Dynamic Pricing of Limited Inventories for Multi-Generation Products

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Abstract

In this research, we consider a retailer selling products from two different generations, both with limited inventory over a predetermined selling horizon. Due to the spatial constraints or the popularity of a given product, the retailer may only display goods from one specific generation. If the transaction of the displayed item cannot be completed, the retailer may provide an alternative from another generation. We analyze two models - Posted-Pricing-First model and Negotiation-First model. The former considers negotiation as being allowed on the price of the second product only and in the latter, only the price of the first product is negotiable. Our results show that the retailer can adopt both models effectively depending on the relative inventory levels of the products. In addition, the retailer is better off compared to the take-it-or-leave-it pricing when the inventory level of the negotiable product is high.

Keywords: Revenue management; multi-generation products; bargaining; dynamic programming.

1. Introduction

In a customer-oriented retail market, manufacturers constantly develop new products that satisfy a variety of customer needs. As a result, the life cycle of a product becomes shorter and the co-existence of multi-generation products is a prevailing phenomenon. For example, a car dealership of Toyota may exhibit 2011 Camry cars in the showroom but stock a number of 2010 Camrys in the warehouse. Apple retail stores mainly display iPhone 4G phones but still maintain a certain inventory level of iPhone 3GS phones upon request. Both generation products are available for customers to purchase, depending on their personal preferences. When the latest version of the product is released into the market, the retailer tends to promote the new generation product because of its popularity, while in the case of the previous generation, the retailer would rather

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reduce the list price and/or allow customers to bargain the price down. For instance, before the latest Macbook Pro 15.4-inch Laptop (MB985LL/A) became available in the market in June 2009, the previous generation MacBook Pro 15.4-inch Laptop (MB470LL/A) was sold at a price of $1,999 based on the data on Apple's official website. Since then, one finds the price of the previous model starts from $1,500, which implies that retailers cut the price of the previous generation product so as to clear their inventories. Not surprisingly, this brings about the problem of determining price and inventory for multi-generation products in the current retailing environment.

From the retailers' point of view, displaying the full range of multi-generation products at the same time may not be the best option. Instead, responding to either lack of space on showroom floors or shelves or the specific requirements of manufacturers, retailers will limit displays to those products with relatively high profit margins [1]. Therefore, a customer considering purchasing a Toyota Camry can only observe the 2011 model when visiting the dealership. If the transaction fails, the dealer will seek to keep the transaction alive by offering the 2010 Camry. However, not all customers will consider the salesman's second offer, simply because they have a strong preference for the latest edition of the car. Similarly, certain customers have a predilection for previous generation products. As a result, in the case of some retailers, the price of the previous generation, such as Air Jordan series shoes, is even higher than that of the latest generation.

When selling the latest generation products, retailers can use a take-it-or-leave-it pricing strategy under which customers and the retailer make a transaction based on the posted price announed by the retailer. For example, the latest generation of MacBook Pro 15.4-inch Laptop (MB985LL/A) is sold at a price of $1,999 in US and there does not exist space for customers to bargain the price down. However, for the automobile industry, negotiation is common so that customers arriving in the dealership may initiate negotiation via a series of offers and counteroffers to get a better deal. Differences in sales formats basically depend on industry characteristics and the products themselves. If the transaction of the latest generation product fails, the sales formats of the previous generation product may change. One could observe that the retailer adopts various sales formats when selling two products from different generations. Apple franchised retailers do not allow customers to bargain for laptop products due to the request of that manufacturer. For previous generation products, however, negotiation will be offered by some retailers, and helps the retailers clear their inventories. For the automobile industry, on the other hand, a Toyota dealer may allow

[1] The latest generation products generally utilize the advantages of new technology and operations process. Thus, the reduction in costs of production and materials will result in an increase in the profit margin of the latest products. For instance, it is reported that the cost of all parts inside the new generation of iPod nano drops about 18% per unit compared with the previous generation (Hesseldahl, 2007).

negotiation on the product of 2011 Toyota Camry. Compared to the latest version of the cars, the dealer will set a discount price for the 2010 model. When the customer asks for the previous generation, room for bargaining is relatively limited, which is regarded as take-it-or-leave-it pricing.

In this research, we model the pricing decisions of the retailer when she sells products from two different generations for a given period of time, each with limited inventories. We consider a dynamic programming model that characterizes the pricing policies of both products under a pair of different inventory levels and the time remaining to the end of the selling season. To analyze realistic pricing decisions in a variety of industries, we consider two general models, namely the Posted-Pricing-First and the Negotiation-First models. In both cases, we assume customers can only observe one product at a time due to limited showroom space or the market condition in which the retailer hopes to promote one product rather than another. In the Posted-Pricing-First model, the retailer does not allow negotiation on the price of the first product. If the transaction fails, customers may consider purchasing the other product whose price is negotiable. In the Negotiation-First model, however, the retailer reverses the order of the sales formats by accepting the request for negotiation on the price of the first product. Similarly, the retailer offers the second product based on take-it-or-leave-it pricing to customers who do not purchase the first product.

Customers are categorized into two types – price-takers and bargainers. If the price of the product is negotiable, a price-taker simply buys the product if his reservation price is above the posted price, whereas the bargainer will bargain the price for a discount. Furthermore, when the retailer disallows negotiation, all customers, whether price-takers or bargainers, follow the take-it-or-leave-it policy. We use the generalized Nash bargaining solution (GNBS) to model the outcome of negotiation between bargainers and the retailer. Under GNBS, bargainers with higher willingness-to-pay will pay for a product at a higher price up to the posted price. For both models, we find that the posted price of the negotiable product increases when bargainers make up the majority of the customer population and/or the retailer plays a leading role in negotiation. These observations, however, do not necessarily hold for the other product. In addition, to compare the posted prices of the second product under both models, the price of the negotiable product is higher than that under take-it-or-leave-it pricing, in that the retailer tends to raise the posted price and collect excess revenue from bargainers with higher reservation prices. We have two interesting findings from our numerical study. The posted price of a product may decrease in the time remaining to the end of the selling season which is different from the observation in traditional dynamic pricing literature. Also in certain circumstances, negotiation may hurt the retailer.

We extend our general models in two directions. In the first, we consider a situation under which the retailer adjusts the belief of the customer's reservation price distribution on the second product after the transaction of the first product fails. This adjusted model is mostly applied to the case when the latest generation is always preferable (e.g., iPhone 4G, and Toyota Camry 2011 model). In the second, we focus on the case where the prices of both products are negotiable. We investigate the optimal pricing decisions and the associated revenue of the retailer and compare them with the general models. The extended models are analyzed in the appendix.

The remainder of this paper is organized as follows. Section 2 provides a survey of the relevant literature. Sections 3 and 4 describe our two dynamic programming models and derive analytical results. We propose the results of the numerical study in Section 5. A discussion of results and future research directions is included in Section 6. The discussions of extended models and all the proofs are relegated to the appendix.

2. Literature Review

We review the literature from two aspects: (1) dynamic pricing of limited inventories, and (2) negotiation between two parties. Research in dynamic pricing with limited inventories has been intensively conducted in the last decade since the pioneering works by Gallego and van Ryzin (1994), and Bitran and Mondschein (1997). In their papers, when facing limited inventories of a product over a short selling season, the seller will constantly adjust the price so that his expected total revenue is maximized. Extensive reviews on this subject are provided in Elmaghraby and Keskinocak (2003) and Bitran and Caldentey (2003). Researchers have explored dynamic pricing topics from various perspectives. Among them, Netessine et al. (2006) and Aydin and Ziya (2008) consider cross-selling. Aydin and Ziya (2009) consider the scenario under which the seller offers a personalized price based on the information of individual customer's willingness-to-pay. Ziya (2009) extends the work of Aydin and Ziya (2009) to more general conditions which lead to simple optimal pricing policies. In addition, Popescu and Wu (2007) discuss the optimal pricing policies when consumers form a reference price while Ahn et al. (2007) consider a scenario in the setting of multiple periods where demand is affected by the price. Transchel and Minner (2009) combine the issues of dynamic pricing and economic order quantity (EOQ) and Koenig and Meissner (2010) compare dynamic and listing pricing policies when selling multiple products that consume a single resource and investigate the risk of adopting listing pricing.

