Optimal Pricing Strategies under Co-Existence of Price-Takers and Bargainers in a Supply Chain

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Abstract

We investigate how co-existence of two types of customers, price-takers and bargainers, influences the pricing decisions in a supply chain. We consider a stylized supply chain which includes one manufacturer and one retailer, and we characterize the optimal prices of the retailer and the manufacturer. We further discuss the effects of the fraction of the bargainers in the customer population and the relative bargaining power of the bargainers on these optimal prices. Our results show that, given the wholesale price, the lowest price at which the retailer is willing to sell (i.e., cut-off price) increases with the relative bargaining power of the bargainers. Both posted and cut-off prices increase in the fraction of the bargainers in the customer population. Moreover, depending on the type of negotiation cost, the variations of both prices will vary. In equilibrium, both posted and cut-off prices do not monotonically increase with the fraction of the bargainers in the customer population. When the maximum reservation price of the customers is low, and/or the negotiation costs are high, and/or the relationship between the bargainer's negotiation cost and reservation price is high, the retailer may reduce both posted and cut-off prices as the fraction of the bargainers increases.

Key words: Game theory; Pricing; Supply chain management; Negotiation.

1. Introduction

In many product selling channels, negotiation plays an important role during daily transactions. The prices of cars, furniture, and electronic appliance that a customer pays are for example negotiated from the posted price. Negotiation is also quite common in the traditional markets in Asia. An interesting phenomenon which may be observed is the amount of negotiations undertaken by customers, who barter with the vendors over almost all products including food, clothes, and daily necessities. After observing the posted price, customers haggle with the vendor in order to receive a discount; customers and retailers will bargain over the price until an agreement has been

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reached. Recently, negotiation has become a more acceptable practice for retailers who sell small-ticket items. Customers can get discounts from the retailers for products ranging from makeup and perfume to Blu-ray discs. In this way customers at a Los Angeles BestBuy, can even get a discount of $10 for a Blu-ray disc [1]. These examples show that negotiation can be a critical sales format in determining the transaction price between the customers and the retailer.

When determining the price of a transaction, retailers may face customers with different purchasing manners: customers who negotiate for a discount and customers who do not. Retailers do not know which category a given customer fits into until they have observed their behavior. For instance, at a car dealership, after observing the posted price (MSRP), some customers will purchase directly without any haggling. However, other customers may haggle with the dealer, before reaching a final agreement on a price below that which is posted. For such retailers, when making pricing decisions, they have to take into account both customers who are not willing to bargain and those who intend to negotiate for a discount.

A retailer may be willing to use negotiation as a sales format since negotiation can bring extra benefit from price discrimination based on customers with heterogeneous reservation price, compared to the posted pricing strategy. However, customers and the retailer often have to spend time and effort reaching an agreement under negotiation and the benefit received from price discrimination may be offset. In particular, when the negotiation cost of each customer is not identical and a high reservation price customer has a high negotiation cost, this cost will influence not only the willingness of the high reservation price customers who negotiate, but also the benefit that the retailer is able to capture through negotiation. Furthermore, the retailer's pricing decisions affect not only his own profit but also the manufacturer's associated profit in a supply chain. The manufacturer, however, can influence the retailer's decision, for example, through a simple contract such as one asserting acceptance of wholesale prices only.

In this research, we consider a stylized supply chain including only one manufacturer and one retailer. The manufacturer produces the product at the unit cost and sells it to the retailer at the wholesale price. The retailer then sells the products to the end customers that are divided into two types: those customers who are not willing to bargain, and customers who intend to negotiate the price down. Each customer has heterogeneous reservation price for the product (i.e., the highest price at which the customer is willing to purchase), which is not observed by the retailer. The retailer only knows the distribution of the customers' reservation prices. In the supply chain, after

[1] "Shoppers haggle for deals from retailers", The Associated Press, December 2008. Source: http://www.msnbc.msn.com/id/28351832/ns/business-small-business.

the manufacturer has determined the wholesale price, the retailer sets the posted price. If the retailer faces customers who are not willing to bargain, the behavior of customers is based on the take-it-or-leave-it principle. If the retailer faces customers who intend to negotiate the price down, the retailer must determine a cut-off price, that being the minimum price at which the retailer is willing to sell the product. Under negotiation, a customer with a high enough reservation price will buy at a final purchase price that splits the surplus according to the generalized Nash bargaining solution. The final purchase price is determined by the cut-off price, the customer's reservation price, each party's relative bargaining power, and the cost of negotiation incurred by both parties.

We solve our model backward by first considering the retailer's pricing decisions. For a given wholesale price, we characterize the unique pair of the retailer's optimal posted and cut-off prices that maximize the retailer's profit. Based on the retailer's best response, we then show the optimal wholesale price of the manufacturer when customer's reservation price follows a specific distribution, and discuss the effects of the model characteristics on these optimal prices. We especially pay our attention to the effects of the fraction of the bargainers in the customer population and the relative bargaining power of the bargainers on the optimal prices. In order to gain deep managerial insights from the analysis, we conduct a numerical study with a wide spectrum of model parameters. There are some interesting results from the numerical study in which the optimal posted price and the optimal cut-off price are not necessarily monotonic in the fraction of the bargainers. Although one may expect that both posted and cut-off prices are increasing with the fraction of the bargainers so that the retailer can do better price discrimination while pricing out the bargainers with low reservation prices, our results show that, in equilibrium, when the maximum reservation price of the customers is low, and/or the negotiation costs incurred by both parties are high, and/or the relationship between the bargainer's negotiation cost and reservation price is high, the retailer reduces both posted and cut-off prices accordingly. This is because the effect of the fraction of the bargainers is dominated by the aforementioned parameters. For the retailer, reducing both prices to attract more customers to buy is more beneficial.

The rest of the paper is organized as follows. Section 2 discusses the relevant literature and position of our research. Section 3 describes the model in detail. Analysis of the relationships between the optimal prices and model parameters, and numerical study are contained in Sections 4 and 5. Section 6 provides some extensions and concludes the paper. All proofs are relegated to the appendix.

2. Literature Review

Bargaining theory has been studied extensively in economics, and there is a wealth of research that discusses the negotiation outcome under several negotiation processes and information structures. Two famous models that discuss negotiation structures are Rubinstein's alternating offers game and the Nash bargaining solution. Rubinstein's alternating offers game (Rubinstein 1982), which is a classic model of the negotiation process, describes the division of a pie between two players. At the beginning of the game, one of the two players offers a partition of the pie and the other chooses whether to accept the offer, or reject it and then makes a counteroffer. The pie will be partitioned only after the players have reached an agreement. The negotiation is identified with finite time interval, and the payoff of each player is discounted over time. The discount rate, which can vary between the two players, stands for their respective negotiation cost. Rubinstein models the process of the negotiation and shows the unique sub-game perfect equilibrium.

The Nash bargaining solution (See Muthoo 1999 for a detailed discussion) is the other classic model for discussion of the outcome of negotiation. Under the Nash bargaining solution, two players have the same bargaining power. Both parties conduct a cooperative game where two players maximize their individual surplus and split the total surplus equally. The Nash bargaining solution, different from the Rubinstein's alternating offers game, ignores the process of negotiation and focuses on modeling the negotiation outcome. In this paper, we use the generalized Nash bargaining solution which is the extension of the classic Nash bargaining solution to model the outcome of the negotiation between the retailer and the end customers. In the generalized Nash bargaining solution, the two players each have different bargaining power and the proportion that each player can get is related to their bargaining power. Alexander and Ledermann (1994) further provide conditions for the uniqueness of constrained generalized Nash bargaining solution.

Significant amount of research in economics and operations management uses bargaining theory and compares the optimal pricing policies under bargaining with the traditional setting - posted pricing, in which the seller charges a take-it-or-leave-it price. Some of this research uses the same modeling approach as ours, the generalized Nash bargaining solution, to model the outcome of negotiation. Among these papers, Wang (1995) identifies major selling methods and finds that when negotiation costs are low enough, negotiation is always the better selling method. Bester (1993) stresses the role of quality uncertainty for the determination of pricing rules. The paper considers the negotiation between the seller and the customers. In addition, Desai and Purohit (2004) develop a model that analyzes two competing retailers' strategic incentives to select their optimal choices of pricing policies - negotiation or take-it-or-leave-it pricing. Roth et al. (2006) show the service providers are more willing to apply negotiation on customized services. In addition, other researchers who have addressed the question of posted pricing versus bargaining include Arnold and Lippman (1998) and Adachi (1999). Unlike all of these earlier research works, our paper adopts the generalized Nash bargaining solution but considers the negotiation between the retailer and the customers. We focus on the co-existence of two types of customers - price-takers and bargainers, and discuss how the fraction of the bargainers in the customer population and the relative bargaining power of the bargainers influence the pricing strategies as well as the profits of the retailer and the manufacturer. The main contribution of our paper is to characterize how the parties in a supply chain choose the optimal price strategies when both price-takers and bargainers co-exist, and show how these price strategies influence the party's profit in a supply chain.

Several papers have considered negotiation in supply chain management. Nagarajan and Bassok (2002) analyze how the structure of the supplier alliance depends on the assembler's bargaining power. Iyer and Villas-Boas (2003) consider negotiation issues in distribution channels. Dukes and Gal-Or (2003) use Nash bargaining solution to model the outcome of negotiation between advertisers and media outlets. In addition, Gurnani and Shi (2006) consider a supply contract for a first-time interaction between a single buyer and a single supplier. The buyer orders a customized product from the single supplier with consideration of supply uncertainty in the channel. Kim and Kwak (2007) consider bargaining process over a replenishment contract in a supply chain. Terwiesch et al. (2005) consider negotiation in revenue management context. Besides, Nagarajan and Sosic (2008) give a thorough survey of some applications of cooperative game theory to supply chain management.

3. Model Description

We consider a supply chain in which there exists one manufacturer and one retailer. The manufacturer produces the product at the unit cost, c , and sells it to the retailer at the wholesale price, w. The retailer sells the product to the end customers who are divided into two types – price-takers and bargainers. Let $q \in [0,1]$ be the fraction of the bargainers in the customer population and $1-q$ be the fraction of the price-takers. Each customer, whether a price-taker or a bargainer, has heterogeneous reservation price for the product (i.e., the highest price at which the customer is willing to purchase), which is not observed by the retailer. For the retailer, the customer's reservation price is a random variable, R_v, with the cumulative density function (cdf), $F(x)$, and the associated probability density function (pdf), $f(x)$. Define $\overline{F}(x) := 1-F(x)$. In addition, we use a

to represent the size of customer population. Therefore, $a\overline{F}(x)$ can be regarded as the total number of customers whose reservation prices are higher than x.

Within a supply chain, the manufacturer sells the items to the retailer at the wholesale price, and collects revenue from the retailer on items sold. After the manufacturer has determined the wholesale price, the retailer sets the posted price, p, and the cut-off price, p_m, which will be defined later. Upon arrival, all customers observe the posted price, p, but not all customers will purchase the item at the posted price. The final purchase price that a customer pays depends on the customer type, price-taker or bargainer, and the individual customer's reservation price. In the following subsections, we describe the behavior of price-takers and bargainers, respectively.

