of the manufacturer to expand the capacity of the second period by manufacturing the products using overtime. In fact, adopting capacity expansion can encourage the manufacturer to reduce the amount of production in the first period, which indirectly alleviates the cost of holding inventory. In addition, the manufacturer can take advantage of producing a larger batch (due to the economics of scale) in the second period to further reduce the unit cost of products because of the property of convexity of the production cost function, $c(y) \times y$. Let $\Delta k \geq 0$ be the amount of the incremental capacity, which is a decision variable of the manufacturer in the second period. For each unit of capacity expanded, the manufacturer has to pay $g > 0$.

The timing of events under the capacity expansion strategy is as follows: At the beginning of the first period, the manufacturer sets the wholesale price, w, and the amount of the incremental capacity, Δk, together with the production quantities in period i, Q_i. At the beginning of the second period, the retailer determines the order quantity, q. At the end of the second period, q units of the product are shipped to the retailer, then the end customer demand is realized. Under the capacity expansion strategy, we also discuss the expected profits for the manufacturer and the retailer based on whether the order quantity is greater than the capacity, K, or not.

When the order quantity from the retailer can simply be satisfied by producing in one period, the manufacturer chooses to start the production in the second period and has no incentive to expand the capacity. The expected profits functions for both parties are the same as those in Equations (2). However, when the order quantity is greater than K, the manufacturer has an alternative to expand the capacity if the cost reductions in the holding cost and the unit production cost dominate the unit expansion cost, g. In such a case, the manufacturer determines Δk for the second period and pays the capacity expansion cost, $g\Delta k$. The production quantities in these two periods are $Q_1 = q - K - \Delta k$ and $Q_2 = K + \Delta k$, respectively. Thus, the expected profits of the manufacturer and the retailer denoted as $\pi_{c,m}$ and $\pi_{c,r}$, respectively, can be formulated as follows:

$$\begin{aligned}\pi_{c,m} &= (w - m - c(q - K - \Delta k))(q - K - \Delta k) \\ &\quad +(w - c(K + \Delta k))(K + \Delta k) - g\Delta k, \\ \pi_{c,r} &= pS(q) - wq. \qquad (8)\end{aligned}$$

Note that under the capacity expansion strategy, the only variant from the basic strategy is the manufacturer's expected profit. For the manufacturer, the holding cost that will be incurred due to the production in the first period can be reduced by the amount of $m\Delta k$, as the manufacturer can meticulously shift parts of the production to the second period. Furthermore, for every unit produced in the second period, the manufacturer enjoys a lower unit production cost as a result of the economics of scale. As capacity expansion is an internal decision for the manufacturer and does not alter the structure of the expected profit function for the retailer, the retailer's expected profit (Equation (8)) is identical to that under the basic strategy (Equation (3)).

Based on the settings of the three strategies, we are mainly interested in the impacts of the three factors on the decisions of both parties: the manufacturer's holding cost (m) and capacity expansion cost (g), and the retailer's holding cost (h). Given these parameters, in the following section, we investigate the optimal decisions for the manufacturer and the retailer of each strategy and compare the expected profits of these strategies.

4. Analysis

In this section, we analyze the optimal pricing and quantity ordering decisions of the manufacturer and the retailer under the wholesale price rebate strategy in Section 4.1 and under the capacity expansion strategy in Section 4.2, respectively.

4.1 Wholesale Price Rebate Strategy

Under the wholesale price rebate strategy, when the order quantity from the retailer is not greater than K and the wholesale price is set at w^*, the decisions and the associated profits for these two parties are identical to those under the basic strategy.

If the order quantity from the retailer is greater than K, the manufacturer has to determine the wholesale price, w, and the price rebate, r. We define the manufacturer's profit as $\pi^v_{r,m} = (w - r - c(q - K))(q - K) + (w - c(K))K$. Given the wholesale price, w, we obtain that $r = w + h - p\overline{F}(q)$

based on the first-order condition of the retailer's profit function defined in Equation (7), and that r and q have one-to-one correspondence. By substituting r into the manufacturer's profit $\pi^v_{r,m}$, we can obtain:

$$\pi^v_{r,m} = (p\overline{F}(q) - h - c(q-K))(q-K) + (w - c(K))K. \tag{9}$$

From the above equation, the wholesale price w only influences the profit received in the second period. To generate high revenue, the manufacturer is intent to set the wholesale price as high as the upper bound $\bar{p}$. Let w^v_r be the optimal wholesale price under the wholesale price rebate strategy. Then $w^v_r = \bar{p}$. The optimal price rebate, r^v, can be directly derived from the relationship of $r^v = w^v_r + h - p\overline{F}(q)$. Therefore, with the optimal price pair (w^v_r, r^v), the retailer's optimal order quantity, q^v_r, is such that

$$r^v = \bar{p} + h - p\overline{F}(q) \quad \text{or} \quad q^v_r = F^{-1}(\frac{r^v - h - \bar{p} + p}{p}). \tag{10}$$