Some articles study the effect of the presence of strategic customers on dynamic pricing. The presence of forward looking customers is considered by Aviv and Pazgal (2008) and advanced selling is considered in Liu and van Ryzin (2008). Elmaghraby et al. (2008) determine a markdown pricing mechanism while buyers purchase multiple units and Dasu and Tong (2010) consider customers anticipate price policies. Others that investigate pricing decisions when customers are strategic include Su (2007), and Kuo et al. (2009a). Regarding dynamic pricing for multiple products, several articles have appeared, such as Zhang and Cooper (2005), Maglaras and Meissner (2006), and Suh and Aydin (2009). In addition, Lin and Sibdari (2009) adopt the multinomial logit (MNL) demand model to consider dynamic pricing competition under different information structures. Sibdari and Pyke (2010) analyze dynamic pricing with the stockpile of the products. Differing from the existing literature, our problem assumes that the retailer sells two products, and an arriving customer can only observe one product at a time due to limited storage or exhibition space. The retailer can decide on the price of which product negotiation is allowed. Our main contribution is to provide a comparison between two models to determine whether retailers would be better off by allowing negotiation on specific types of products. In addition, under our setting, it can be shown that negotiation is not necessarily beneficial to the retailer.

Bargaining is a major research topic in relation to our work, and has been intensively studied in literature, especially in the fields of economics and operations management. A detailed introduction of bargaining theory is provided by Fudenberg and Tirole (1991) as well as Muthoo (1999). One of the classic models in bargaining theory is the Nash bargaining solution. Under the Nash bargaining solution, two parties with equal bargaining power negotiate for a pie that is split based on their surplus and the disagreement payoffs. The Nash bargaining solution can be extended to a more general version – the generalized Nash bargaining solution, in which two parties have different bargaining powers and the one who owns a relatively larger bargaining power gains a larger fraction of the total surplus. Another classic model in bargaining theory is the Rubinstein alternating-offers model (Rubinstein 1982) in which two parties split a pie based on a series of offers and counteroffers. Unlike the Nash bargaining solution that only focuses on the outcome of negotiation, Rubinstein's model emphasizes the bargaining process and contains a detailed discussion. This bargaining process leads to the same equilibrium as the generalized Nash bargaining solution when the time intervals between two subsequent offers are close to zero and the discount factor is close to one. As a result, the generalized Nash bargaining solution can be simply applied to a problem that discusses the negotiation process between two parties.

In our study, the generalized Nash bargaining solution is being used for exploring the negotiation between the retailer and the arriving bargainer. The total surplus between the two parties is determined by the customer's valuation, the posted prices, and the marginal revenues of the products; the latter of which depends on the inventory levels of the two products and the time remaining to the end of the selling season. The marginal revenues of the products form as the disagreement payoffs and the retailer will adjust the posted prices of two products based on time and inventories on hand. When the bargainer's valuation of the product is below the posted price, the total surplus is determined by the difference between the valuation and the marginal revenue. When the valuation is above the posted price, however, the total surplus is the difference between the posted price and the marginal revenue due to the fact that the bargainer is reluctant to reveal his real valuation and to share excess surplus with the retailer. Therefore, the price that a bargainer pays is always less than the posted price which is the exact price paid by the price-takers. Under our setting, bargainers always gain a larger surplus compared to price-takers with the same valuation of the product.

Several papers consider negotiation with a single retailer (e.g., Riley and Zeckhauser, 1983, Wang, 1995, Arnold and Lippman, 1998, Roth et al., 2006, Kuo et al., 2009b, Kuo et al., 2009) and with competing retailers (e.g., Bester, 1993, Adachi, 1999, Desai and Purohit, 2004). The Nash bargaining solution we adopt is also commonly used in literature. Among these papers, Wang (1995) shows negotiation is always a better mechanism when implementing cost is low enough. Bester (1993) investigates the bargaining between the seller and the end customers with quality uncertainty. Desai and Purohit (2004) consider a model with two competing retailers who choose either posted pricing or negotiation whereas Roth et al. (2006) consider a scenario in which customization services are offered by service providers. Kuo et al. (2009b) analyze how the manufacturer uses the wholesale-price-only contract to induce the retailer to adopt either posted pricing or negotiation. Kuo et al. (2009) discuss the optimal pricing strategies in a supply chain when price-takers and bargainers co-exist.

A fair amount of research includes negotiation within the supply chain management context. Nagarajan and Sosic (2006) provides a detailed review of cooperative bargaining theory in supply chains. In addition, Nagarajan and Bassok (2002) consider supplier alliance in the assembly problem. Iyer and Villas-Boas (2003) study negotiation in distribution channel relationships. Dukes and Gal-Or (2003) apply the Nash bargaining solution to advertising. Wu (2004) uses the Rubinstein's model to find the coordinating contracts. Furthermore, Gurmami and Shi (2006) discuss the design of a supply chain contract when the supplier is unreliable in delivery, and Lovejoy (2010) includes negotiation in a series supply chain.

3. Model Description

We consider a scenario in which a retailer is selling two products (denoted by product i, $i = 1, 2$), each with limited inventories over a fixed selling horizon. The selling horizon can be divided into T periods short enough so that at most one arrival occurs during each period. We assume the probability that a single arrival occurs in one period is $\lambda \in [0, 1]$. An arriving customer can only observe product 1 due to limit of display space or the retailer's selling strategy. If the customer fails to buy product 1, the retailer will offer the alternative – product 2. The timing of the events in each period is as follows. At the beginning of each period, the retailer offers product 1 and announces the posted price, p_1, to the arriving customer. If the customer purchases the item, then the period ends, otherwise, the retailer provides product 2 at the posted price, p_2. The customer population consists of two types - price-takers and bargainers. Let $q \in (0, 1)$ be the fraction of bargainers in the customer population, and $1 - q$ be the fraction of price-takers. In this research, we consider the case where the retailer is able to decide whether she allows the customers to bargain for a specific product (either product 1 or product 2, but not both). If the retailer allows negotiation for the price of a specific product, then bargainers seek to negotiate for a price discount from the posted price, and price-takers, on the other hand, either accept the posted price and purchase the item or simply reject the offer. If a specific product is not allowed for negotiation, the purchase decisions of both bargainers and price-takers follow the take-it-or-leave-it policy. From the retailer's perspective, for each product, there exists an opportunity cost, c_i, below which she would never sell. Such cost associated with each product may vary in each period and is determined by the remaining time of the selling season, and the inventory levels of both products. The goal of the retailer is to set the posted prices p_1 and p_2 in each period to maximize the expected total revenue over the selling horizon.

Each customer, whether a bargainer or a price-taker, has a respective reservation price for each product (the highest price a customer is willing to spend on the product). Let r_i be the reservation price of an arriving customer for product i. The reservation price of each individual customer is the information which remains unrevealed to the retailer. To the retailer, the reservation prices of the customer population are regarded as a random variable, R_i, over the interval $[0, b_i]$ with the cumulative density function, $F_i(\cdot)$ and the probability density function, $f_i(\cdot)$. We assume F_i is strictly increasing and define $\overline{F}_i(\cdot) := 1 - F_i(\cdot)$. Notice that the preference of each individual customer for both products may vary to the extent that one may prefer the product of the latest generation (e.g., iPhone 4G) while others would rather purchase old-fashioned generations (e.g., Air

Jordan series)[2]. As a result, for the purpose of generality, we do not impose any specific assumptions on the reservation price distribution of each product. In the appendix, we will consider a model under which all customers prefer the product of the latest generation (product 1). In this case, the posted price of product 1 forms the upper bound on the customer's reservation price distribution for product 2, provided that customers' valuation for the first product is always better than that for the other.

For each product, the retailer can determine the sales format – allow or disallow negotiation. For product i, if the retailer does not allow negotiation, each customer, whether a price-taker or a bargainer, compares the observed posted price, p_i, with his reservation price r_i. The customer with r_i will accept the posted price and purchase the product if $p_i \leq r_i$, and reject otherwise. If the retailer allows negotiation for the price of product i, the price-taker will purchase the product if $p_i \leq r_i$. The bargainer, however, will try to negotiate the price down and the final price that he actually pays for the product depends on his reservation price r_i. Therefore, not all bargainers purchase the product at the posted price. In this model, we assume all customers are *myopic* in the sense that he will purchase product 1 as long as the surplus of purchasing product 1 is non-negative.