3.1 Price-takers

When the price-takers arrive and observe the posted price, p, only the price-takers whose reservation prices are higher than p will purchase the product. Therefore, the aggregate demand at the price p is given by $a(1-q)\overline{F}(p)$. Given the posted price, p, and the wholesale price, w, we can write the retailer's profit, $\Pi_{RP}(p,w)$, and the manufacturer's profit, $\Pi_{MP}(w,p)$, from the price-takers:

$$\begin{aligned} \Pi_{RP}(p,w) &= a(1-q)(p-w)\overline{F}(p), \text{ and} \\ \Pi_{MP}(w,p) &= a(1-q)(w-c)\overline{F}(p). \end{aligned} \tag{1}$$

3.2 Bargainers

When facing the bargainers, the retailer will determine not only the posted price, p, but also the cut-off price (i.e., the minimum price at which the retailer is willing to sell), denoted by p_m. During negotiation, both the retailer and the bargainer spend time and effort to negotiate for the product and the costs of negotiation are incurred by both parties. Let $c_r(>0)$ be the cost of negotiation incurred by the retailer. For those customers who negotiate, there exists a cost of negotiation, $c_e+\alpha r$, where $c_e>0$ is base negotiation cost, r is bargainer's reservation price, and $0 \leq \alpha < 1$. Notice that the negotiation cost of the bargainer is linearly increasing in her reservation price, which is not restrictive given that a higher reservation price bargainer might have a higher opportunity cost of negotiation compared to a lower reservation price customer [2]. For the retailer, the decisions of the cut-off price can be regarded as the disagreement payoff: the retailer will not sell the product to the bargainers below the cost, which is the wholesale price plus the cost of negotiation, $w+c_r$.

[2] Our model assumes that the cost of negotiation incurred by the bargainer is based on the opportunity cost each bargainer values. With different reservation price, each bargainer tends to have different cost of negotiation for the time and effort spent on negotiation. One may simplify our model by considering a case where the cost of negotiation is identical for each bargainer. Such assumption is a special case of our model by setting $\alpha = 0$.

Note that the retailer can set the cut-off price equal to $w+c_r$, and sell the products to the bargainers at the reservation price $w+c_r$ or higher, but this cut-off price is not guaranteed to be the optimum. On the other hand, notice that the final price agreed by the retailer and a bargainer should not exceed the posted price, p. Therefore, the final price that a bargainer will pay for the product is between p and p_m. In other words, the posted price and the cut-off price form respective upper and lower bounds of the price that the retailer gathers during negotiation. Notice that the decision of the lower bound (i.e., the cut-off price) will normally benefit the retailer by pricing out bargainers with lower reservation prices.

To model the outcome of negotiation, we use the generalized Nash bargaining solution (See Muthoo 1999 and discussion in the appendix) to capture the final agreement between the bargainers and the retailer. Based on the generalized Nash bargaining solution, the surplus is split between the retailer and the bargainer based on their relative bargaining power. Let p_N be the final purchase price after negotiation. For the retailer, any final price which is less than the cut-off price will be rejected, so the surplus of the retailer is given by $p_N - p_m$ [3]. For a bargainer, a negotiation cost, $c_e + \alpha r$, must be taken into account during the negotiation process. Therefore, the bargainer with reservation price less than $p_N + c_e + \alpha r$ will not be able to purchase the product. That is, the surplus of the bargainer with reservation price is $r - p_N - c_e - \alpha r$ (or equivalently, $(1-\alpha)r - p_N - c_e$). Let $\beta \in (0, 1)$ be the bargainer's relative bargaining power and $1 - \beta$ be the retailer's. Following the generalized Nash bargaining solution, the retailer with $p_m \geq w + c_r$ and the bargainer with $r \geq p_m + c_e + \alpha r$ (or equivalently, $r \geq \frac{p_m+c_e}{1-\alpha}$) will bargain for the final purchase price, p_N, that maximizes the following objective function:

$$\max_{p_m \leq p_N \leq \min(p,r)} ((1-\alpha)r - p_N - c_e)^{\beta}(p_N - p_m)^{1-\beta}$$

Note that the final purchase price after negotiation, p_N, should be set between the cut-off price, p_m, and the bargainer's reservation price, r. Moreover, p_N can never exceed the posted price, p. It follows from the above expression that the final price, $p_N^*(p_m, r, p)$, is either the posted price, p, or the convex combination of $(1-\alpha)r - c_e$ and p_m, whichever is smaller. As a result, for any $\beta \in (0, 1)$, the final purchase price can be expressed in the form:

$$\begin{aligned} p_N^*(p_m, r, p) &= \arg \max_{p_m \leq p_N \leq \min(p,r)} \{((1-\alpha)r - p_N - c_e)^{\beta}(p_N - p_m)^{1-\beta}\} \\ &= \min\{p, (1-\alpha)(1-\beta)r + \beta p_m - (1-\beta)c_e\} \end{aligned} \quad (2)$$

[3] As aforementioned discussion, the cost of negotiation incurred by the retailer, c_r, is implicitly considered in p_m, where $p_m \geq w + c_r$, as the retailer will not choose the cut-off price below $w + c_r$. The case under which the retailer must sell to bargainers who pay at least the retailer's cost, $w + c_r$, is a special case of our model.

From (2), we can derive that if a bargainer's reservation price is higher than $\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}$, she will pay the posted price p for the product. Given the cut-off price, the bargainer with reservation price $r \in [\frac{p_m+c_e}{1-\alpha}, \frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)})$ will pay $(1-\alpha)(1-\beta)r+\beta p_m-(1-\beta)c_e$. We summarize the individual bargainer's final purchase price in terms of her reservation price in the following equation:

$$p_N^*(p_m, r, p) = \begin{cases} p, & \text{if } r \geq \frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}, \\ (1-\alpha)(1-\beta)r+\beta p_m-(1-\beta)c_e, & \text{if } r \in [\frac{p_m+c_e}{1-\alpha}, \frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}) \end{cases}$$

Based on the equation above, we characterize the retailer's profit function, $\Pi_{RN}(p, p_m, w)$, and the manufacturer's profit function, $\Pi_{MN}(w, p, p_m)$, from the bargainers:

$$\begin{aligned} \Pi_{RN}(p, p_m, w) &= aq\Bigg[(p-w-c_r)\overline{F}(\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}) \\ &\quad + \int_{\frac{p_m+c_e}{1-\alpha}}^{\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}} [(1-\alpha)(1-\beta)x+\beta p_m-(1-\beta)c_e-w-c_r]f(x)dx\Bigg] \\ \Pi_{MN}(w, p, p_m) &= aq(w-c)\overline{F}(\frac{p_m+c_e}{1-\alpha}) \end{aligned} \quad (3)$$

3.3 Retailer's and the Manufacturer's Profit Functions

Based on equations (1) and (3), we can formulate the retailer's profit function, $\Pi_R(p, p_m, w)$, and the manufacturer's profit function, $\Pi_M(w, p, p_m)$, when both price-takers and bargainers co-exist in the customer population:

$$\begin{aligned} \Pi_R(p, p_m, w) &= \Pi_{RP}(p, w) + \Pi_{RN}(p, p_m, w) \\ &= a(1-q)(p-w)\overline{F}(p) + aq\Bigg[(p-w-c_r)\overline{F}(\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}) \\ &\quad + \int_{\frac{p_m+c_e}{1-\alpha}}^{\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}} [(1-\alpha)(1-\beta)x+\beta p_m-(1-\beta)c_e-w-c_r]f(x)dx\Bigg] \\ \Pi_M(w, p, p_m) &= \Pi_{MP}(w, p) + \Pi_{MN}(w, p, p_m) \\ &= a(1-q)(w-c)\overline{F}(p) + aq(w-c)\overline{F}(\frac{p_m+c_e}{1-\alpha}) \end{aligned} \quad (4)$$

Let $p^*(w)$ and $p_m^*(w)$ denote, respectively, the optimal posted price and the optimal cut-off price of $\Pi_R(p, p_m, w)$; anticipating the best response of the retailer, the manufacturer will choose the optimal wholesale price, w^*, to maximize $\Pi_M(w, p^*(w), p_m^*(w))$.

4. Analysis

In this section, we focus on the case in which both price-takers and bargainers co-exist (i.e., $0 < q < 1$) simultaneously. We characterize the optimal posted price and the optimal cut-off price for a given wholesale price, w, and also investigate how the optimal posted price and the optimal cut-off price depend on the fraction of the bargainers, q, the relative bargaining power of the bargainers, β, and the wholesale price, w. At the end of this section, we add one assumption in which the customer's reservation price follows a uniform distribution. Under this assumption, we express the closed form solutions of the optimal posted price and the optimal cut-off price of the retailer, as well as the optimal wholesale price of the manufacturer, respectively. In addition, we show how these prices are affected by the model characteristics and discuss the managerial insights. In the appendix, we discuss two special cases under which all customers are price-takers (i.e., $q = 0$) and all customers are bargainers (i.e., $q = 1$), respectively.

4.1 Price-takers and bargainers co-exist

In this subsection, we consider a general case in which price-takers and bargainers co-exist within the customer population. The retailer sets not only the posted price, p, but also the cut-off price, p_m, when facing both types of the customers. To impose a structure on $\Pi_R(p, p_m, w)$ so that it is amenable to analysis, we have the following assumption:

Assumption 1. *The cdf of R_v, $F(\cdot)$, is defined over the interval $(0, b)$ for some $0 < b < \infty$ where b is the maximum reservation price of the customers* [4]. *Also, $F(\cdot)$, has increasing failure rate. That is, $\frac{f(\cdot)}{\bar{F}(\cdot)}$ is increasing.*

With Assumption 1, the following lemma characterizes that, for a given wholesale price, w, there exists a unique pair of the posted price and the cut-off price that maximizes the retailer's expected profit:

Lemma 1. *Given the wholesale price, w, suppose the fraction of the bargainers is less than a half (i.e., $q \leq 1/2$), there exists a unique pair of the posted price, p, and the cut-off price, p_m, that satisfy the first order conditions for $\Pi_R(p, p_m, w)$, and this pair maximizes $\Pi_R(p, p_m, w)$.*

Notice that the assumption of $q \leq 1/2$ is not unpractical given that about one-quarter of customers would like to bargain for items at a BestBuy store in Minnesota (Richtel, 2008). In particular, as the following lemma shows, if one assumes that the customer's reservation price follows a uniform distribution, then Lemma 1 holds without assuming the fraction of the bargainers being less than a half.

[4] We assume b is finite to reflect the fact that customers will not pay unreasonably high price for a product. In addition, one may envision a situation where the lower bound of the customer reservation price is positive, instead of zero. Adding such a feature to our model will not change qualitative results and derived managerial insights, but only raise computational complexity.