Under the wholesale price rebate strategy, the manufacturer will choose a wholesale price equal to $\bar{p}$ that gains the maximum surplus that the manufacturer can collect in the second period. From the retailer's perspective, in addition to the profit margin generated in both periods, additional benefit can be realized in the first period only if the rebate offered by the manufacturer can at least cover the holding cost of the retailer. On the other hand, the manufacturer is willing to provide such a rebate to induce the retailer for ordering a quantity larger than the single period capacity, K. Doing it simultaneously benefits the manufacturer in the second period from the revenue side (since the wholesale price is set at the maximum level) and the cost side (due to economics of scale in the production cost). In this circumstance, large bargaining power on the wholesale price decision motivates the manufacturer to take maximum supply chain profit to be obtained in the second period if a high order quantity from the retailer is expected. However, the existence of the price rebate results in a shift of partial bargaining power to the retailer in the first period, because the manufacturer has to simultaneously raise the price rebate to some level. We summarize the optimal decisions of the manufacturer and the retailer in the following proposition.

Proposition 1. *Under the wholesale price rebate strategy, the manufacturer sets the wholesale price equal to the upper bound wholesale price, i.e.,* $w^v_r = \bar{p}$ *and the rebate,* $r^v = \bar{p} + h - p\overline{F}(q)$. *Consequentially, the retailer orders the quantity,* q^v_r*, where* $q^v_r = F^{-1}(\frac{r^v - h - \bar{p} + p}{p})$.

4.2 Capacity Expansion Strategy

Similar to the wholesale price rebate strategy, the capacity expansion strategy provides no incentive for the manufacturer to adjust the capacity in the second period when the order quantity is less than the one period capacity, K. Under this case, the analytical results under this strategy are the same as those under the basic strategy. Contrarily, if the order quantity is above K, then the capacity from K to $K+\Delta k$ in the second period may be increased to reduce product inventory holding costs. Under capacity expansion, the manufacturer determines the amount of capacity to increase, Δk, in the second period and the associated wholesale price, w. Note that the expected profit functions of the retailer under the basic strategy (Equation (3)) and the capacity expansion strategy (Equation (8)) are the same in the sense that raising the capacity in the second period does not influence the amount ordered from the retailer. That is, capacity expansion is an internal decision of the manufacturer, and thus, the order quantity of the retailer, q, should satisfy $w = p\overline{F}(q)$ given the wholesale price, w.

Then we determine the optimal decisions of the manufacturer. Let $\pi^v_{c,m}$ be the manufacturer's profit function. We then obtain:

$$\pi^v_{c,m} = (w - m - c(q - K - \Delta k))(q - K - \Delta k) + (w - c(K + \Delta k))(K + \Delta k) - g\Delta k. \tag{11}$$

We substitute the retailer's response function $w = p\overline{F}(q)$ into $\pi^v_{c,m}$ and obtain:

$$\begin{aligned} \pi^v_{c,m} \quad = \quad & (p\overline{F}(q) - m - c(q - K - \Delta k))(q - K - \Delta k) \\ & +(p\overline{F}(q) - c(K + \Delta k))(K + \Delta k) - g\Delta k. \end{aligned} \tag{12}$$

Taking q as constant, $\pi^v_{c,m}$ is convex in Δk, and thus, the optimal capacity expansion Δk is such that:

$$\Delta k = 0, \text{ or } \Delta k = q - K.$$

Note that the capacity expansion strategy follows the basic strategy when $\Delta k = 0$, because the manufacturer does not raise any capacity in the second period. On the other hand, if $\Delta k = q - K$, all productions are conducted in the second period to avoid the holding costs that will be incurred if production starts in the first period. Consider the two Δk's and solve for the optimal q. Let q^v_c be the optimal order quantity of the retailer. In fact, q^v_c cannot be uniquely determined, because the manufacturer's function is too complicated. Therefore, in the next section, we add one condition by assuming that the market demand distribution follows a uniform distribution. In such a case, we can characterize the optimal order quantity and the associated decisions of the manufacturer and the retailer in closed forms.