In this paper, the detailed process of negotiation is not the major concern, instead we simply consider the negotiation outcome based on the generalized Nash bargaining solution (GNBS) (for details, see Muthoo, 1999). In fact, a large stream of research adopts GNBS to describe the outcome of negotiation (Bester, 1993, Wang, 1995, Arnold and Lippman, 1998, and Kuo et al., 2009a). Under the generalized Nash bargaining solution, the total surplus (the difference between the bargainer's reservation price and the opportunity cost of the product) will be split by the retailer and a bargainer based on the relative bargaining power. Let $\beta \in (0, 1)$ be the relative bargaining power of the retailer and $1 - \beta$ be the relative bargaining power of bargainers. If a bargainer's reservation price is less than the opportunity cost of the product, c_i, then the agreement between both parties is not achieved. On the other hand, if the reservation price of a bargainer exceeds the opportunity cost of the product, his negotiation behavior will depend on whether his reservation price is higher than the posted price of the product. For those bargainers whose reservation prices are lower than the posted price, the final price is determined by the bargainer's reservation price, the cost of the product, and the relative bargaining power between the two parties. According to the generalized Nash bargaining solution, when the bargainer's reservation price is $r_i \in [c_i, p_i)$, the

[2]To the extreme, if a specific customer is only willing to buy product 2, then his reservation price for product 1 can be denoted by $r_1 = 0$. Same logic can be applied to product 1 as well.

relationship between these parameters is expressed as the following function:

$$\begin{aligned} \max_{p_{\mathbf{N}}} \quad & (r_i - p_{\mathbf{N}})^{1-\beta}\ (p_{\mathbf{N}} - c_i)^{\beta} \\ \text{subject to} \quad & c_i \leq p_{\mathbf{N}} \leq r_i \end{aligned} \tag{1}$$

where $p_{\mathbf{N}}$ is the price agreed after negotiation. Note that for a bargainer who succeeds in negotiation, a surplus $r_i - p_{\mathbf{N}}$ is gained. For the retailer, on the other hand, an item is sold at a price higher than c_i, and the surplus is $p_{\mathbf{N}} - c_i$. Based on GNBS, the transaction is always committed when $r_i \geq c_i$, and the party who has relatively higher bargaining power will gain more surplus.

Consider now that the bargainer's reservation price is higher (i.e., $r_i \geq p_i$). After he has observed the posted price, he tends to adjust his reservation price down to p_i and negotiate the price with the retailer. The final price that the bargainer pays will maximize the following problem:

$$\begin{aligned} \max_{p_{\mathbf{N}}} \quad & (p_i - p_{\mathbf{N}})^{1-\beta}\ (p_{\mathbf{N}} - c_i)^{\beta} \\ \text{subject to} \quad & c_i \leq p_{\mathbf{N}} \leq p_i \end{aligned} \tag{2}$$

Note that the setting of negotiation outcome reflects two facts. First, a bargainer with a reservation price higher than the posted price is reluctant to share excess surplus $(r_i - p_i)$ with the retailer under negotiation. In addition, the posted price not only serves as the upper bound of the final price that a bargainer pays, but also has effects on the outcome of negotiation. Let $p^*_{\mathbf{N}}(c_i, r_i, p_i)$ be the final price agreed after negotiation. Based on the aforementioned discussion (equations (1) and (2)), we can characterize the optimal price agreed by both parties after negotiation:

$$\begin{aligned} p^*_{\mathbf{N}}(c_i, r_i, p_i) &= \min\left\{ \arg\max_{c_i \leq p_{\mathbf{N}} \leq r_i} \left[(r_i - p_{\mathbf{N}})^{1-\beta} (p_{\mathbf{N}} - c_i)^{\beta} \right] \ ,\ \arg\max_{c_i \leq p_{\mathbf{N}} \leq p_i} \left[(p_i - p_{\mathbf{N}})^{1-\beta} (p_{\mathbf{N}} - c_i)^{\beta} \right] \right\} \\ &= \min\{\beta r_i + (1-\beta)c_i, \beta p_i + (1-\beta)c_i\} \\ &= \begin{cases} \beta p_i + (1-\beta)c_i & \text{if } r_i \geq p_i; \\ \beta r_i + (1-\beta)c_i & \text{if } c_i \leq r_i < p_i. \end{cases} \end{aligned} \tag{3}$$

Notice that if $\beta \to 1$, the negotiation outcome is dominated by the retailer, in which the final price agreed by both parties is either the bargainer's reservation price, r_i, or the posted price of the product, p_i, whichever is smaller. If $\beta \to 0$, the bargainer has full bargaining power so that he can drive the final price down to c_i. Also note that the final price that a bargainer pays is never above the posted price. That is, under negotiation, bargainers always purchase the products at a lower price compared with price-takers. In fact, bargainers with reservation prices lying within this range $(c_i \leq r_i < p_i)$ are the target customers, from which the retailer discriminates prices better.

It is because under negotiation the final price of the product sold to the bargainer is based on his individual reservation price instead of a fixed posted price. Observe that the retailer can always set a higher posted price in order to receive more revenue from the bargainers under negotiation; doing this will however exclude the price-takers with lower reservation prices. This is the tradeoff the retailer faces when determining the posted price. In the following two subsections, we will consider two models – Posted-Pricing-First Model and Negotiation-First Model, depending on whether the price of the product is allowed for negotiation.

3.1 Posted-Pricing-First Model

We consider a scenario in which the retailer only allows negotiation for the price of product 2. When transacting with the retailer for product 1, all customers follow the take-it-or-leave-it principle (purchase if $r_1 \geq p_1$). If a customer successfully buys product 1, the period ends; otherwise, the retailer will offer product 2 whose price is negotiable. The price-taker will pay p_2 if $r_2 \geq p_2$. If the customer is a bargainer, the final price that he pays is based on the result shown in equation (3), in which he pays $\beta r_2 + (1-\beta)c_2$ if $r_2 \in [c_2, p_2)$ and $\beta p_2 + (1-\beta)c_2$ if $r_2 \geq p_2$.

The retailer's revenue is determined by the number of remaining selling periods and the inventory levels of the products. Let t denote the number of the remaining selling periods where $t = 1, \ldots, T$, and $x \geq 0$ and $y \geq 0$ denote the inventory levels of product 1 and product 2, respectively. For each t, the retailer determines p_1 and p_2 given x units of product 1 and y units of product 2 so as to maximize her expected total revenue. Let $V_{\text{P}}(x, y, t)$ be the retailer's expected total revenue with t periods to go and the inventory levels x and y for products 1 and 2, respectively. As a result, the retailer's revenue maximization problem is modeled based on the dynamic programming setting:

$$V_{\text{P}}(x, y, t) = \max_{p_1, p_2} J_{\text{P}}(p_1, p_2, x, y, t) \text{ for } x, y > 0, t = 1, \ldots, T,$$

where

$$\begin{aligned} J_{\text{P}}(p_1, p_2, x, y, t) \quad = \quad & \lambda \overline{F}_1(p_1)(p_1 + V_{\text{P}}(x-1, y, t-1)) \\ & + \lambda q F_1(p_1)[\textstyle\int_{p_2}^{b_2} [\beta p_2 + (1-\beta)c_2] f_2(x) dx + \int_{c_2}^{p_2} [\beta x + (1-\beta)c_2] f_2(x) dx \\ & + \overline{F}_2(c_2) V_{\text{P}}(x, y-1, t-1)] + \lambda(1-q) F_1(p_1) \overline{F}_2(p_2)(p_2 + V_{\text{P}}(x, y-1, t-1)) \\ & + (1 - \lambda \overline{F}_1(p_1) - \lambda q F_1(p_1) \overline{F}_2(c_2) - \lambda(1-q) F_1(p_1) \overline{F}_2(p_2)) V_{\text{P}}(x, y, t-1). \end{aligned}$$

$$\text{Boundary condition: } V_{\text{P}}(x, y, 0) = 0 \text{ for } x, y \geq 0, \text{ and } V_{\text{P}}(0, 0, t) = 0 \text{ for } t = 1, \ldots, T. \tag{4}$$

Note that when the inventory level of either product is zero, the terms in the revenue function, $J_{\text{P}}(p_1, p_2, x, y, t)$, with respect to that product will vanish as the retailer only considers single-product revenue optimization problem (see Bitran and Mondschein, 1997, and Kuo et al., 2009a). Furthermore, for the retailer, the opportunity cost of product 1 with t periods to go is equivalent to the marginal revenue of the product, that is, $V_{\text{P}}(x, y, t-1) - V_{\text{P}}(x-1, y, t-1)$. Similarly, the cost of product 2 is $V_{\text{P}}(x, y, t-1) - V_{\text{P}}(x, y-1, t-1)$. Then the costs of products 1 and 2 with t periods to go, depending on the inventory levels x and y, can be expressed as follows:

$$\begin{aligned} c_1 = c_1(x, y, t) &:= V_{\text{P}}(x, y, t-1) - V_{\text{P}}(x-1, y, t-1), \quad \text{and} \\ c_2 = c_2(x, y, t) &:= V_{\text{P}}(x, y, t-1) - V_{\text{P}}(x, y-1, t-1). \end{aligned} \tag{5}$$