Lemma 2. *If the reservation price distribution, F, is uniform over the interval $(0, b)$, then for any $q \in (0, 1)$, there exists a unique pair of the posted price, p, and the cut-off price, p_m, that satisfy the first order conditions for $\Pi_R(p, p_m, w)$, and this pair maximizes $\Pi_R(p, p_m, w)$.*

The following proposition shows how the wholesale price, w, influences the optimal prices, $p^*(w)$ and $p^*_m(w)$, respectively [5].

Proposition 1. *Both $p^*(w)$ and $p^*_m(w)$ are increasing in the wholesale price, w, that is, $\frac{dp^*(w)}{dw} \geq 0$ and $\frac{dp^*_m(w)}{dw} \geq 0$.*

The results in Proposition 1 are intuitive. When the manufacturer raises the wholesale price, w, the retailer's cost of ordering items from the manufacturer increases, and thus, the profit margin decreases. In order to maintain the profit margin, the retailer would like to raise the posted price to price out a proportion of the customers with lower reservation prices. The retailer also increases the cut-off price in order to maintain profit margin which is high enough to cover the cost of negotiation.

In addition to the wholesale price, w, two other important factors that influence the optimal prices $p^*(w)$ and $p^*_m(w)$ are the fraction of the bargainers, q, and the relative bargaining power of the bargainers, β. The following proposition shows the relationships between the optimal prices and the model parameters:

Proposition 2. *Given the wholesale price, w,*
(a) The optimal posted price $p^*(w)$ is increasing in the fraction of the bargainers, q.
(b) The optimal cut-off price $p^*_m(w)$ is increasing in both the fraction of the bargainers, q, and the bargainer's relative bargaining power, β.

First, we consider the effect of the fraction of the bargainers on the optimal prices. As the fraction of the bargainers increases, the retailer puts more emphasis on the bargainers. As a result, the retailer will raise the posted price to enable better price discrimination among bargainers. By setting such a high posted price, the retailer can raise the cut-off price to increase the final purchase price. On the other hand, note that the cut-off price and the relative bargaining power of the bargainers behave in the opposite directions: a higher cut-off price and/or a lower relative bargaining power of the bargainers will result in a higher final purchase price (observed from (4)), and thus, the retailer will receive higher profits. In fact, one can show that the retailer's profit function, $\Pi_R(p, p_m, w)$, is supermodular [6] in the cut-off price and in the bargainer's relative

[5] As the proof reveals, the results in Propositions 1 and 2 hold under general conditions that do not require F being uniform distribution.

[6] Note that a function $f : R^m \to R$ is supermodular if $f(x \wedge y) + f(x \vee y) \geq f(x) + f(y)$ for $x, y \in R^m$, where $x \wedge y := (\min(x_1, y_1), \min(x_2, y_2), \ldots, \min(x_m, y_m))$ and $x \vee y := (\max(x_1, y_1), \max(x_2, y_2), \ldots, \max(x_m, y_m))$.

bargaining power. Thus, the cut-off price and the bargaining power are economic complements. If the relative bargaining power of the bargainers decreases, the retailer already takes advantage of the lower bargainers' bargaining power and gathers a higher profit. Therefore, the retailer would rather lower the cut-off price in order to attract more bargainers.

4.2 Uniform distribution of the customer's reservation price

Note that the co-existence of both price-takers and bargainers complicates the decision of the wholesale price set by the manufacturer. In fact, our result shows that there may not exist the unique wholesale price that maximizes the manufacturer's profit. Therefore, in this subsection, we add one assumption in that the customer's reservation price follows a uniform distribution over the interval $(0, b)$. The assumption of uniform not only facilitates the analysis of the optimal prices in the supply chain, but also benefits the retailer if the retailer does not have enough information regarding the shape of the customer's reservation price distribution. In particular, in the next section, we conduct a numerical study to further explore the effects of model parameters on the optimal prices. For some commonly-used reservation price distributions with unbounded domain, such as exponential and Weibull, we need to use their truncated versions to fit the assumption of domain being $(0, b)$. The truncated reservation price distributions may result in an effect on the optimal prices, which distorts the analysis. As a result, we use uniform distribution assumption for the following sections.

Under uniform distribution assumption, we can characterize the optimal wholesale price[7], w^*, the optimal posted price, $p^*(w^*)$, and the optimal cut-off price, $p^*_M(w^*)$ in equilibrium, in a closed form, respectively:

$$
\begin{aligned}
w^* &= \frac{-qc_r[q(\beta-1)+2(1-\alpha+q\alpha)]}{2(1-\alpha+q\alpha)^2} + \frac{c_r q\beta - c_e q - b\alpha + b}{2(1-\alpha+q\alpha)} + \frac{c}{2} \\
p^*(w^*) &= \frac{c_r q\beta - c_e q - b\alpha + b}{q(\beta-1)+2(1-\alpha+q\alpha)} + \left(1 - \frac{1-\alpha+q\alpha}{q(\beta-1)+2(1-\alpha+q\alpha)}\right) w^* \\
p^*_m(w^*) &= c_r(1-\beta) + \frac{(c_r q\beta - c_e q - b\alpha + b)\beta}{q(\beta-1)+2(1-\alpha+q\alpha)} + \left(1 - \frac{(1-\alpha+q\alpha)\beta}{q(\beta-1)+2(1-\alpha+q\alpha)}\right) w^*
\end{aligned}
\qquad (5)
$$

Note that the optimal wholesale price, w^*, the optimal posted price, $p^*(w^*)$, and the optimal cut-off price, $p^*_m(w^*)$, are all functions of the relative bargaining power of the bargainers, β, and the fraction of the bargainers, q. The following proposition shows the effect of the relative bargaining power of the bargainers on the optimal wholesale price:

[7]When the customer's reservation price follows a uniform distribution over the interval $(0, b)$, one can show that the manufacturer's profit, $\Pi_M(w, p^*(w), p^*_m(w))$ is concave in w, and thus the optimal wholesale price, w^*, can be uniquely determined.

Proposition 3. *If the reservation price distribution, F, follows a uniform distribution over the interval $(0, b)$, then the optimal wholesale price, w^*, is increasing in the customer's relative bargaining power, β.*

As the relative bargaining power of the bargainers decreases, the retailer, given the same wholesale price, raises the posted price to obtain higher prices from bargainers, and reduces the cut-off price to attract more customers. In order to capture more revenues, the manufacturer charges a low wholesale price so that more customers will purchase, and thus, benefits the manufacturer more than charging a high wholesale price in order for a high profit margin.

Based on Propositions 1-3, we can characterize the effect of the relative bargaining power of the bargainers on the optimal cut-off price in equilibrium:

Corollary 1. *If the reservation price distribution, F, follows a uniform distribution over the interval $(0, b)$, then in equilibrium, the optimal cut-off price, $p_m^*(w^*)$ is increasing in the customer's relative bargaining power, β.*

Now, we discuss the effects of negotiation costs on the optimal prices. The following proposition formally states, the cost of negotiation incurred by the retailer or by the bargainers has opposite impact on the optimal pricing strategies of the retailer. On the other hand, the costs of negotiation induce the manufacturer to reduce the wholesale price in equilibrium.

Proposition 4. *Consider the reservation price distribution, F, follows a uniform distribution over the interval $(0, b)$.*
(a) Given the wholesale price, w, both the optimal posted price, $p^*(w)$, and the optimal cut-off price, $p_m^*(w)$, are increasing in the negotiation cost of the retailer, c_r, and decreasing in the base negotiation cost of the bargainers, c_e.
(b) The optimal wholesale price, w^*, is decreasing in both negotiation cost of the retailer, c_r, and base negotiation cost of the bargainers, c_e.

Note from (4) that an increase in c_r enhances the cost of the retailer from bargainers, thereby the retailer has to maintain a fairly reasonable profit margin by selling the product to high reservation price bargainers, and in the meantime, dropping out low reservation price bargainers. As a result, given the wholesale price, w, the retailer tends to raise both posted and cut-off prices with a high c_r: raise the posted price to enable better price discrimination, in particular, from high reservation price bargainers, and raise the cut-off price to increase the barrier of successful purchase so that low reservation price bargainers are not able to buy. However, the negotiation cost incurred by the bargainers, c_e, reverses the retailer pricing decisions of posted and cut-off prices. To wit, the retailer sets a lower cut-off price to attract more bargainers and a lower posted price to balance the profit from both types of the customers. On the other hand, the variations of the wholesale price to both costs of negotiation are consistent. When either negotiation cost increases, the manufacturer will reduce the wholesale price to absorb a portion of the cost from negotiation.

Based on both results in Proposition 4, it implies that, in equilibrium, both the posted and cut-off prices, $p^*(w^*)$ and $p_m^*(w^*)$, in the supply chain will be reduced as the negotiation cost of the bargainers, c_e, increases. The following corollary formally states this observation.

Corollary 2. *If the reservation price distribution, F, follows a uniform distribution over the interval $(0, b)$, then in equilibrium, both optimal posted price, $p^*(w^*)$, and the optimal cut-off price, $p_m^*(w^*)$, are decreasing in the base negotiation cost of the bargainers, c_e.*

5. Numerical Study

In this section, we conduct a numerical study to gain further managerial insights. We focus our analysis on the relationships between the optimal prices and the model parameters which are untraceable through theoretical discussions in the previous sections. We analyze the effects of model parameters on these optimal prices under the assumption of the customers' reservation prices being uniform distributed over the interval $(0, b)$. We investigate how optimal prices change under different scenarios.

We consider several different combinations of parameter values. We use three different values for the base negotiation cost of the bargainers ($c_e \in \{0.5, 2, 200\}$), two for the maximum reservation price of the customer ($b \in \{120, 1000\}$), three for the customer's bargaining power ($\beta \in \{0.3, 0.6, 0.7\}$), and three for the negotiation cost of the retailer ($c_r \in \{0.5, 2, 228\}$). We also set five different values for α ($\alpha \in \{0.001, 0.004, 0.01, 0.1, 0.2\}$), and nine different values for the fraction of the bargainers ($q \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$). Note that this parameter set results in 2430 different combinations of $c_e, b, \beta, c_r, \alpha$, and q [8].

5.1 The Effect of relative bargaining power

We first discuss the effect of relative bargaining power on the optimal posted price. Among 2430 different scenarios, we find that, in equilibrium, the optimal posted price, $p^*(w^*)$, decreases in the relative bargaining power, β. Notice that when the relative bargaining power of the bargainers,

[8] Notice that these 2430 combinations show similar patterns for the following analysis; thereby, the figures and the associated parameters in Section 5 are depicted for demonstration.

β, decreases (or the relative bargaining power of the retailer, $1 - \beta$, increases), the retailer can increase the posted price and use the high bargaining power to the full in order to carry out price discrimination. Notice that the posted price forms as a ceiling for the retailer. Therefore, charging a higher posted price increases the ceiling and in turn benefits the retailer to price discriminate among bargainers especially in the case of bargainers with reservation prices which are too high.