5. When Demand is Uniformly Distributed

In this section, we assume that the market demand follows a uniform distribution with support on $[0, R]$. With this specific distribution, we can obtain the optimal decisions of both parties in closed forms. Lemma 1 characterizes the optimal decisions of both parties under the basic strategy.

Lemma 1. *Under the basic strategy, let (q^*, w^*) and (q^{**}, w^{**}) be the optimal order quantity and wholesale price to the problems defined in Equations* (2) *and* (3)*, we have:*

$$(q^*, w^*) = \left(\min\left\{K, \max\left\{\frac{(p-\bar{p})R}{p}, \frac{(p-a)R}{2(p-bR)}\right\}\right\}, \max\left\{p - \frac{Kp}{R}, \min\left\{\bar{p}, p - \frac{(p-a)p}{2(p-bR)}\right\}\right\}\right), \ \textit{and}$$

$$\begin{aligned}(q^{**}, w^{**}) = \Bigg(&\max\left\{K^+, \frac{(p-\bar{p})R}{p}, \min\left\{2K, \frac{(p-a-m-2bK)R}{2(p-bR)}\right\}\right\},\\ &\min\left\{p\left(1-\frac{K^+}{R}\right), \bar{p}, \max\left\{p\left(1-\frac{2K}{R}\right), p\left(1-\frac{p-a-m-2bK}{2(p-2bR)}\right)\right\}\right\}\Bigg).\end{aligned}$$

Proposition 2 presents a condition in which the retailer will order a quantity greater than the single period capacity.

Proposition 2. *Under the basic strategy, the retailer's optimal order quantity will never be less than the single period capacity, K, if*

$$(p-a)R - 2(p-bR)K \geq 0.$$

Earlier discussions have pointed out that if the retailer orders a quantity below the single period capacity, the manufacturer need not adopt the wholesale price rebate and the capacity expansion strategies. In other words, all three strategies provide the same profit with order quantity $q \leq K$ to the manufacturer. In fact, Proposition 2 implies that a higher demand (i.e., higher R) and a higher retail price significantly induce a higher order quantity if the associated cost increment is moderate. The aforementioned condition signals the manufacturer to choose a strategy other than the basic one to mitigate the effect of the holding cost that resulted from the production in the first period, which is the main focus of our paper.

Under the uniform distribution of demand, we can also obtain the optimal decisions in closed forms for each of the two strategies. Lemma 2 depicts the detailed results of both strategies.

Lemma 2. *Given $(p-a)R - 2(p-bR)K > 0$, the optimal decisions for the wholesale price rebate strategy and the capacity expansion strategies are as follows:*
(i) Under the wholesale price rebate strategy, the optimal order quantity, q_r^v, the optimal wholesale price, w_r^v, and the optimal price rebate, r^v, are:

$$\begin{aligned} q_r^v &= \max\left\{K^+, \min\left\{\frac{(p-a-h)R + (p-2bR)K}{2(p-bR)}, 2K\right\}\right\}, \\ w_r^v &= \bar{p}, \ \textit{and} \quad r^v = \bar{p} + h - \frac{p(R - q_r^v)}{R}. \end{aligned} \tag{13}$$

(ii) Under the capacity expansion strategy, the optimal order quantity, q_c^v, the optimal wholesale price, w_c^v, and the optimal amount of capacity increment, Δk^v, are:

$$\begin{aligned} q_c^v &= \max\left\{K^+, \frac{(p-a-g)R}{2(p-bR)}, \frac{(p-\bar{p})R}{p}\right\}, \\ w_c^v &= \min\left\{p\left(1 - \frac{K^+}{R}\right), p\left(1 - \frac{(p-a-g)}{2(p-bR)}\right), \bar{p}\right\}, \ \textit{and} \quad \Delta k^v = q_c^v - K. \end{aligned} \tag{14}$$

Proposition 3 below states that if the manufacturer's single period capacity is large enough, then the resultant order quantity of the retailer under the wholesale price rebate strategy is the largest among the three strategies.