For simplicity, we use c_i as the shorthand notation of $c_i(x, y, t)$ for $i = 1, 2$. Therefore, the function $J_{\text{P}}(p_1, p_2, x, y, t)$ can be rewritten as

$$\begin{aligned} J_{\text{P}}(p_1, p_2, x, y, t) \;=\;& \lambda \overline{F}_1(p_1)(p_1 - c_1) \\ &+ \lambda q F_1(p_1) \left[\int_{p_2}^{b_2} [\beta p_2 + (1-\beta) c_2] f_2(x) dx + \int_{c_2}^{p_2} [\beta x + (1-\beta) c_2] f_2(x) dx \right. \\ &\left. - \overline{F}_2(c_2) c_2 \right] + \lambda (1-q) F_1(p_1) \overline{F}_2(p_2)(p_2 - c_2) + V_{\text{P}}(x, y, t-1). \end{aligned} \tag{6}$$

3.2 Negotiation-First Model

In this model, we suppose the retailer allows negotiation for the price of product 1 only. For product 1, bargainers and price-takers follow their individual purchasing behavior, respectively. If an arriving customer is a price-taker, he will purchase product 1 only if $r_1 \geq p_1$. If the customer is a bargainer, he will pay $\beta p_1 + (1-\beta) c_1$ if $r_1 \geq p_1$ and $\beta r_1 + (1-\beta) c_1$ if $r_1 \in [c_1, p_1)$. If customers successfully buy product 1, then the period ends; otherwise, the retailer will offer product 2 at the take-it-or-leave-it price, p_2. Customers will buy product 2 if their reservation price is higher than the posted price p_2. Similarly, given the inventory levels x and y, in each period, the retailer will set the posted prices, p_1 and p_2, to maximize the expected total revenue denoted by $V_{\text{N}}(x, y, t)$. As a result, the retailer's revenue maximization problem is given by

$$V_{\text{N}}(x, y, t) = \max_{p_1, p_2} J_{\text{N}}(p_1, p_2, x, y, t) \text{ for } x, y > 0, t = 1, \ldots, T,$$

where

$$\begin{aligned} J_{\text{N}}(p_1, p_2, x, y, t) \;=\;& \lambda q \left[\textstyle\int_{p_1}^{b_1} [\beta p_1 + (1-\beta) c_1] f_1(x) dx + \int_{c_1}^{p_1} [\beta x + (1-\beta) c_1] f_1(x) dx \right. \\ &\left. + \overline{F}_1(c_1) V_{\text{N}}(x-1, y, t-1) \right] + \lambda (1-q) \left[p_1 \overline{F}_1(p_1) + \overline{F}_1(p_1) V_{\text{N}}(x-1, y, t-1) \right] \\ &+ \lambda \left[q F_1(c_1) + (1-q) F_1(p_1) \right] \overline{F}_2(p_2) \left[p_2 + V_{\text{N}}(x, y-1, t-1) \right] \\ &+ \left[1 - \lambda (q \overline{F}_1(c_1) + (1-q) \overline{F}_1(p_1) + (q F_1(c_1) + (1-q) F_1(p_1)) \overline{F}_2(p_2)) \right] V_{\text{N}}(x, y, t-1). \end{aligned}$$

Boundary condition: $V_{\text{N}}(x, y, 0) = 0$ for $x, y \geq 0,$ and $V_{\text{N}}(0, 0, t) = 0$ for $t = 1, \ldots, T.$ (7)

Using the same expression as in equation (5), $J_{\text{N}}(p_1, p_2, x, y, t)$ can be simplified as follows:

$$
\begin{aligned}
J_{\text{N}}(p_1, p_2, x, y, t) \quad = \quad & \lambda q \left[\int_{p_1}^{b_1} \left[\beta p_1 + (1-\beta)c_1\right] f_1(x)dx + \int_{c_1}^{p_1} \left[\beta x + (1-\beta)c_1\right] f_1(x)dx - \overline{F}_1(c_1)c_1 \right] \\
& + \lambda(1-q)\overline{F}_1(p_1)(p_1 - c_1) + \lambda \left[qF_1(c_1) + (1-q)F_1(p_1)\right] \overline{F}_2(p_2)(p_2 - c_2) \\
& + V_{\text{N}}(x, y, t-1). \qquad (8)
\end{aligned}
$$

Similar to the Posted-Pricing-First model, the retailer will consider the single-product revenue maximization problem when she has no inventory of either product in stock.

Remark 1. Our models are inherently applicable to a scenario under which the latest generation products do not have limited inventories by assuming $x > T$. In this case, the opportunity cost of product 1, c_1, is equal to zero, and the associated posted price should be lower compared to that in the limited inventory level case. In addition, our analysis in the next section remains unaltered. On the other hand, one may relax the assumption of the fixed time horizon in both models. In fact, when the number of remaining selling periods is sufficiently large, the opportunity costs of both products are relatively high, and hence, the retailer always targets the customers with high reservation price. In this sense, negotiation provides less benefit to the retailer compared to dynamic pricing.

Remark 2. Our setting can be applied to the case where all arriving customers are aware of the fact that both generation products are available for purchase or both products are displayed simultaneously, while the retailer only announces the price of the second product after the purchase decision of the arriving customer for the first product has been made. Furthermore, the potential customers may know the average selling prices of the products, such as automobiles and consumer electronic products, in the local area from the Internet. However, the posted prices tend to vary from retailer to retailer based on their cost structures and the scales of the channel. If the price of each product is revealed to the arriving customer at the same time, the purchase decision made by the customer is according to comparing the surplus between two products. Such strategic customer behavior may result in the issues of price competition and product line selection, and thus, is beyond the scope of this paper.

Remark 3. In our model, we focus on the case under which each customer is quoted a price of product 1 upon arrival. This assumption is appropriate under dynamic pricing settings in which the retailer intends to sell products under a relatively short selling horizon and the cost of changing the price tags is negligible. We acknowledge that there exists some limitations in our model, for example, for cases where price-takers simply leave quietly without asking for product 2. Such scenario is beyond the scope of the paper and we leave this possibility as a fruitful direction for future work.

4. Analysis

In this section, we characterize the optimal prices of the retailer that maximize the expected total revenue in both models. For the Negotiation-First model, observe from equation (8) that if all customers are bargainers (i.e., $q \to 1$), the retailer has the incentive to set the posted price of product 1 as high as possible; this price allows the retailer to do better price discrimination. On the other hand, if the customer population only consists of price-takers (i.e., $q \to 0$), the posted price of product 1 only depends on its opportunity cost, c_1, and the customer's reservation price distribution, $F_1(x)$. In this scenario, the problem can be simply solved and has been discussed widely in the dynamic pricing literature (see Gallego and van Ryzin 1994, Bitran and Mondschein 1997, and Kuo et al. 2009a). The decision on the posted price of product 2 is irrelevant to the posted price of product 1, p_1, but the retailer's expected revenue as accrued from product 2 is indeed affected by the value of p_1. With a higher p_1, fewer customers can afford product 1. Therefore, a portion of the customers, $\lambda\left[qF_1(c_1) + (1-q)F_1(p_1)\right]\overline{F}_2(p_2)$, will successfully buy product 2. Similarly, a lower p_1 induces more purchases of product 1, and thus, fewer customers will buy product 2.

Likewise, in the Posted-Pricing-First model, the retailer disallows price negotiation for product 1. That is, all customers can be regarded as price-takers. If the retailer raises the posted price of product 1, p_1, too high, then more customers cannot afford product 1 and $\lambda F_1(p_1)$ of the customers will consider buying product 2. Note that the retailer can always reduce p_1 if clearing the inventory of product 1 is a major concern, though doing so will result in a lower profit margin. When determining p_1, the retailer also needs to consider the potential revenue accrued from product 2, received from both price-takers and bargainers. A higher fraction of bargainers induces the retailer to set a higher posted price of product 2, enabling the retailer to gain more surplus under negotiation among bargainers with high reservation prices. The decision of the posted price for product 2 is similar to the case when the retailer chooses the posted price of product 1 in the Negotiation-First model. Notice that if $q \to 0$ in both models, these two models are identical, in that, negotiation does not exist for both products.

Before characterizing the technical propositions of the posted prices of the two products, we require the following assumptions on the customer's reservation price distributions and the fraction of bargainers in the customer population:

Assumption 1. *The cdf of the customer's reservation price distribution for product i, $F_i(\cdot)$, has increasing failure rate. That is, $\frac{f_i(\cdot)}{\bar{F}_i(\cdot)}$ is increasing.*

Assumption 2. *The fraction of bargainers in the customer population, q, is less than $\frac{1}{2}$.*

The first assumption provides some regularity to the revenue function. Several probability distributions which are commonly used have increasing failure rates such as uniform, Erlang, normal, weibull with shape parameter greater than one, and their truncated versions. The restriction of q in the second assumption remains consistent with Kuo et al. (2009a), in which as in our model, the retailer has the latitude to give a discount to retain bargainers but does not necessarily use it as a major sales mode.