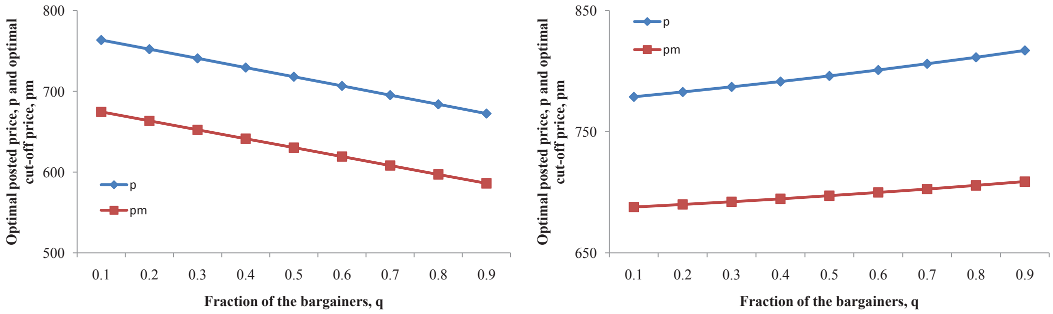
5.2 The Effect of the fraction of the bargainers

In this subsection, we further investigate the effect of the fraction of the bargainers on all the prices. First, our results show that the manufacturer tends to reduce the wholesale price as bargainers become the majority of the customer population (i.e., higher q). Note that it is conceivable that as the fraction of the bargainers increases, the retailer puts more emphasis on the bargainers. The benefit from price discrimination induces the retailer to sell the items to more bargainers with heterogeneous reservation prices, and hence, collect more revenue. At the same time, the manufacturer reduces the wholesale price as the profit generated from selling more products to the customers outweigh the situation when the manufacturer raises the wholesale price for a higher profit margin but less customers are able to purchase. This move successfully leads an decrease in the cut-off price and then more bargainers are able to purchase. The results are more apparent when the fraction of the bargainers is very high.

We next move our attention to the effect of the fraction of the bargainers, q, on the optimal posted price, $p^*(w^*)$, as well as the optimal cut-off price, $p_m^*(w^*)$, in equilibrium. As represented in the following subsections, the effects of q on both posted and cut-off prices are not monotonic, depending on model characteristics such as c_e, b, c_r, and α. These driving forces may influence the strategic move of the retailer from different angles. As a result, we identify these results in each subsection, respectively.

5.2.1 The effect of base negotiation cost of the bargainers

Note that the base negotiation cost, c_e, can be regarded as the fixed cost to a bargainer as long as she, with a high enough reservation price, negotiates and successfully purchases, and thus, is not related to her reservation price directly. As a result, an increase in the base negotiation cost has the same effect on each bargainer who purchases the item. On the other hand, the base negotiation cost negatively influences the final purchase price agreed by the retailer and the bargainers, especially on those bargainers with modest reservation prices. Although a higher fraction of the bargainers allows the retailer to focus on the bargainers by enabling better price discrimination, the negative effect of high cost of negotiation, c_e, will also lead to a lower final purchase price, both of which form as the trade-off of the retailer's when implementing negotiation.



(a) Ce=200

(b) Ce=0.5

Figure 1: The figure illustrates the effect of the fraction of the bargainers on the optimal posted price and the optimal cut-off price at two different base negotiation costs, c_e: (a) $c_e = 200$ (left) and (b) $c_e = 0.5$ (right). Here, $a = 1, c = 100, \beta = 0.6, c_r = 2, \alpha = 0.01$ and we assume the customer's reservation price follows a uniform distribution over an interval $(0, 1000)$.

Fig. 1 shows that both posted and cut-off prices decrease in the fraction of the bargainers when the base negotiation cost of the bargainers is high, and the opposite is true with a low base negotiation cost. That is, more bargainers in the customer population do not necessarily allow the retailer to ignore price-takers, especially when bargainers incur high cost of negotiation, and thus, the profit margin generated from them is not high. In fact, we observe from the left panel of Fig. 1 that the expected selling price to bargainers is 679.98 when $q = 0.3$ (compared to the expected selling price to price-takers 740.88) [9] and reduces to 644.55 when $q = 0.7$ (compared to the expected selling price to price-takers 695.31). In addition, a reduction in the wholesale price as more bargainers exist (earlier result in this section) provides the retailer an incentive to decrease both prices in order to allow more sales to take place. Thus, when the fraction of the bargainers increases, both prices decrease accordingly. On the other hand, with a low base negotiation cost (right panel of Fig. 1), more bargainers can afford to purchase. The retailer can increase both prices to enable better price discrimination: increase the posted price to raise the ceiling of the price and target the bargainers with high reservation prices, and increase the cut-off price to enlarge

[9] Based on our negotiation outcome, bargainers with reservation price $r \geq \frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}$ pay at p and those with $r \in [\frac{p_m+c_e}{1-\alpha}, \frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)})$ pay at $(1-\alpha)(1-\beta)r + \beta p_m - (1-\beta)c_e$. The expected selling price to bargainers is $\frac{1}{1-F(\frac{p_m+c_e}{1-\alpha})}[\int_{\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}}^{b} pf(x)dx + \int_{\frac{p_m+c_e}{1-\alpha}}^{\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}} [(1-\alpha)(1-\beta)x + \beta p_m - (1-\beta)c_e)]f(x)dx]$ while the expected selling price to price-takers is simply p. For $q = 0.3$, we have $p = 740.88$ and $p_m = 652.48$, and hence, the expected selling price to bargainers and price-takers are 679.98 and 740.88, respectively. Same logic applies for $q = 0.7$.

the final purchase price and focus on the bargainers with modest reservation prices.

5.2.2 The effect of maximum reservation price of the customers

In our model, b is represented as the maximum reservation price of the end customers. A higher b means the retailer has more space to do better price discrimination and receives higher profits from bargainers with higher reservation prices. We show the effect of the maximum reservation price of the customers on the optimal posted price, $p^*(w^*)$, and the cut-off price, $p_m^*(w^*)$, in equilibrium, in Fig. 2.

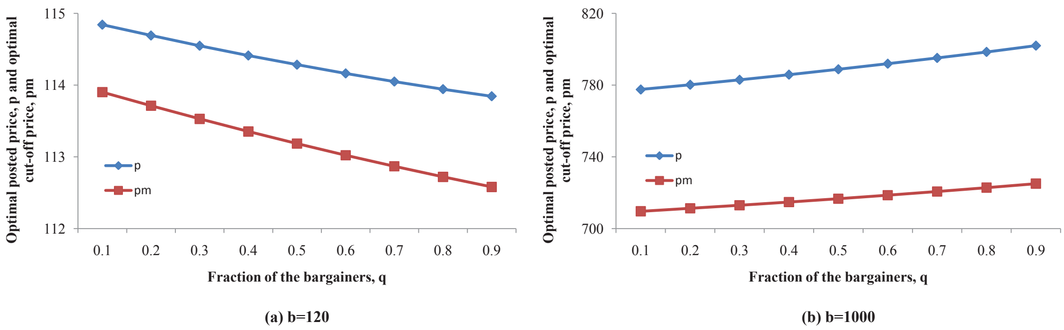


Figure 2: The figure illustrates the effect of the fraction of the bargainers on the optimal posted price and the optimal cut-off price at two different highest reservation prices of the customers, b: (a) $b = 120$ (left) and (b) $b = 1000$ (right). Here, $a = 1, c = 100, \beta = 0.7, c_r = 2, c_e = 2, \alpha = 0.01$ and we assume the customer's reservation price follows a uniform distribution over an interval $(0, b)$.

As Fig. 2 shows, both posted price and cut-off price decrease in the fraction of the bargainers with a low maximum reservation price of the customers, and the result is reversed otherwise. Notice that a lower maximum reservation price of the end customers basically squeezes the space for the retailer to price discriminate. It can be observed from the result (left panel of Fig. 2) that the difference between the optimal posted price and the optimal cut-off price is narrow, in that the retailer sells the items to the bargainers close to the take-it-or-leave-it pricing. Although an increase in the fraction of the bargainers should potentially induce high posted and cut-off prices so that the final purchase price can be raised accordingly, a lower maximum reservation price implies that there is seldom or no customer with a very high reservation price, and thus, it is rather difficult for the retailer to receive excess profit through price discrimination. In addition, we find through the earlier numerical result that the wholesale price decreases in q as well, which provides the retailer some space to reduce both prices to attract more customers, whether they be price-takers or bargainers. As a result, rather than raising the optimal prices to enjoy a moderate profit margin, lowering both posted and cut-off prices to attract more customers is considered a better strategy for the retailer.

On the other hand, when b is high, the retailer's ability to price discriminate enhances and will receive higher profit from the bargainers with higher reservation prices. In particular, as the fraction of the bargainers increases, the retailer places more emphasis on the bargainers. Therefore, the retailer raises both posted and cut-off prices instead. We observe this effect from the right panel of Fig. 2. The difference between the posted price and the cut-off price is relatively large so that the retailer has ample space to price discriminate based on the bargainers' reservation prices and receives a higher profit.

5.2.3 The effect of negotiation cost of the retailer

Note that the existence of the negotiation cost of the retailer, c_r, decreases the willingness of the retailer to allow negotiation with the bargainers. Based on the numerical outcome, we observe from Fig. 3 that the cut-off price increases in the fraction of the bargainers under a low c_r. However, when such negotiation cost is high, the relationship between the cut-off price and the fraction of the bargainers is not obvious. The same pattern can be found for the posted price (Fig. 4). When the fraction of the bargainers, q, is low, the results are similar to what we had in Section 5.2.1 (effect of the base negotiation cost of the bargainers) as the retailer does not necessarily increase both prices in q since the effect of retailer's negotiation cost dominates his pricing decisions, and thus, the associated profit. However, with a high fraction of the bargainers, the retailer's pricing decisions are primarily influenced by an increase in the fraction of the bargainers in that the retailer is willing to increase both prices for finer price discrimination.

5.2.4 The effect of α

In our model, the negotiation cost of the bargainers is divided into two parts. In addition to the base negotiation cost, we also consider the part of the negotiation cost related to the bargainers' reservation prices, αr. A higher α means that the negotiation cost of a bargainer is highly related to her reservation price. We show the effect of α on both posted and cut-off prices in Fig. 5.