Proposition 3. *The optimal order quantity under the wholesale price rebate strategy is larger than that under the other two strategies if*

$$K \geq \max\left\{\frac{(h-m)R}{p}, \frac{(h-g)R}{p-2bR}, \frac{R(p-\bar{p})}{p} + \frac{R(a+h-\bar{p})}{p-2bR}, \frac{(p-a-g)R}{4(p-bR)}\right\}.$$

Proposition 3 indicates that raising the single period capacity can result in a larger order quantity from the retailer when the wholesale price rebate strategy is used. Under the wholesale price rebate strategy, the manufacturer provides an attractive rebate to the retailer. Therefore, the retailer tends to order more to enjoy the benefits in the first period, even though the capacity is adequate. The larger order quantity indirectly yields a higher profit for the manufacturer, indicating that the manufacturer is willing to adopt the wholesale price rebate strategy when the single period capacity is sufficiently high. Proposition 4 formalizes the statement.

Proposition 4. *The wholesale price rebate strategy outperforms the other two strategies from the manufacturer's viewpoint if*

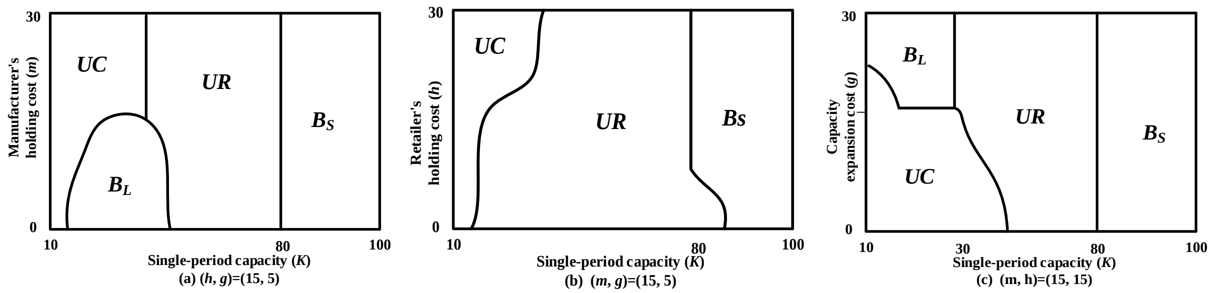
$$K \geq \max\left\{\frac{(p-a-m)R}{2p}, \frac{(p-a-g)R}{2(p-bR)}\right\}.$$

Proposition 4 summarizes the result of comparing the profits of the manufacturer under the three strategies. The aforementioned result documents the rationale of Gree, the largest air conditioner manufacturer in China, to provide a price discount to its retailers if they receive the inventory during the off-peak season. Proposition 4 also provides an operational strategy to seasonal product manufacturers if they consider using a price rebate to encourage the earlier movement of their inventories or increase capacity to satisfy a high demand in the coming season.

6. Numerical Study

We conduct a numerical study to gain further managerial insights regarding the three strategies. We focus on three driving forces of the manufacturer's holding cost, m, the retailer's holding cost, h, and the capacity expansion cost, g, on the strategic move of the manufacturer and the associated profits of both parties in the supply chain. Figure 1 shows that the supply chain adopts the capacity expansion strategy when the single period capacity is limited and adopts the wholesale price rebate strategy as the capacity becomes higher. When the capacity is limited, the manufacturer can use the incremental resource to satisfy the order by increasing the capacity in the second period. However, aggregating the production in one period can decrease the unit production cost because of the economics of scale. With a high capacity, if the wholesale price rebate strategy is adopted, the manufacturer will set a wholesale price equal to a high level, subtract most surplus from the retailer in the second period, but will only give a price rebate in the first period. The effect of rebate paid to the retailer diminishes as the capacity becomes larger. As a result, offering a price rebate becomes the best option for the manufacturer. Finally, when the capacity level is sufficiently high, adopting any of the three strategies (basic, wholesale price rebate, and capacity expansion) are the same.

Figure 2 presents the profits of both parties in terms of capacity. From the manufacturer's perspective, adopting the capacity expansion strategy can improve the profit compared to the other two strategies, in particular, when the single period capacity is low. Under the capacity expansion strategy, on the other hand, the retailer has the highest profit among the three strategies and the profit is independent of the single period capacity (see Figure 2(a)). Based on the earlier discussion, the capacity expansion strategy is an internal decision for the manufacturer, who can make use of the economics of scale to reduce the unit production cost through aggregating all the production in the second period. Further, the manufacturer offers a more attractive wholesale price



B_S: basic strategy with $q \leq K$, ***B_L***: basic strategy with $q > K$, ***UR***: wholesale price rebate strategy, ***UC***: capacity expansion strategy.