Throughout the paper, we use model P for the Posted-Pricing-First model and model N to stand for the Negotiation-First model. The following lemma shows that under the given assumptions, the retailer's expected total revenue functions are well-behaved so that the optimal posted prices can be determined uniquely for both models.

Lemma 1. *Suppose that the retailer has x and y units of inventory for products 1 and 2, respectively, with t periods to go until the end of the selling horizon. For model s, $s = P, N$, if Assumptions 1 and 2 hold, there exists a unique pair of the posted prices, p_1 and p_2, that satisfies the first order conditions of $J_s(p_1, p_2, x, y, t)$ and maximizes $J_s(p_1, p_2, x, y, t)$.*

Notice that if the customer's reservation prices for both products are uniformly distributed, the assumption of q being less than $\frac{1}{2}$ is not necessary. In fact, when distributions for both products are uniform, one can check that for any $q \in (0, 1)$, both posted prices are uniquely determined and satisfy the first order conditions of the retailer's expected revenue function. Let $p^*_{s1}(x, y, t)$ and $p^*_{s2}(x, y, t)$ be, respectively, the optimal posted prices of product 1 and product 2 with inventory levels x and y with t periods to go for model s, $s = P, N$. In the remainder of this section, we characterize the properties of the optimal posted prices as well as the associated expected total revenue of the retailer in both models.

First, we discuss the effects of the fraction of bargainers, q, and the relative bargaining power of the retailer, β, on the posted prices, $p^*_{s1}(x, y, t)$ and $p^*_{s2}(x, y, t)$ for model $s, s = P, N$.

Proposition 1. *Suppose that the retailer adopts model P with x and y units of inventory for products 1 and 2, respectively, and t periods to go until the end of the selling horizon. Then:*

(a) The optimal posted price of product 2, $p^*_{\mathbf{P2}}(x, y, t)$, increases in the fraction of bargainers, q.

(b) The optimal posted prices of products 1 and 2, $p^*_{\mathbf{P1}}(x, y, t)$ and $p^*_{\mathbf{P2}}(x, y, t)$, both increase in the relative bargaining power of the retailer, β.

When the retailer adopts model P, the fraction of bargainers, q, influences not only the posted price of product 2, but also the posted price of product 1. A high fraction of bargainers enables the retailer to set a high posted price, which expands the range of the final purchase price paid by bargainers, and in turn increases the retailer's revenue. On the other hand, when the retailer determines the posted price of product 1, she has to compare the revenue of product 1 with that of product 2, which in turn, is influenced by the fraction of bargainers, q. In particular, if the retailer experiences a larger revenue from product 2 due to an increase in q, she will raise the posted price of product 1 as a means to induce more bargainers to purchase product 2. In that sense, the posted price of product 1 increases in q. However, the posted price of product 1 may decrease if an increase in q is negatively related to the revenue of product 2.

The effect of bargaining power, β, on the posted price of product 2 is intuitive: the retailer can take advantage of higher relative bargaining power and set a higher price for better price differentiation. Furthermore, an increase in the bargaining power will also increase the posted price of product 1. The retailer sets a higher price for the first product which targets high reservation price customers. If the transaction fails, the retailer can still use high bargaining power to generate excess revenue on the second product.

Proposition 2. *Suppose that the retailer adopts model N with x and y units of inventory for product 1 and 2, respectively, and t periods to go until the end of the selling horizon. Then:*

(a) The optimal posted price of product 1, $p^*_{\mathbf{N1}}(x, y, t)$, increases in the fraction of bargainers, q, but the optimal posted price of product 2, $p^*_{\mathbf{N2}}(x, y, t)$, is independent of q.

(b) The optimal posted price of product 1, $p^*_{\mathbf{N1}}(x, y, t)$, increases in the relative bargaining power of the retailer, β, but the optimal posted price of product 2, $p^*_{\mathbf{N2}}(x, y, t)$, is independent of β.

Observe from equation (8) that as the posted price of product 1, p_1, increases, the expected revenue of the retailer from bargainers also increases. This revenue increment is derived from two sources. First, notice that the bargainers with reservation prices $r \in [c_1, p_1)$ are the major group from which the retailer does price discrimination. Setting a higher p_1 provides a wider range for the retailer to achieve this goal. Secondly, the final price paid by the bargainer, $\beta p_1 + (1 - \beta)c_1$, is also augmented if his reservation price is larger than the opportunity cost of the product. At the same time, when the fraction of bargainers, q, increases, the retailer can raise p_1 without worrying about excluding more price-takers. As a result, the retailer has more tendency to set p_1 higher as q increases. However, the change of q only affects the proportion of the customers who consider buying the second product, but does not lead the decision of p_2.

As β increases, setting a higher p_1 enables the retailer to take advantage of negotiation for better price discrimination, but this price increment sacrifices the revenue gained from price-takers. In fact, the former benefit resulted from bargainers due to the increases of both β and p_1 basically compensates for the revenue loss from price-takers. However, such higher bargaining advantage cannot be extended to the posted price decision of the second product, which only relates to the distribution of the customer's reservation price and the opportunity cost of the product.

Proposition 3. *Consider the opportunity costs of products 1 and 2 with t periods to go and inventories x and y for both products, respectively, $c_1(x, y, t)$ and $c_2(x, y, t)$ for model s, $s = P, N$. Then:*
(a) The optimal posted price of product 1, $p^*_{s1}(x, y, t)$ increases in $c_1(x, y, t)$, but decreases in $c_2(x, y, t)$.
(b) The optimal posted price of product 2, $p^*_{s2}(x, y, t)$ increases in $c_2(x, y, t)$, but is independent of $c_1(x, y, t)$.

Apparently, a higher cost of a product will drive the retailer to raise its posted price for both models. However, the magnitude of the increase in the posted price depends on the customer's reservation price distribution and whether the retailer allows negotiation. Furthermore, when the cost of product 2, $c_2(x, y, t)$, increases, the retailer will decrease the posted price of product 1, p_1. It is because an increase in $c_2(x, y, t)$ drives the retailer to raise the posted price of product 2, p_2, so that a smaller proportion of customers can afford to purchase product 2. Therefore, the retailer tends to lower p_1 to increase the probability of a successful transaction. For both models, however, the change of $c_1(x, y, t)$ has no effect on p_2. Notice that $c_1(x, y, t)$ only has direct effect on the price that each customer pays for the first product (for both models) and on the fraction of customers considering purchasing product 2 (model N only). When the customer considers purchasing product 2, $c_1(x, y, t)$ is not a factor that determines p_2.

Proposition 4. *Suppose that the retailer adopts model s with x and y units of inventory for products 1 and 2, respectively, and with t periods to go until the end of the season for $s = P, N$.*
(a) If the retailer's bargaining power, β, increases, then the retailer's optimal expected revenue, $V_s(x, y, t)$, increases.
(b) If bargainers have full bargaining power (i.e., $\beta \to 0$), then the retailer's optimal expected revenue, $V_s(x, y, t)$, decreases in the fraction of bargainers, q.

Note that higher bargaining power has no effect on the revenue from price-takers and from the other product without negotiation, but increases revenue accrued from bargainers. As a result, when the retailer allows negotiation for the price of a product, everything being equal, the retailer can realize more revenue if she has higher bargaining power. On the other hand, when the bargainers have full bargaining power, the final transaction price paid by the bargainers is equal to the cost (i.e., marginal revenue) of the product. Therefore, the larger fraction of the bargainers is, the less revenue the retailer is able to collect. Note that when the retailer has full bargaining power, $\beta \to 1$, the retailer's optimal expected revenue does not necessarily increase in the fraction of bargainers. The logic behind the conclusion is that the retailer is able to price discriminate among bargainers with reservation price between the cost of the product and the posted price. If price discrimination from bargainers can compensate for the revenue loss due to price-takers being priced out, then the retailer is better off with more bargainers. Similarly, if the effect of losing too many price-takers dominates the effect of price discrimination, having more bargainers may not be an advantage to the retailer.

In the rest of this section, we further consider the relationship between both models. The following proposition shows that the posted price of product 2 under model P is always higher than that under model N.