We may learn from Fig. 5 that when α is high, the posted price and the cut-off price are decreasing in relation to the fraction of the bargainers. The result is opposite when α is low. This pattern and logic behind the retailer's strategic move are similar to the one which occurred when we discussed the effect of the maximum reservation price in Section 5.2.2. In fact, a lower b or a higher α limits the space of price discrimination since the retailer is not able to receive a higher

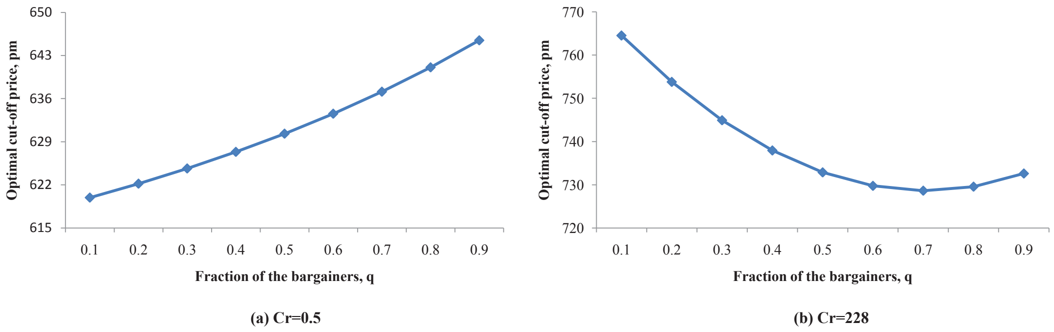


Figure 3: The figure illustrates the effect of the fraction of the bargainers on the optimal cut-off price at two different negotiation costs of the retailer, c_r: (a) $c_r = 0.5$ (left) and (b) $c_r = 228$ (right). Here, $a = 1, c = 100, \beta = 0.3, c_e = 2, \alpha = 0.004$ and we assume the customer's reservation price follows a uniform distribution over an interval $(0, 1000)$.

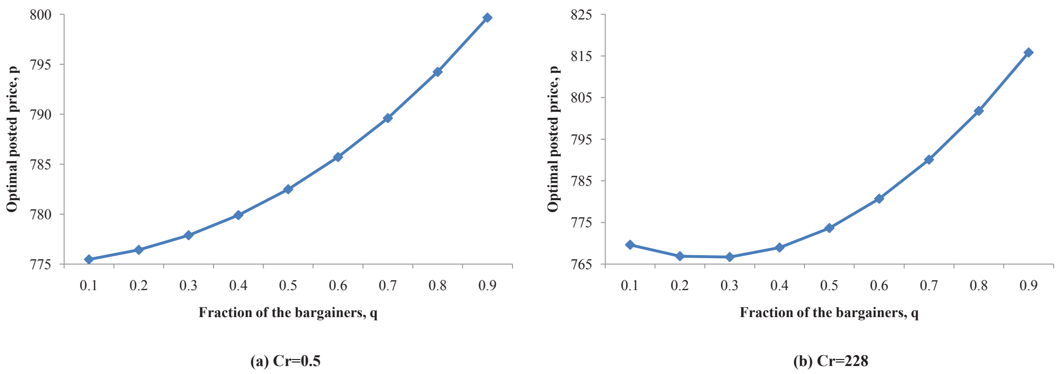


Figure 4: The figure illustrates the effect of the fraction of the bargainers on the optimal posted price at two different negotiation costs of the retailer, c_r: (a) $c_r = 0.5$ (left) and (b) $c_r = 228$ (right). Here, $a = 1, c = 100, \beta = 0.3, c_e = 2, \alpha = 0.1$ and we assume the customer's reservation price follows a uniform distribution over an interval $(0, 1000)$.

profit from higher reservation prices bargainers. This effect forces the retailer to reduce both prices so as to attract more customers. However, the pattern is reversed as b becomes higher and/or α lower.

6. Conclusion

This paper considers a supply chain where the retailer orders the products from the manufacturer at the wholesale price and sells the items to two types of customers - price-takers and bargainers. The price that a customer pays for the product depends on their individual type. The behavior

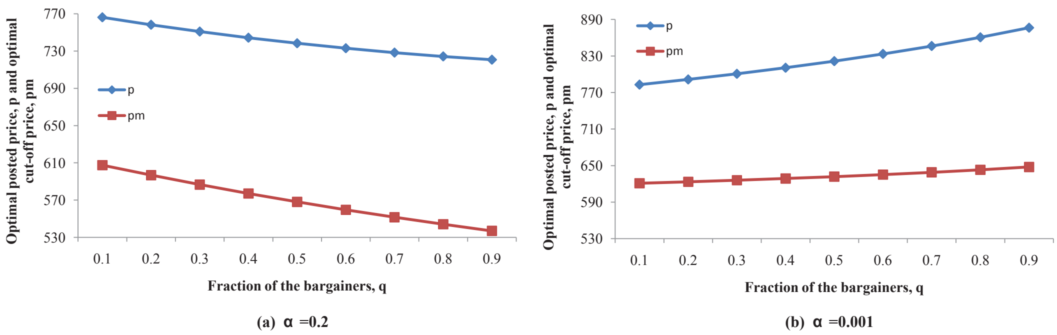


Figure 5: The figure illustrates the effect of α on the optimal posted price and the optimal cut-off price at two different α: (a) $\alpha = 0.2$ (left) and (b) $\alpha = 0.001$ (right). Here, $a = 1, c = 100, \beta = 0.3, c_r = 2, c_e = 2$ and we assume the customer's reservation price follows a uniform distribution over an interval $(0, 1000)$.

of the price-takers follows the take-it-or-leave-it principle. However, the behavior of the bargainers depends on their reservation prices and subsequent negotiation outcome, which is derived by the generalized Nash bargaining solution. In this bargaining model, the final purchase price splits the difference between the bargainer's net reservation price after negotiation (i.e., reservation price minus negotiation cost) and the minimum price at which the retailer is willing to sell. Our model characterizes the optimal posted and cut-off prices of the retailer, and the optimal wholesale price of the manufacturer, which depend on the fraction of the bargainers and the relative bargaining power of the bargainers. Furthermore, we identify four factors which affect the equilibrium of the optimal prices.

Our results show that, given the wholesale price, there exists a unique pair of the posted price and the cut-off price that maximizes the retailer's profit. In addition, we show that the higher the wholesale price is, the higher the posted price and the cut-off price are. We also show the effects of the fraction of the bargainers and the relative bargaining power of the bargainers on the optimal prices. In order to further explore the effects of model parameters on the optimal prices, we assume that the customer's reservation price follows a uniform distribution. We provide the closed forms of the optimal wholesale price, the optimal posted price, and the optimal cut-off price in equilibrium, respectively. We find that the optimal cut-off price and the optimal wholesale price increase with the relative bargaining power of the bargainers. However, both posted and cut-off prices may behave differently depending on the negotiation cost incurred by the party under negotiation. In addition, we conduct a numerical study to get more managerial insights. We observe the fact that the posted price decreases in the relative bargaining power of the bargainers, and the wholesale price decreases in the fraction of the bargainers. We also find an interesting result in which the effects of the fraction of the bargainers on the posted and the cut-off prices are not monotonic. When the maximum customer's reservation price is low, and/or the negotiation costs are high, and/or the relationship between the bargainer's negotiation cost and reservation price is high, the retailer will decrease both posted and cut-off prices even when the fraction of the bargainers increases, doing which can attract more customers, no matter price-takers or bargainers to purchase. This finding shows that as more bargainers are in the customer population, the retailer may not necessarily focus only on bargainers by charging high posted and cut-off prices so as to do better price discrimination.

There are several future research directions. For example, in the real world, there are multiple sellers who sell the same product. In this case, the price that rival companies set will definitely influence the price of a specific seller. If there is a price war in place within the industry, then the other companies' posted prices form the upper bound of the selling price that a specific seller can charge. The other direction is to relax the assumption we provided in this model: the negotiation cost of the bargainers is linearly related to their reservation price. For example, the negotiation costs of the bargainers might be related to the reservation price in the polynomial principle. Under different assumptions of the negotiation costs of the bargainers, the retailer's optimal pricing strategy might change. In addition, another direction of the paper is to investigate cases in which the retailer is allowed to keep a fraction of the products in advance and sells it to the bargainers. Thus, the question becomes: what is the effect of keeping these products on the optimal posted price and the optimal cut-off price? Another interesting line of research would be to analyze the effect of negotiation when customers can learn the prices paid by previous customers. That is, the customer who negotiates later has higher bargaining power than the former bargainers. The relative bargaining power should be seen as the function of the bargaining sequence.

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APPENDIX

In this appendix, we first introduce the concept of generalized Nash bargaining solution. We then provide the detailed derivations of our technical results.

Generalized Nash Bargaining Solution

Consider there are two players, A and B, who try to come to an agreement over alternatives in some set S. Each player i has a utility function, $u_i, i = A, B$ defined over $S \cup D$, where D is the outcome when the agreement is not reached by both players. In addition, define the disagreement pair $d = (d_A, d_B)$ where $d_i = u_i(D)$. Then the set of all pairs of utility from the agreement is given by:

$$\Omega = \{[u_A(x), u_B(x)] \in R^2 : x \in S\}$$

Assume the set Ω is convex and compact and for some $\omega \in \Omega$, we have $\omega_i > d_i, i = A, B$. In addition, define β and $1-\beta$ as the relative bargaining power of player A and player B, respectively where $\beta \in (0, 1)$. Then the generalized Nash bargaining solution, $f(\Omega, d)$, is the unique solution to the following maximization problem

$$max \quad (u_A - d_A)^{\beta}(u_B - d_B)^{1-\beta}$$

that satisfies the three axioms: (1) Invariance to equivalent utility representations, (2) Independence of irrelevant alternatives, and (3) Pareto efficiency. Note that if $\beta = \frac{1}{2}$, then the generalized Nash bargaining solution reduces to Nash bargaining solution which satisfies the axiom of symmetry.

Proof of Lemma 1

We first prove that, for a given p, $\Pi_R(p, p_m, w)$ is strictly unimodal in p_m. To do so, we prove the following claims: (i) $\left.\frac{\partial \Pi_R(p,p_m,w)}{\partial p_m}\right|_{p_m=w+c_r} \geq 0$, (ii) $\frac{\partial^2 \Pi_R(p,p_m,w)}{\partial p_m^2} < 0$ whenever $\frac{\partial \Pi_R(p,p_m,w)}{\partial p_m} = 0$, and (iii) $\left.\frac{\partial \Pi_R(p,p_m,w)}{\partial p_m}\right|_{p_m=p} \leq 0$.