Figure 1: Manufacturer's best strategy with respect to (a) manufacturer's holding cost, m, (b) retailer's holding cost, h, and (c) capacity expansion cost, g. Here, we use $R = 200, p = 60, \bar{p} = 48, a = 20$ and $b = 0.01$.

to induce the retailer to order more. Hence, adopting the capacity expansion strategy is profitable for both parties, but the effect is diminished as the single period capacity becomes higher. Hence, capacity expansion is the equilibrium when the capacity is low. When the single period capacity is high, on the other hand, the wholesale price rebate strategy benefits the manufacturer more significantly. With high capacity, however, the retailer's profit decreases in capacity because the manufacturer grabs most of the supply chain profit in period two by setting the wholesale price equal to its upper bound and this pattern is more apparent as the capacity increases. Therefore, the wholesale price rebate strategy yields the highest profit to the manufacturer, but the lowest to the retailer when the single period capacity is high. In this case, the retailer is reluctant to accept the wholesale price rebate strategy but the manufacturer, acting as a stackelberg leader, can dominate the determination of the supply chain. As a result, when the capacity is high, the manufacturer simply uses the dominant role in the supply chain to force the adoption of the wholesale price rebate strategy.

Remark. In our model, the unit production cost incurred by the manufacturer is $c(y) = a - by$ where b is strictly positive to model the effect of economics of scale. One may consider the case where $b = 0$, representing the fact that such effect no longer exists. In order to further study the case $b = 0$, we also conduct a numerical experiment and compare it with earlier results. In fact, based on aforementioned discussion, the capacity expansion strategy is mainly influenced by the effect of economics of scale. That is, the manufacturer using the capacity expansion strategy can benefit from this effect by extending the capacity of the second period and aggregating all the production in this period. As a result, if such effect disappears, both profits of the manufacturer and the retailer under the capacity expansion strategy will be reduced more significantly compared to the other two

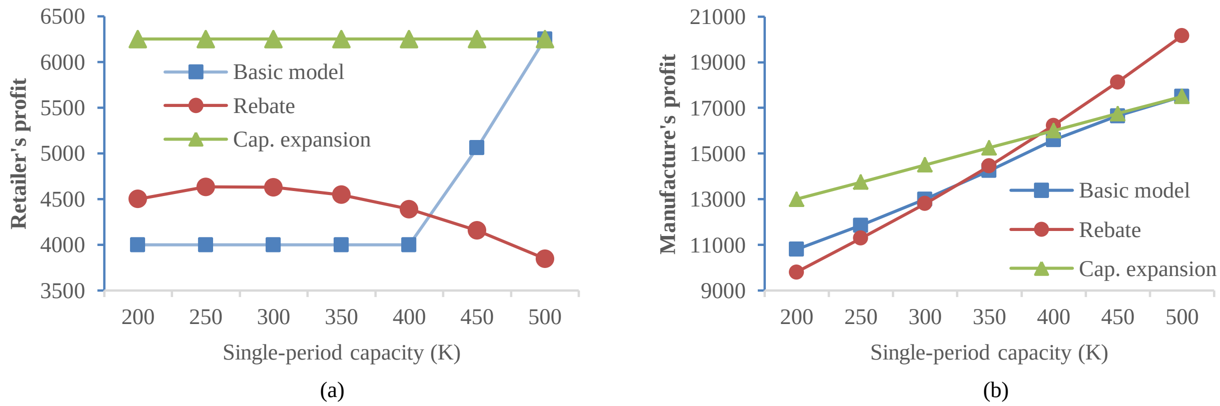
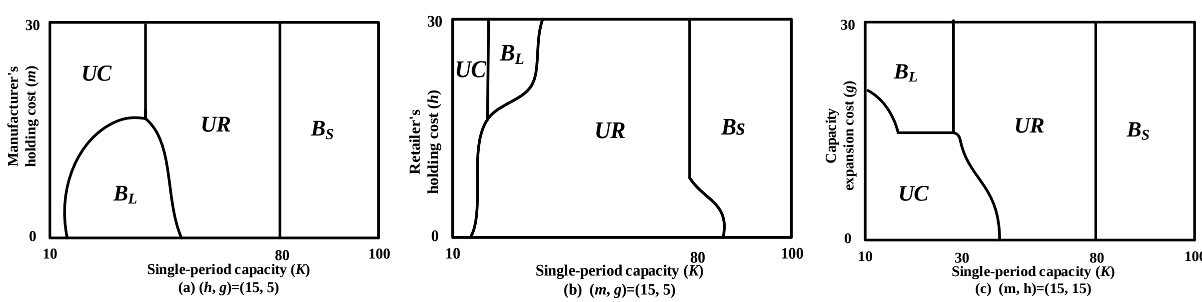


Figure 2: Retailer's and manufacturer's optimal profits. Here, we use $R = 2000, p = 100, \bar{p} = 80, a = 45, b = 0.01, m = 20, h = 25$ and $g = 15$.

strategies as shown in Figure 4 (compared to Figure 2). Furthermore, for some parameter sets the regions in which the capacity expansion strategy is best strategy of the manufacturer with $b > 0$ will no longer be the best with $b = 0$ (see Figure 3 for $b = 0$ vs. Figure 1 for $b = 0.01$).