Proposition 5. *Suppose that in each of both models, the retailer has x and y units of inventory for products 1 and 2, respectively with t periods to go until the end of the season. The optimal posted price for product 2 in model P is higher than that in model N, that is, $p^*_{\mathbf{P2}}(x, y, t) \geq p^*_{\mathbf{N2}}(x, y, t)$.*

From the retailer's point of view, the decision for the posted prices of product 2, p_2, in both models is equivalent to that of only one product being sold when she chooses to use negotiation or not, given the same cost. When negotiation is allowed, p_2 tends to be higher, which enables better price discrimination. This observation is consistent with Kuo et al. (2009a), in which the posted price is higher under negotiation relative to take-it-or-leave-it pricing when only one product is sold. Furthermore, the comparison between the optimal posted prices for product 1 in both models is complex; both posted prices are sensitive to the retailer's bargaining power, β, and the customer's reservation price distributions for both products, $F_i, i = 1, 2$. As a result, there does not seem to exist a particular structure.

Proposition 6. *Assume the customer's reservation price for product i is uniformly distributed over the interval $[0, b_i]$. Given the cost of product i, c_i, the retailer adopts model s with x and y units of inventory for products 1 and 2, respectively, and with t periods to go until the end of the season.*

Suppose the optimal posted price for product i in model s is $p^*_{si}(x,y,t)$ for $s = P, N$ and $i = 1, 2$.
(a) The difference in the optimal posted prices for product 1 between model P and model N, $p^*_{\mathbf{P1}}(x,y,t) - p^*_{\mathbf{N1}}(x,y,t)$, increases in c_1.
(b) The difference in the optimal posted prices for product 2 between model P and model N, $p^*_{\mathbf{P2}}(x,y,t) - p^*_{\mathbf{N2}}(x,y,t)$, decreases in c_2.

In model P, when the cost of product 1 increases by one unit, the retailer will incur a marginal loss by exactly one unit if she does not adjust the posted price for product 1. In model N, on the other hand, the final negotiation price agreed by the bargainer and the retailer will be increased by $1 - \beta$ unit even though the retailer keeps the posted price the same (see equation (8)). More specifically, an increase in c_1 already supports the retailer to compensate for part of the loss when the negotiation is allowed. Therefore, with the use of negotiation in model N, the retailer will not set the posted price too high compared to model P. This intuition can be applied to the case of product 2 with the reversed result.

5. Numerical Study

We conduct a numerical study to further explore the effects of negotiation and dynamic pricing on the retailer's expected revenue when she sells multiple generations products. In our numerical study we use three different values for each of the probability that a customer arrives in each period ($\lambda \in \{0.2, 0.5, 0.8\}$), the bargaining power of the retailer ($\beta \in \{0.2, 0.5, 0.8\}$), and the fraction of bargainers ($q \in \{0.2, 0.5, 0.8\}$). We consider different combinations of parameter values, λ, β, and q in a 10-period selling horizon and range the beginning inventory level of each product from 1 to 10. For each combination and the beginning inventory level, we solve the dynamic programs in (6) and (8) and determine the optimal posted price of each product and the retailer's expected revenue.

Dynamic pricing literature suggests that, the retailer tends to raise the posted price of a product as the inventory level decreases and/or the time remaining to the end of the selling season increases regardless of using dynamic pricing only (Gallego and van Ryzin, 1994, Bitran and Mondschein, 1997) or dynamic pricing with negotiation (Kuo et al., 2009a). The conclusion was reached under the assumption that the retailer offers only one product. In our models in which the retailer provides two products, one at a time, these conclusions are not necessarily valid. Based on the numerical study we conducted, the retailer may decrease the posted price of a product when she has more time to sell no matter whether the price of this product is negotiable. The optimal posted price of product 1 over the selling season for given inventory levels of both products in both models is illustrated in Figure 1.

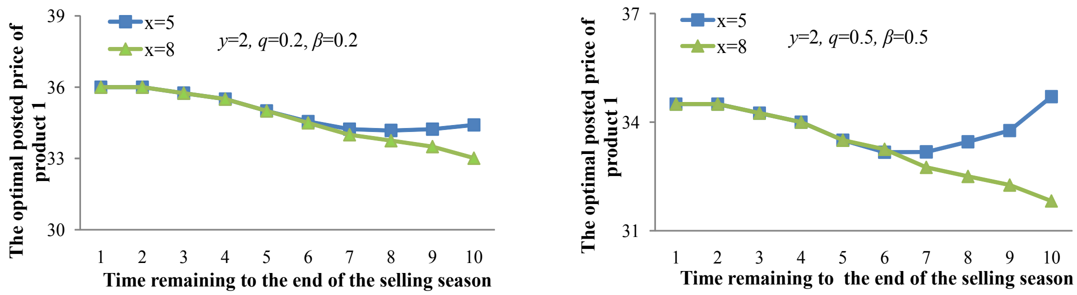

Figure 1: The optimal posted price of product 1 over the remaining time to the end of the selling season under model N (left) and model P (right). Here, $\lambda = 0.8$ and $F_1(\cdot)$ Weibull over the internal $[0, 150]$ with shape parameter 2 and scale parameter 40 and $F_2(\cdot)$ Weibull over the internal $[0, 150]$ with shape parameter 2 and scale parameter 30.

Compared with the single product case under which the retailer determines the posted price based on only the opportunity cost (i.e., marginal revenue) of the product, in the multiple-product case, the decision of the posted price depends on not only the cost of the product, but also the relative magnitude of the costs for both products. If one product brings less benefit to the retailer (i.e., the marginal revenue of the retailer by retaining the product for another period is lower), then the retailer tends to lower the posted price in a way to promote the product at that period. It can be the driver that marks the price down as there is more time until the end of the selling season. According to our numerical study, this counterintuitive result mainly occurs when the retailer sets the posted price of product 1 regardless of whether negotiation is allowed. In our setting in which the customer can only observe one product at a time, the retailer can use the posted price of product 1 as a tool to induce the customers for purchasing one product as opposed to the other. For example, the retailer reduces the posted price of product 1 so as to attract the customers to purchase product 1, doing which may result from the fact that at the current period it is considered more valuable to retain product 2 for another period.

How the posted price of a product is affected by the inventory level of the other product is shown in Figure 2. First, we consider the effect of the inventory level of product 1 on the posted price of the second product. Notice that given the inventory level of product 2, the posted price of product 2 decreases with the inventory level of the first product for both models (see Figure 2 right panel). When setting the price of product 2, the retailer is more cautious of the possibility of failed transaction for product 2, in particular, when the total inventory level (i.e., $x + y$ units) is higher, irrespective of the relative magnitude of the inventory level of each product. In other words, the retailer is facing the pressure of reducing the posted price of product 2 with the increase in the inventory level of the first product since the retailer's immediate concern is successfully making the deal and clearing the inventory level of the current product (i.e., product 2), rather than collecting more revenues.

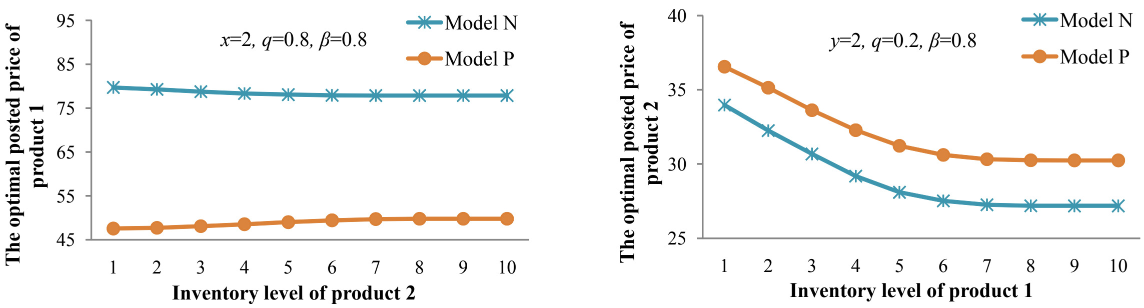

Figure 2: The optimal posted price of a product over the inventory level of the other product under both models. Here, $\lambda = 0.8$ and $F_1(\cdot)$ Weibull over the internal $[0, 150]$ with shape parameter 2 and scale parameter 40 and $F_2(\cdot)$ Weibull over the internal $[0, 150]$ with shape parameter 2 and scale parameter 30.

The effect of the inventory level for the second product on the posted price of product 1 under model N is similar to the observations in the right panel of Figure 2, as the retailer reduces the posted price of product 1 when the inventory level of product 2 increases. Although the retailer can seek a successful deal of product 2 by setting a lower price even if the transaction of product 1 fails, under model N, take-it-or-leave-it pricing narrows the flexibility of selling the second product to the customers with lower reservation prices, especially if the retailer has excess inventory. As a result, the retailer is not able to adopt an aggressive pricing strategy on product 1 as the inventory level of the second product is high. On the other hand, under model P where the price of the second product is negotiable, the retailer can utilize negotiation in two ways: gaining excess revenues from high reservation price customers and liquidating excess inventory to low reservation price customers. That is, the retailer can set a high posted price of product 1 without being worried about excess inventory of the second product. Therefore, the posted price pattern of product 1 with respect to the inventory level of the second product under each model is reversed.