The first and second derivatives of $\Pi_R(p, p_m, w)$ with respect to p_m are

$$\frac{\partial \Pi_R(p,p_m,w)}{\partial p_m} = aq\left[\frac{w+c_r-p_m}{1-\alpha} f\left(\frac{p_m+c_e}{1-\alpha}\right) + \beta\left(F\left(\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right) - F\left(\frac{p_m+c_e}{1-\alpha}\right)\right)\right] \tag{A-1}$$

$$\begin{aligned}\frac{\partial^2 \Pi_R(p,p_m,w)}{\partial p_m^2} = \; & aq\left[\frac{-(1+\beta)}{1-\alpha} f\left(\frac{p_m+c_e}{1-\alpha}\right) + \frac{w+c_r-p_m}{(1-\alpha)^2} f'\left(\frac{p_m+c_e}{1-\alpha}\right)\right. \\ & \left. -\frac{\beta^2}{(1-\beta)(1-\alpha)} f\left(\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)\right].\end{aligned} \tag{A-2}$$

Claims (i) and (iii) follow directly from (A-1). To show claim (ii), note from (A-1) and (A-2)

$$\left.\frac{\partial^2\Pi_R(p,p_m,w)}{\partial p_m^2}\right|_{\frac{\partial \Pi_R(p,p_m,w)}{\partial p_m}=0} = \frac{aq}{1-\alpha}\Bigg[-(1+\beta)f\left(\frac{p_m+c_e}{1-\alpha}\right) - \frac{\beta\left(F\left(\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)-F\left(\frac{p_m+c_e}{1-\alpha}\right)\right)}{f(\frac{p_m+c_e}{1-\alpha})}$$
$$\times f'\left(\frac{p_m+c_e}{1-\alpha}\right) - \frac{\beta^2}{(1-\beta)}f\left(\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)\Bigg]. \tag{A-3}$$

If $f'(\frac{p_m+c_e}{1-\alpha}) \geq 0$, then all three terms in the brackets of (A-3) are negative. Hence, claim (ii) holds if $f'(\frac{p_m+c_e}{1-\alpha}) \geq 0$. Now, consider the case that $f'(\frac{p_m+c_e}{1-\alpha}) < 0$. First, note that

$$\beta\left(F\left(\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right) - F\left(\frac{p_m+c_e}{1-\alpha}\right)\right) = \beta\left(\overline{F}\left(\frac{p_m+c_e}{1-\alpha}\right) - \overline{F}\left(\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)\right)$$
$$< \beta\overline{F}\left(\frac{p_m+c_e}{1-\alpha}\right).$$

Thus, we have

$$-(1+\beta)f\left(\frac{p_m+c_e}{1-\alpha}\right) - \frac{\beta\left(F\left(\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)-F\left(\frac{p_m+c_e}{1-\alpha}\right)\right)}{f(\frac{p_m+c_e}{1-\alpha})}f'\left(\frac{p_m+c_e}{1-\alpha}\right)$$
$$\leq -(1+\beta)f\left(\frac{p_m+c_e}{1-\alpha}\right) - \frac{\beta\overline{F}\left(\frac{p_m+c_e}{1-\alpha}\right)}{f(\frac{p_m+c_e}{1-\alpha})}f'\left(\frac{p_m+c_e}{1-\alpha}\right) \leq 0$$

where the last inequality is from the fact that F has increasing failure rate (IFR) and, thus, $f^2(\cdot) + f'(\cdot)\overline{F}(\cdot) \geq 0$. Hence, the first two terms in the brackets of (A-3) add up to a negative number, and the third term in the brackets of (A-3) is also negative, concluding the proof of claim (ii).

Exploiting the strict unimodality of $\Pi_R(p,p_m,w)$ in p_m for given p, let $p_m^*(p)$ be the unique optimal value of p_m at a given p. Define the induced function $\Pi_R^*(p,w) := \Pi_R(p,p_m^*(p),w)$. We now show that $\Pi_R^*(p,w)$ is strictly unimodal in p.

We prove the unimodality of $\Pi_R^*(p,w)$ by showing (iv) $\left.\frac{d\Pi_R^*(p,w)}{dp}\right|_{p=w} \geq 0$, (v) $\frac{d^2\Pi_R^*(p,w)}{dp^2} < 0$ whenever $\frac{d\Pi_R^*(p,w)}{dp} = 0$, and (vi) $\left.\frac{d\Pi_R^*(p,w)}{dp}\right|_{p=b} \leq 0$.

The partial derivative of $\Pi_R(p,p_m,w)$ with respect to p is

$$\frac{\partial\Pi_R(p,p_m,w)}{\partial p} = aq\overline{F}\left(\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right) + a(1-q)\left[\overline{F}(p) - (p-w)f(p)\right] \tag{A-4}$$

Claims (iv) and (vi) directly follow (A-4). In what follows, we write p_m^* as a shorthand notation for $p_m^*(p)$. To conclude that $\Pi_R^*(p,w)$ is strictly unimodal in p, it remains to prove claim (v). To that end, first note that:

$$\frac{d^2\Pi_R^*(p,w)}{dp^2} = \left.\frac{\partial^2\Pi_R(p,p_m,w)}{\partial p^2}\right|_{p_m=p_m^*} + \frac{dp_m^*}{dp}\left.\frac{\partial^2\Pi_R(p,p_m,w)}{\partial p\partial p_m}\right|_{p_m=p_m^*} \tag{A-5}$$

Using the implicit function theorem, we have

$$\frac{dp_m^*}{dp} = -\frac{\left.\frac{\partial^2 \Pi_R(p,p_m,w)}{\partial p \partial p_m}\right|_{p_m=p_m^*}}{\left.\frac{\partial^2 \Pi_R(p,p_m,w)}{\partial p_m^2}\right|_{p_m=p_m^*}} \tag{A-6}$$

Substituting (A-6) in (A-5), we obtain

$$\frac{d^2\Pi_R^*(p,w)}{dp^2} = \frac{\left.\frac{\partial^2 \Pi_R(p,p_m,w)}{\partial p^2}\right|_{p_m=p_m^*} \left.\frac{\partial^2 \Pi_R(p,p_m,w)}{\partial p_m^2}\right|_{p_m=p_m^*} - \left(\left.\frac{\partial^2 \Pi_R(p,p_m,w)}{\partial p \partial p_m}\right|_{p_m=p_m^*}\right)^2}{\left.\frac{\partial^2 \Pi_R(p,p_m,w)}{\partial p_m^2}\right|_{p_m=p_m^*}} \tag{A-7}$$

Since $\Pi_R(p, p_m, w)$ is strictly unimodal in p_m for given p, as we proved in the first part of this lemma, the denominator is always negative. Therefore, it suffices to show that the numerator in (A-7) is strictly positive. Note that

$$\frac{\partial^2 \Pi_R(p,p_m,w)}{\partial p^2} = -a(1-q)[2f(p)+(p-w)f'(p)] - aq\frac{f\left(\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)}{(1-\beta)(1-\alpha)}, \tag{A-8}$$

$$\frac{\partial^2 \Pi_R(p,p_m,w)}{\partial p_m^2} = aq\left[\frac{-(1+\beta)}{1-\alpha}f\left(\frac{p_m+c_e}{1-\alpha}\right) + \frac{w+c_r-p_m}{(1-\alpha)^2}f'\left(\frac{p_m+c_e}{1-\alpha}\right) - \frac{\beta^2}{(1-\beta)(1-\alpha)}f\left(\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)\right] \tag{A-9}$$

$$\frac{\partial^2 \Pi_R(p,p_m,w)}{\partial p \partial p_m} = \frac{aq\beta}{(1-\beta)(1-\alpha)}f\left(\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right) \tag{A-10}$$

Using the expressions above, one can check that the numerator in (A-7) can be written as

$$-a(1-q)[2f(p)+(p-w)f'(p)]\left.\frac{\partial^2 \Pi_R(p,p_m,w)}{\partial p_m^2}\right|_{p_m=p_m^*}$$
$$-\frac{a^2q^2}{(1-\beta)(1-\alpha)}f\left(\frac{p-\beta p_m^*+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)\left[\frac{-(1+\beta)}{1-\alpha}f\left(\frac{p_m^*+c_e}{1-\alpha}\right) + \frac{w+c_r-p_m^*}{(1-\alpha)^2}f'\left(\frac{p_m^*+c_e}{1-\alpha}\right)\right]$$

Notice that, when $p_m = p_m^*$, $\frac{\partial \Pi_R(p,p_m,w)}{\partial p_m} = 0$. Using this fact, we can utilize (A-1) to substitute for $w+c_r-p_m^*$ in the above expression and show the terms $\left[\frac{-(1+\beta)}{1-\alpha}f\left(\frac{p_m^*+c_e}{1-\alpha}\right) + \frac{w+c_r-p_m^*}{(1-\alpha)^2}f'\left(\frac{p_m^*+c_e}{1-\alpha}\right)\right]$ is negative (see the earlier argument where we prove (A-3) is negative). Furthermore, $\left.\frac{\partial^2 \Pi_R(p,p_m,w)}{\partial p_m^2}\right|_{p_m=p_m^*}$ is negative (since $\Pi_R(p, p_m, w)$ is strictly unimodal in p_m for given p). Thus, to conclude the proof of claim (v), it suffices to show that $-a(1-q)[2f(p)+(p-w)f'(p)]$ is also negative when $\frac{d\Pi_R^*(p,w)}{dp} = \left.\frac{\partial \Pi_R(p,p_m,w)}{\partial p}\right|_{p=p_m^*} = 0$. Given $\left.\frac{\partial \Pi_R(p,p_m,w)}{\partial p}\right|_{p=p_m^*} = 0$, we can utilize (A-4) to substitute for $p-w$ in $-a(1-q)[2f(p)+(p-w)f'(p)]$ and we obtain:

$$-a(1-q)[2f(p)+(p-w)f'(p)] = -a(1-q)[2f(p) + \frac{q\overline{F}\left(\frac{p-\beta p_m^*+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)+(1-q)\overline{F}(p)}{(1-q)f(p)}f'(p)]. \tag{A-11}$$

If $f'(p) \geq 0$, then (A-11) is clearly negative and the proof is done. Consider now the case that $f'(p) < 0$. Therefore:

$$
\begin{aligned}
-a(1-q)[2f(p)+(p-w)f'(p)] &= -a(1-q)\left(2f(p)+\frac{q\overline{F}\left(\frac{p-\beta p_m^*+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)+(1-q)\overline{F}(p)}{(1-q)f(p)}f'(p)\right)\\
&\leq -a(1-q)\left(2f(p)+\frac{\overline{F}(p)f'(p)}{(1-q)f(p)}\right)\\
&\leq -a(1-q)\left(2f(p)+2\frac{\overline{F}(p)f'(p)}{f(p)}\right)\\
&< 0
\end{aligned}
$$

where the first inequality holds because $p_m^* \leq p$ and, thus, $\overline{F}\left(\frac{p-\beta p_m^*+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right) \leq \overline{F}(p)$; the second inequality holds because $q \leq \frac{1}{2}$; and the last inequality holds because F is IFR and, thus, $f^2(\cdot)+f'(\cdot)\overline{F}(\cdot) \geq 0$. Thus, we have shown that (A-11) is negative, which concludes the proof of claim (v), which in turn concludes the proof of the lemma[10].

Proof of Lemma 2

Observe from the proof of Lemma 1 that we only use the restriction $q \leq \frac{1}{2}$ to prove (A-11) is negative. When, F is uniform over $(0, b)$, $f' = 0$ and, thus, the result of (A-11) being negative directly holds without the assumption of $q \leq \frac{1}{2}$.