B_S: basic strategy with $q \leq K$, ***B_L***: basic strategy with $q > K$, ***UR***: wholesale price rebate strategy, ***UC***: capacity expansion strategy.

Figure 3: Manufacturer's best strategy with respect to (a) manufacturer's holding cost, m, (b) retailer's holding cost, h, and (c) capacity expansion cost, g when $b = 0$. Here, we use $R = 200, p = 60, \bar{p} = 48$ and $a = 20$.

7. Conclusion

This paper analyzes a model for maximizing the expected profit for a manufacturer who produces seasonal products in two periods with limited capacity. We consider two strategies that further increase the expected profit for the manufacturer: wholesale price rebate and capacity expansion. With the wholesale rebate strategy, the retailer is provided with a wholesale price discount by the manufacturer, who induces a higher order quantity by sharing part of the risk of the retailer

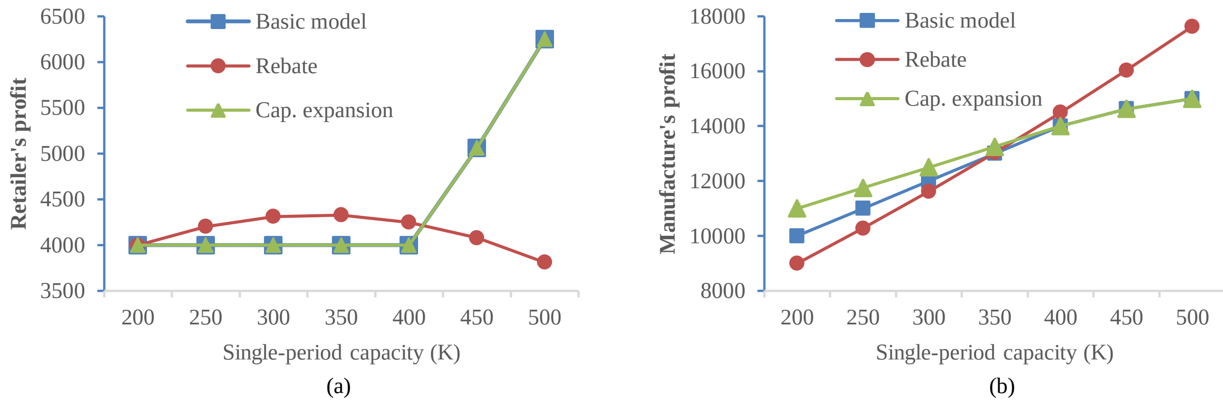


Figure 4: Retailer's and manufacturer's optimal profits when $b = 0$. Here, we use $R = 2000, p = 100, \bar{p} = 80, a = 45, m = 20, h = 25$ and $g = 15$.

to increase profit. Through the capacity expansion strategy, the manufacturer aggregates the production in one period to take advantage of the economics of scale, and mitigates the inventory holding costs by paying a cost-effective amount of expansion fees.

We characterize the expected profits for the manufacturer and the retailer and the optimal decisions for both parties when the manufacturer adopts the wholesale rebate strategy or the capacity expansion strategy. The best strategy for the manufacturer is determined by the following driving forces: the unit costs of holding inventory for the manufacturer, the unit costs of holding inventory for the retailer, and the unit costs of capacity expansion. Under the wholesale price rebate strategy, the manufacturer can always seize the maximum surplus from the retailer in the second period by setting the wholesale price to its upper bound. Consequently, the best strategy for the manufacturer is the wholesale price rebate with high capacity. When the single period capacity is relatively low, the manufacturer tends to choose the capacity expansion so that the order quantity can be augmented. Furthermore, adopting the capacity expansion strategy compared with the basic and the wholesale price rebate strategies can benefit both parties. Our research provides an insightful guideline for the parties in the supply chain to adopt the wholesale price rebate or the capacity expansion strategy which maximizes the overall profit of the chain.

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