We also compare the revenues of both models with the one when negotiation is not allowed on the prices of both products (No-neg. model), respectively, as shown in Figure 3. According to Kuo et al. (2009a) in the case of single product, negotiation is especially beneficial for the retailer when the inventory level of the product is high. The explanation behind this observation is that when the retailer needs to clear the inventory, she can always price discriminate among customers

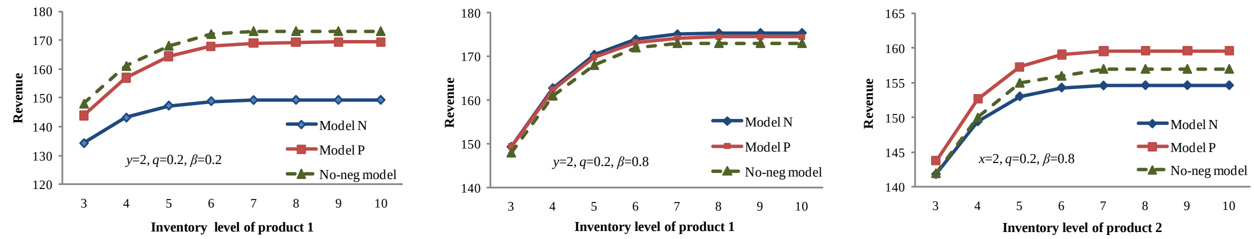

Figure 3: The effect of negotiation on the expected total revenue of model N, model P, and no negotiation model. Here, $\lambda = 0.8$ and $F_1(\cdot)$ Weibull over the internal $[0, 150]$ with shape parameter 2 and scale parameter 40 and $F_2(\cdot)$ Weibull over the internal $[0, 150]$ with shape parameter 2 and scale parameter 30.

with high reservation prices by means of negotiation. However, the exposition can be extended to the two-product case only when the retailer's relative bargaining power is high, and the results are reversed when the bargainer has higher bargaining power. In our model settings, negotiation serves as an impetus for the retailer to collect excess revenues from high reservation price bargainers when her relative bargaining power is high. On the contrary, lower relative bargaining power makes negotiation become resistant since low reservation price bargainers may purchase the product at the price close to the opportunity cost of the product. As a result, for the case of the bargaining power, β, being 0.2 (Figure 3 left panel), when the inventory level of product 1 is higher, the expected revenue of model N is the lowest among these models. On the other hand, for the case when bargaining power is 0.8 (Figure 3 central panel), the retailer can collect more revenue in model N compared to the other two scenarios. Similarly, the expected revenue of model P is the highest among three models as the retailer has higher bargaining power and a higher inventory level of product 2 (Figure 3 right panel). These results suggest that *the retailer with high bargaining power should adopt model N when she has a larger amount of product 1 in stock and adopts model P when the inventory level of product 2 is significant.*

6. Conclusion

This paper studies dynamic pricing among multi-generation products with limited inventories when the retailer allows negotiation over the price of just one product. In particular, we consider two models in which the retailer determines whether the price of the product is negotiable or based on a take-it-or-leave-it principle. Every arriving customer only observes the first product and transacts with the retailer accordingly. The second product is offered only if the transaction of the first product fails. In fact, the decision on a whether the price of the product is negotiable serves as a driver that influences the retailer's optimal pricing strategies and the associated revenue. In this study, we adopt the generalized Nash bargaining solution to describe the bargaining behavior initiated between the retailer and bargainers. The results derived from our settings characterize one phenomenon: regardless of the bargainers' reservation prices, the transaction price paid by the bargainers is always lower than their price-taking counterparts.

For both models, we show that a unique pair of the posted prices for both products can be uniquely determined to maximize the retailer's expected revenue. When bargainers make up the majority of the customer population, our model suggests the retailer can raise the posted price of the product allowed for negotiation. By so doing, the retailer can collect more revenue for better price discrimination. The relative bargaining power of the retailer plays a more complex role in influencing the decision of the posted prices. Higher bargaining power augments the posted price of the negotiable product, which in turn raises the ceiling of the price that bargainers pay. The resultant revenues received from bargainers can compensate for the portion of price-takers being left out. The effect of the relative bargaining power on the other (non-negotiable) product depends on whether negotiation is allowed for the price of the second product. In such a case (model P in our setting), the impact of the bargaining power is extended to the decision of the posted price for the first product.

According to our computational study, we present results that differ from traditional dynamic pricing literature. Given the inventory levels of both products, we show that the posted price of a product may not increase with the time remaining to the end of the selling season. Furthermore, the retailer may be worse off when adopting negotiation. Depending on the relative bargaining power, negotiation can be a double-edged sword. When the bargaining power is high, the retailer can benefit from negotiation. If the bargainer leads the negotiation, the retailer abandons negotiation as a tool for price differentiation.

Our main objective is to study the effects of dynamic pricing and negotiation on the retailer's optimal pricing strategies. In pursuit of this goal, two models are constructed in compliance with the current practice. In the automobile industry, car dealers already offer an attractive deal for the previous generation, and thus, the range for negotiation is relatively small compared with the latest one. On the other hand, due to the short lifespan of electronic products, retailers need to liquidate the inventories of the previous generations so as to avoid obsolescence. In this situation, negotiation becomes an adoptable scheme for the retailer. For the tractability of the problem, we assume negotiation is allowed on the price of one product in each model. This setting allows us to better understand the effects of negotiation and sales format sequences on the retailer's profit. One may consider the case where the prices of both products are negotiable. In fact, adopting negotiation on two products simply increases complexity of the problem in that the optimal posted price in each period may not be uniquely determined.

Several research directions extend out from this work. One could extend our work by considering a case in which the retailer sets not only the posted prices, but also the cut-off prices, below which the retailer does not sell. The decision of the cut-off price may help the retailer do better price discrimination without selling products to the bargainers with reservation prices too low. Another interesting research direction is to consider a case when each arriving customer can observe all the products at the same time and determine which to purchase depending on their relative surplus.

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Online Supplement

A. Adjusted Belief of Reservation Price Distributions

Here, we extend the model to the case under which all customers, whether bargainers and price-takers, prefer the first product to the other. More specifically, the willingness-to-pay of the customers for product 2 is no higher than that for product 1 (i.e., $r_2 \leq r_1$). Therefore, if the transaction of product 1 fails, the retailer will adjust the reservation price distribution of the customers for product 2. Notice that this phenomenon is of practical relevance, in particular, for automobiles and consumer electronic products. We consider model P first. At the beginning of each period, the retailer sets the non-negotiable price, p_1, for product 1. An arriving customer will purchase the product if his reservation price, r_1, is higher than p_1. If the transaction fails, the retailer knows the reservation price of the arriving customer for product 2 will not exceed p_1, and adjusts the belief of the customer's reservation price. That is, p_1 forms as the upper bound of the reservation price distribution from the retailer's perspective, which implies that $p_1 \geq p_2$. Let $F_2^a(\cdot)$ be the cumulative density function of the adjusted reservation price for product 2 with its pdf, $f_2^a(\cdot)$, and $\overline{F}_2^a(\cdot) := 1 - F_2^a(\cdot)$. We have

$$f_2^a(x) = \begin{cases} \frac{f_2(x)}{F_2(p_1)} & \text{if } 0 \leq x \leq p_1; \\ \\ 0 & \text{otherwise.} \end{cases} \tag{A-1}$$

Therefore, for $p_1 \geq c_2$, we can rewrite the function $J_{\text{P}}(p_1, p_2, x, y, t)$ in equation (6) as follows:

$$\begin{aligned} J_{\text{P}}(p_1, p_2, x, y, t) &= \lambda \overline{F}_1(p_1)(p_1 - c_1) \\ &\quad + \lambda q F_1(p_1) \left[\int_{p_2}^{p_1} [\beta p_2 + (1-\beta) c_2] f_2^a(x) dx + \int_{c_2}^{p_2} [\beta x + (1-\beta) c_2] f_2^a(x) dx \right. \\ &\quad \left. - \overline{F}_2^a(c_2) c_2 \right] + \lambda (1-q) F_1(p_1) \overline{F}_2^a(p_2)(p_2 - c_2) + V_{\text{P}}(x, y, t-1). \end{aligned} \tag{A-2}$$