Proof of Proposition 1

It follows from Lemma 1 that the optimal posted price, $p^*(w)$ and the optimal cut-off price, $p_m^*(w)$, are given by the unique pair of p and p_m that satisfy the first order conditions of $\Pi_R(p, p_m, w)$, that is,

$$\frac{\partial \Pi_R(p^*(w), p_m^*(w), w)}{\partial p} = 0 \text{ and} \frac{\partial \Pi_R(p^*(w), p_m^*(w), w)}{\partial p_m} = 0.$$

Implicit differentiation of the two equalities with respect to w yields

$$
\begin{aligned}
(A+B)\frac{dp^*(w)}{dw}+\sqrt{AC}\frac{dp_m^*(w)}{dw} &= -a(1-q)f(p^*(w)),\\
\sqrt{AC}\frac{dp^*(w)}{dw}+(C+D)\frac{dp_m^*(w)}{dw} &= \frac{-aq}{1-\alpha}f\left(\frac{p_m^*(w)+c_e}{1-\alpha}\right).
\end{aligned}
$$

[10]Notice from equation (4) that there may exist a case in which the optimal prices are given by $p^*(w) = p_m^*(w) = w + c_r$, and thus the profit from the bargainers is equal to zero. In this case, the retailer's best strategy is to allow only the posted pricing (no cut-off price decision) and the profit function is given by $\Pi_R(p, p_m, w) = \Pi_R(p, w) = a(p-w)\overline{F}(p)$, in which case Lemma 1 gives the same result. Thus, we focus only on the case $w + c_r < p_m^*(w) \leq p^*(w)$.

where

$$
\begin{aligned}
A &= -\frac{aq}{(1-\beta)(1-\alpha)} f\left(\frac{p^*(w)-\beta p_m^*(w)+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right) \le 0, \\
B &= -a(1-q)f'(p^*(w))(p^*(w)-w) - 2a(1-q)f(p^*(w)) \le 0, \\
C &= -\frac{aq\beta^2}{(1-\beta)(1-\alpha)} f\left(\frac{p^*(w)-\beta p_m^*(w)+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right) \le 0, \text{ and} \\
D &= -\frac{aq(1+\beta)}{1-\alpha} f\left(\frac{p_m^*(w)+c_e}{1-\alpha}\right) + \frac{aq}{(1-\alpha)^2}(w+c_r-p_m^*(w))f'\left(\frac{p_m^*(w)+c_e}{1-\alpha}\right) \le 0.
\end{aligned}
$$

Notice that the fact that B and D are both negative is directly from Lemma 1 (see the earlier argument where we prove (A-11) and (A-3) are negative, respectively). With simple algebra, we obtain

$$
\begin{aligned}
\frac{dp^*(w)}{dw} &= \frac{\frac{aq}{1-\alpha}\sqrt{AC} f\left(\frac{p_m^*(w)+c_e}{1-\alpha}\right) - a(1-q)(C+D)f(p^*(w))}{AD+BC+BD} \ge 0, \text{ and} \\
\frac{dp_m^*(w)}{dw} &= \frac{a(1-q)\sqrt{AC} f(p^*(w)) - \frac{aq}{1-\alpha}(A+B) f\left(\frac{p_m^*(w)+c_e}{1-\alpha}\right)}{AD+BC+BD} \ge 0,
\end{aligned}
$$

where the inequalities are directly from that A, B, C, and D are all negative, which concludes the proof.

Proof of Proposition 2

In this proof, we first prove that both the optimal posted price and the optimal cut-off price are increasing in the fraction of the bargainers, q. Then we prove the optimal cut-off price is increasing in the relative bargaining power of the bargainers, β. Notice from Lemma 1 that the optimal posted price, $p^*(w)$ and the optimal cut-off price, $p_m^*(w)$ satisfy the first order conditions of $\Pi_R(p, p_m, w)$, that is,

$$
\frac{\partial \Pi_R(p^*(w), p_m^*(w), w)}{\partial p} = 0 \text{ and} \frac{\partial \Pi_R(p^*(w), p_m^*(w), w)}{\partial p_m} = 0.
$$

Implicit differentiation of the two equalities with respect to q yields

$$
\begin{aligned}
(A+B)\tfrac{dp^*(w)}{dq} + \sqrt{AC}\tfrac{dp_m^*(w)}{dq} &= a\left(\overline{F}(p^*(w)) - \overline{F}\left(\tfrac{p^*(w)-\beta p_m^*(w)+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)\right. \\
&\quad \left. -(p^*(w)-w)f(p^*(w))\right). \\
\sqrt{AC}\tfrac{dp^*(w)}{dq} + (C+D)\tfrac{dp_m^*(w)}{dq} &= a\beta\left(\overline{F}\left(\tfrac{p^*(w)-\beta p_m^*(w)+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right) - \overline{F}\left(\tfrac{p_m^*(w)+c_e}{1-\alpha}\right)\right) \\
&\quad -\frac{a}{1-\alpha}(w+c_r-p_m^*(w))f\left(\frac{p_m^*(w)+c_e}{1-\alpha}\right).
\end{aligned}
$$

where

$$
\begin{aligned}
A &= -\frac{aq}{(1-\beta)(1-\alpha)} f\left(\frac{p^*(w)-\beta p_m^*(w)+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right) \leq 0, \\
B &= -a(1-q)f'(p^*(w))(p^*(w)-w) - 2a(1-q)f(p^*(w)) \leq 0, \\
C &= -\frac{aq\beta^2}{(1-\beta)(1-\alpha)} f\left(\frac{p^*(w)-\beta p_m^*(w)+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right) \leq 0, \text{ and} \\
D &= -\frac{aq(1+\beta)}{1-\alpha} f\left(\frac{p_m^*(w)+c_e}{1-\alpha}\right) + \frac{aq}{(1-\alpha)^2}(w+c_r-p_m^*(w))f'\left(\frac{p_m^*(w)+c_e}{1-\alpha}\right) \leq 0.
\end{aligned}
$$

The fact that B and D are both negative is directly from Lemma 1 (see the earlier argument where we prove (A-11) and (A-3) are negative, respectively). With simple algebra and the fact that

$$
\begin{aligned}
p^*(w) - w &= \frac{q\overline{F}\left(\frac{p^*(w)-\beta p_m^*(w)+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right) + (1-q)\overline{F}(p^*(w))}{(1-q)f(p^*(w))}, \text{ and} \\
p_m^*(w) - w - c_r &= \frac{(1-\alpha)\beta\left(\overline{F}\left(\frac{p_m^*(w+c_e)}{1-\alpha}\right) - \overline{F}\left(\frac{p^*(w)-\beta p_m^*(w)+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)\right)}{f\left(\frac{p_m^*(w)+c_e}{1-\alpha}\right)}
\end{aligned}
$$

based on the first order conditions $\left.\frac{\partial \Pi_R(p,p_m,w)}{\partial p}\right|_{p=p^*(w)} = 0$ and $\left.\frac{\partial \Pi_R(p,p_m,w)}{\partial p_m}\right|_{p_m=p_m^*(w)} = 0$, we obtain

$$
\begin{aligned}
\frac{dp^*(w)}{dq} &= \frac{-a(C+D)\overline{F}\left(\frac{p^*(w)-\beta p_m^*(w)+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)}{(1-q)(AD+BC+BD)} \geq 0, \text{ and} \\
\frac{dp_m^*(w)}{dq} &= \frac{a\sqrt{AC}\overline{F}\left(\frac{p^*(w)-\beta p_m^*(w)+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)}{(1-q)(AD+BC+BD)} \geq 0,
\end{aligned}
$$

where the inequalities are directly from that A, B, C, and D are all negative, which concludes the proof.

We then prove the optimal cut-off price is increasing in β. Following the same logic with subsequent algebraic simplification, we obtain

$$
\begin{aligned}
\frac{dp^*(w)}{d\beta} &= \frac{aqD(p^*(w)-p_m^*(w))f\left(\frac{p^*(w)-\beta p_m^*(w)+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)}{(1-\alpha)(1-\beta)^2(AD+BC+BD)} \\
&\quad - \frac{aq\sqrt{AC}\left(\overline{F}\left(\frac{p^*(w)-\beta p_m^*(w)+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right) - \overline{F}\left(\frac{p_m^*(w)+c_e}{1-\alpha}\right)\right)}{AD+BC+BD}, \\
\frac{dp_m^*(w)}{d\beta} &= \frac{aq(A+B)\left(\overline{F}\left(\frac{p^*(w)-\beta p_m^*(w)+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right) - \overline{F}\left(\frac{p_m^*(w)+c_e}{1-\alpha}\right)\right)}{AD+BC+BD} \\
&\quad - \frac{aq\beta B(p^*(w)-p_m^*(w))f\left(\frac{p^*(w)-\beta p_m^*(w)+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)}{(1-\alpha)(1-\beta)^2(AD+BC+BD)}.
\end{aligned}
$$

Notice first that A, B, C, and D are all negative. In addition, since $p_m^*(w) \leq p^*(w)$, we obtain that $\overline{F}\left(\frac{p^*(w)-\beta p_m^*(w)+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right) \leq \overline{F}\left(\frac{p_m^*(w)+c_e}{1-\alpha}\right)$, and thus the first term of $\frac{dp_m^*(w)}{d\beta}$ is positive. Note that the second term is negative since $p_m^*(w) \le p^*(w)$ and $B \le 0$, and thus, $\beta B(p^*(w) - p_m^*(w)) \le 0$. Thus, both terms of $\frac{dp_m^*(w)}{d\beta}$ add up to a positive number, that is, $\frac{dp_m^*(w)}{d\beta} \ge 0$, which concludes the proof.

Proof of Proposition 3

Note that when the customer's reservation price follows a uniform distribution over the interval $(0, b)$, the optimal wholesale price, w^* is given by

$$w^* = \frac{-qc_r[q(\beta-1)+2(1-\alpha+q\alpha)]}{2(1-\alpha+q\alpha)^2} + \frac{c_r q\beta - c_e q - b\alpha + b}{2(1-\alpha+q\alpha)} + \frac{c}{2}.$$

Take the derivative of w^* with respect to β, we obtain

$$\frac{dw^*}{d\beta} = \frac{qc_r(1-\alpha)(1-q)}{2(1-\alpha+q\alpha)^2} \ge 0,$$

where the inequality is from the fact that $\alpha \in [0, 1)$ and $q \in [0, 1]$, which concludes the proof.

Proof of Corollary 1

To show $\frac{dp_m^*(w^*)}{d\beta} \ge 0$, we have

$$\frac{dp_m^*(w^*)}{d\beta} = \left.\frac{\partial p_m^*(w)}{\partial \beta}\right|_{w=w^*} + \left.\frac{\partial p_m^*(w)}{\partial w}\right|_{w=w^*} \frac{dw^*}{d\beta}.$$

Note that both $\left.\frac{\partial p_m^*(w)}{\partial \beta}\right|_{w=w^*}$ and $\left.\frac{\partial p_m^*(w)}{\partial w}\right|_{w=w^*}$ are positive by Proposition 2(b) and Proposition 1, respectively. In addition, when the customer's reservation price distribution follows a uniform distribution over the interval $(0, b)$, Proposition 3 shows that $\frac{dw^*}{d\beta} \ge 0$. Thus, both terms of $\frac{dp_m^*(w^*)}{d\beta}$ are positive, and thus, $\frac{dp_m^*(w^*)}{d\beta} \ge 0$, which concludes the proof.

Proof of Proposition 4

Note that when the customer's reservation price follows a uniform distribution over the interval $(0, b)$, the optimal wholesale price, w^*, the optimal posted price, $p^*(w)$, and the optimal cut-off price, $p_m^*(w)$, are given by

$$\begin{aligned}
w^* &= \frac{-qc_r[q(\beta-1)+2(1-\alpha+q\alpha)]}{2(1-\alpha+q\alpha)^2} + \frac{c_r q\beta - c_e q - b\alpha + b}{2(1-\alpha+q\alpha)} + \frac{c}{2} \\
p^*(w) &= \frac{c_r q\beta - c_e q - b\alpha + b}{q(\beta-1)+2(1-\alpha+q\alpha)} + \left(1 - \frac{1-\alpha+q\alpha}{q(\beta-1)+2(1-\alpha+q\alpha)}\right) w \\
p_m^*(w) &= c_r(1-\beta) + \frac{(c_r q\beta - c_e q - b\alpha + b)\beta}{q(\beta-1)+2(1-\alpha+q\alpha)} + \left(1 - \frac{(1-\alpha+q\alpha)\beta}{q(\beta-1)+2(1-\alpha+q\alpha)}\right) w
\end{aligned}$$

Take the derivative of w^*, $p^*(w)$, and $p_m^*(w)$ with respect to c_r and c_e, respectively, we obtain

$$\frac{dp^*(w)}{dc_r} = \frac{q\beta}{q(\beta-1)+2(1-\alpha+q\alpha)} \geq 0, \qquad \frac{dp^*(w)}{dc_e} = \frac{-q}{q(\beta-1)+2(1-\alpha+q\alpha)} \leq 0,$$
$$\frac{dp_m^*(w)}{dc_r} = 1-\beta+\frac{q\beta^2}{q(\beta-1)+2(1-\alpha+q\alpha)} \geq 0, \qquad \frac{dp_m^*(w)}{dc_e} = \frac{-q\beta}{q(\beta-1)+2(1-\alpha+q\alpha)} \leq 0,$$
$$\frac{dw}{dc_r} = \frac{-q[(1-\alpha)(2-\beta+q\beta-q)+\alpha q]}{2(1-\alpha+q\alpha)^2} \leq 0, \qquad \frac{dw^*}{dc_e} = \frac{-q}{2(1-\alpha+q\alpha)} \leq 0,$$

where all the inequalities are from the fact that $\alpha \in [0,1)$, $\beta \in (0,1)$, and $q \in [0,1]$, and thus, both $(1-\alpha)(2-\beta+q\beta-q)$ and $q(\beta-1)+2(1-\alpha+q\alpha)$ are non-negative, which concludes the proof.

Proof of Corollary 2

To show $\frac{dp^*(w^*)}{dc_e} \leq 0$, we have

$$\frac{dp^*(w^*)}{dc_e} = \left.\frac{\partial p^*(w)}{\partial c_e}\right|_{w=w^*} + \left.\frac{\partial p^*(w)}{\partial w}\right|_{w=w^*} \frac{dw^*}{dc_e}.$$

Note that $\left.\frac{\partial p^*(w)}{\partial w}\right|_{w=w^*}$ is positive by Proposition 1. In addition, when the customer's reservation price distribution follows a uniform distribution over the interval $(0,b)$, Proposition 4 shows that $\left.\frac{\partial p^*(w)}{\partial c_e}\right|_{w=w^*} \leq 0$ and $\frac{dw^*}{dc_e} \leq 0$, respectively. Thus, both terms of $\frac{dp^*(w^*)}{dc_e}$ are negative, and thus, $\frac{dp^*(w^*)}{dc_e} \leq 0$, which concludes the proof.

The proof of $\frac{dp_m^*(w^*)}{dc_e} \leq 0$ follows the same logic, and thus, omitted.

Lemma 3. *Consider all customers are price-takers ($q = 0$). Given the wholesale price, w, there exists a unique posted price, p, that satisfies the first order condition for $\Pi_R(p,w)$, and this posted price maximizes $\Pi_R(p,w)$.*

Proof of Lemma 3

Note that when $q = 0$, the retailer's profit function in (4) is given by $\Pi_R(p,p_m,w) = \Pi_R(p,w) = a(p-w)\overline{F}(p)$. We prove the unimodality of $\Pi_R(p,w)$ in p by showing (i) $\left.\frac{\partial \Pi_R(p,w)}{\partial p}\right|_{p=w} \geq 0$, (ii) $\frac{\partial^2 \Pi_R(p,w)}{\partial p^2} < 0$ whenever $\frac{\partial \Pi_R(p,w)}{\partial p} = 0$, and (iii) $\left.\frac{\partial \Pi_R(p,w)}{\partial p}\right|_{p=b} \leq 0$.
First note that the first and second partial derivatives of $\Pi_R(p,w)$ in p are

$$\frac{\partial \Pi_R(p,w)}{\partial p} = a\overline{F}(p) - a(p-w)f(p) \text{ and} \quad \text{(A-12)}$$
$$\frac{\partial^2 \Pi_R(p,w)}{\partial p^2} = -2af(p) - a(p-w)f'(p). \quad \text{(A-13)}$$

Claim (i) follows from (A-12) while claim (iii) follows from $\overline{F}(b) = 0$. To show claim (ii), note from (A-12) and (A-13)

$$\left.\frac{\partial^2 \Pi_R(p,w)}{\partial p^2}\right|_{\frac{\partial \Pi_R(p,w)}{\partial p}=0} = -2af(p) - a\frac{\overline{F}(p)}{f(p)}f'(p). \tag{A-14}$$

Since F is strictly increasing and has increasing failure rate (IFR), we have $\overline{F}(p)f'(p) + f^2(p) > 0$ at any p and claim (ii) follows, concluding the proof of unimodality of $\Pi_R(p,w)$ in p.

Lemma 4. *Consider all customers are bargainers ($q = 1$). Given the wholesale price, w, the retailer sets the posted price to the maximum of the customer's reservation price (i.e., $p = b$). Furthermore, there exists a unique cut-off price, p_m, that satisfies the first order condition for $\Pi_R(p_m, w)$ and maximizes the retailer's profit, $\Pi_R(p_m, w)$.*

Proof of Lemma 4

We first prove that when $q = 1$, given the wholesale price, the retailer sets the posted price equal to the upper bound of the customer's reservation price (i.e., $p = b$). Then we show that there exists a unique cut-off price that satisfies the first order condition and maximizes the retailer's profit. Note that

$$\begin{aligned}\Pi_R(p,p_m,w) &= a\left[(p-w-c_r)\overline{F}\left(\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}\right)\right.\\ &\left.+\int_{\frac{p_m+c_e}{1-\alpha}}^{\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}}[(1-\beta)(1-\alpha)x+\beta p_m-(1-\beta)c_e-w-c_r]f(x)dx\right].\end{aligned} \tag{A-15}$$

If $p = b$, then the retailer's profit becomes:

$$\Pi_R(b,p_m,w) = a\int_{\frac{p_m+c_e}{1-\alpha}}^{b}[(1-\beta)(1-\alpha)x+\beta p_m-(1-\beta)c_e-w-c_r]f(x)dx, \tag{A-16}$$

where the equality in (A-16) is from the fact that

$$\begin{aligned}\tfrac{b-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)} - b &= \frac{b-\beta p_m+(1-\beta)c_e-(1-\beta)(1-\alpha)b}{(1-\beta)(1-\alpha)}\\ &\geq \frac{b-\beta p_m+(1-\beta)c_e-(1-\beta)b}{(1-\beta)(1-\alpha)} = \frac{\beta(b-p_m)+(1-\beta)c_e}{(1-\beta)(1-\alpha)} \geq 0.\end{aligned}$$

Therefore, to prove the result, it suffices to show that $\Pi_R(b,p_m,w) - \Pi_R(p,p_m,w) \geq 0$. We have

$$
\begin{aligned}
& \Pi_R(b,p_m,w)-\Pi_R(p,p_m,w) \\
= \ & a\int_{\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}}^{b} [(1-\beta)(1-\alpha)x+\beta p_m-(1-\beta)c_e-p]f(x)dx \\
\geq \ & a\int_{\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}}^{b} [(1-\beta)(1-\alpha)\frac{p-\beta p_m+(1-\beta)c_e}{(1-\beta)(1-\alpha)}+\beta p_m-(1-\beta)c_e-p]f(x)dx \\
\geq \ & 0.
\end{aligned}
$$

We already showed that the retailer sets $p=b$ when $q=1$. Thus, the retailer's profit function is given by

$$
\Pi_R(p,p_m,w)=\Pi_R(b,p_m,w)=a\int_{\frac{p_m+c_e}{1-\alpha}}^{b}[(1-\beta)(1-\alpha)x+\beta p_m-(1-\beta)c_e-w-c_r]f(x)dx.
$$

Now we show that there exists a unique cut-off price that satisfies the first order condition and maximizes the retailer's profit. We prove the unimodality of $\Pi_R(b,p_m,w)$ in p_m by showing (i) $\left.\frac{\partial\Pi_R(b,p_m,w)}{\partial p_m}\right|_{p_m=w+c_r}\geq 0$, (ii) $\frac{\partial^2\Pi_R(b,p_m,w)}{\partial p_m^2}<0$ whenever $\frac{\partial\Pi_R(b,p_m,w)}{\partial p_m}=0$, and (iii) $\left.\frac{\partial\Pi_R(b,p_m,w)}{\partial p_m}\right|_{p_m=b}\leq 0$.

First note that the first and second partial derivatives of $\Pi_R(b,p_m,w)$ in p_m are

$$
\frac{\partial\Pi_R(b,p_m,w)}{\partial p_m} = a\frac{w+c_r-p_m}{1-\alpha}f\left(\frac{p_m+c_e}{1-\alpha}\right)+a\beta\overline{F}\left(\frac{p_m+c_e}{1-\alpha}\right), \text{ and} \qquad \text{(A-17)}
$$

$$
\frac{\partial^2\Pi_R(b,p_m,w)}{\partial p_m^2} = -a\frac{1+\beta}{1-\alpha}f\left(\frac{p_m+c_e}{1-\alpha}\right)+a\frac{w+c_r-p_m}{(1-\alpha)^2}f'\left(\frac{p_m+c_e}{1-\alpha}\right). \qquad \text{(A-18)}
$$

Claim (i) follows from (A-17) while claim (iii) follows from $\overline{F}(b)=0$. To show claim (ii), note from (A-17) and (A-18)

$$
\left.\frac{\partial^2\Pi_R(b,p_m,w)}{\partial p_m^2}\right|_{\frac{\partial\Pi_R(b,p_m,w)}{\partial p_m}=0} = \frac{-a}{1-\alpha}\left[\frac{(1+\beta)f^2\left(\frac{p_m+c_e}{1-\alpha}\right)+\beta\overline{F}\left(\frac{p_m+c_e}{1-\alpha}\right)f'\left(\frac{p_m+c_e}{1-\alpha}\right)}{f\left(\frac{p_m+c_e}{1-\alpha}\right)}\right]
$$

Since F is IFR, and both $\alpha\in[0,1)$, we have $\overline{F}(\cdot)f'(\cdot)+f^2(\cdot)>0$. Hence claim (ii) follows, concluding the proof of unimodality of $\Pi_R(b,p_m,w)$ in p_m.