Notice that if $p_1 < c_2$, the retailer will not offer product 2 due to the fact that the arriving customer cannot afford c_2, which is the marginal revenue of product 2. In this sense, the function $J_{\text{P}}(p_1, p_2, x, y, t)$ in equation (A-2) will be reduced to

$$J_{\text{P}}(p_1, p_2, x, y, t) = \lambda \overline{F}_1(p_1)(p_1 - c_1) + V_{\text{P}}(x, y, t-1).$$

In model N, customers are divided into two groups – price-takers and bargainers. A price-taker or a bargainer will purchase product 1 if his reservation price is above p_1, or c_1, respectively. Similarly, if the arriving customer fails to purchase product 1, the retailer adjusts the belief of the customer's reservation price for product 2. The adjusted reservation price distribution for price-takers is derived based on equation (A-1). For bargainers, the upper bound will be replaced by c_1, and the pdf of the adjusted reservation price distribution of product 2 for bargainers, $f_2^b(x)$, is given by:

$$f_2^b(x) = \begin{cases} \frac{f_2(x)}{F_2(c_1)} & \text{if } 0 \le x \le c_1; \\ \\ 0 & \text{otherwise.} \end{cases} \tag{A-3}$$

For $\max\{c_1, c_2\} \le p_1$, the function in equation (8) can be rewritten by

$$\begin{aligned} J_{\text{N}}(p_1, p_2, x, y, t) \quad = \quad & \lambda q \left[\int_{p_1}^{b_1} [\beta p_1 + (1-\beta)c_1] f_1(x)dx + \int_{c_1}^{p_1} [\beta x + (1-\beta)c_1] f_1(x)dx - \overline{F}_1(c_1)c_1 \right] \\ & + \lambda(1-q)\overline{F}_1(p_1)(p_1 - c_1) + \lambda \left[qF_1(c_1)\overline{F}_2^b(p_2) + (1-q)F_1(p_1)\overline{F}_2^a(p_2) \right] (p_2 - c_2) \\ & + V_{\text{N}}(x, y, t-1). \end{aligned} \tag{A-4}$$

We can observe that if $c_1 \le p_1 \le c_2$, the third term of function $J_{\text{N}}(p_1, p_2, x, y, t)$ in equation (A-4) will disappear in that the retailer has no incentive to offer product 2 knowing that the arriving customer will never pay a price higher than the marginal revenue of product 2.

It should be noted that both adjusted models only apply for the case in which the retailer has the first product in stock (i.e., $x > 0$). When the retailer has product 2 only, she will not be able to use the posted price, p_1, or the opportunity cost, c_1, of product 1 to update the reservation price distribution of the customers for product 2 (i.e., $f_2^a(\cdot) = f_2^b(\cdot) = f(\cdot)$). In that sense, each adjusted model is equivalent to the original model P or N, respectively.

For both models, the adjusted customers' reservation distributions of product 2 are the functions of the posted price, p_1. The revenue to go function of the retailer in each period is complex due to the structure of bimodality, and thus, the optimal posted prices cannot be uniquely determined. We conduct a numerical experiment to further explore their managerial implications.

To test whether the aforementioned results in Sections 4 and 5 are robust in the adjusted model, we vary the arrival rate, the proportion of bargainers, the relative bargaining power, initial inventory levels of both products, and the remaining time to the end of the selling season. In all the cases we considered, we found that Propositions 1 through 4 continue to hold except the effects of the proportion of bargainers, q, the relative bargaining power, β, and the cost of the first product, c_1, on the posted price of product 2, p_2, in particular, under model N. In fact, in the adjusted model, amplifying any of these driving effects expands the space of adjustment for p_2 (due to an increase in p_1), which allows the retailer to set a higher p_2 to subtract more revenue from higher willingness-to-pay customers. Furthermore, the choice of p_1 adjusts the belief of the customer's reservation price distribution for the other product, and thus, indirectly influences the determination of p_2. This complexity of interaction between p_1 and p_2 leads to the conclusion that Propositions 5 and 6 may not hold. On the other hand, all the results in Section 5 still hold. It is worth noting that when the retailer adjusts the belief of the customer reservation price distribution, the posted price of the first product, p_1, is higher while the posted price of product 2, p_2, is lower compared to the original scenarios no matter whether model P or model N is adopted. It is due to the fact that, p_1 now becomes the ceiling that restricts the pricing flexibility of product 2 if p_1 is set too low. With a low p_1, the retailer cannot choose a higher price on the second product since the customers tend to value the second product lower after observing the posted price of the first product in the adjusted model. This effect of limited pricing flexibility directly results in less revenue.

B. Prices of Both Products Are Negotiable

In this section, we consider the case where the prices of both products are negotiable. Follow the same analysis in Section 3, the retailer's expected total revenue can be rewritten as follows:

$$V(x,y,t) = \max_{p_1,p_2} J(p_1,p_2,x,y,t) \text{ for } x,y > 0, t = 1,\ldots,T,$$

$$\begin{aligned}
J(p_1,p_2,x,y,t) \quad = \quad & \lambda q\left[\int_{p_1}^{b_1}\left[\beta p_1 + (1-\beta)c_1\right] f_1(x)dx + \int_{c_1}^{p_1}\left[\beta x + (1-\beta)c_1\right] f_1(x)dx - \overline{F}_1(c_1)c_1\right] \\
& +\lambda(1-q)\overline{F}_1(p_1)(p_1-c_1) \\
& +\lambda q F_1(c_1)\left[\int_{p_2}^{b_2}\left[\beta p_2 + (1-\beta)c_2\right] f_2(x)dx + \int_{c_2}^{p_2}\left[\beta x + (1-\beta)c_2\right] f_2(x)dx - \overline{F}_2(c_2)c_2\right] \\
& +\lambda(1-q)F_1(p_1)\overline{F}_2(p_2)(p_2-c_2) + V(x,y,t-1).
\end{aligned}$$

$$\text{Boundary condition: } V(x,y,0) = 0 \text{ for } x,y \geq 0, \text{ and } V(0,0,t) = 0 \text{ for } t = 1,\ldots,T.$$

Notice that the optimal posted prices of both products in the above model cannot be uniquely determined. To gain further managerial insights, we conduct a numerical study using the combinations of different parameters mentioned in Section 5 and check whether the results are robust in the aforementioned formulations. Based on the results, we conclude that the retailer will raise the prices of both products when the fraction of bargainers, q, and/or the retailer's bargaining power, β, increases. Furthermore, for both products, an increase in the opportunity cost of the product results in an increase in the associated price; though the cost and the price of the other product may be negatively related. When the retailer adopts this model, our analysis shows that the price of product 2 always dominates that of model N, but the outcome may vary when compared with the price of product 2 in model P. Similar patterns can be observed as in Figures 1 to 3, particularly, in Figure 3, the effect of the retailer's bargaining power on the retailer's revenue is more intense than models N and P. The interpretations of these outcomes follow the discussions in Sections 4 and 5.

C. Proofs of Results in Section 4

Proof of Lemma 1

Note that we provide the proof of the result for model N in detail and omit the proof for model P since the proof of model P follows a similar sequence of arguments. We prove that, for a given $p_2 \in \big[c_2, b_2\big)$, $J_{\mathrm{N}}(p_1, p_2, x, y, t)$ is strictly unimodal in p_1 for $p_1 \in [c_1, b_1)$. To do so, we prove the following claims: (i) $\frac{\partial J_{\mathrm{N}}(p_1,p_2,x,y,t)}{\partial p_1}\Big|_{p_1=c_1} \geq 0$, (ii) $\frac{\partial^2 J_{\mathrm{N}}(p_1,p_2,x,y,t)}{\partial p_1^2} < 0$ whenever $\frac{\partial J_{\mathrm{N}}(p_1,p_2,x,y,t)}{\partial p_1} = 0$, and (iii) $\frac{\partial J_{\mathrm{N}}(p_1,p_2,x,y,t)}{\partial p_1}\Big|_{p_1=b_1} \leq 0$.

The first and second derivatives of $J_{\mathrm{N}}(p_1, p_2, x, y, t)$ (given by (8)) with respect to p_1 are

$$\begin{aligned}
\frac{\partial J_{\mathrm{N}}(p_1, p_2, x, y, t)}{\partial p_1} &= \lambda\left[q\beta\overline{F}_1(p_1) + (1-q)\overline{F}_1(p_1) - (1-q)f_1(p_1)(p_1 - c_1)\right] \\
&\quad +\lambda(1-q)f_1(p_1)\overline{F}_2(p_2)(p_2 - c_2) \text{ and} \qquad \text{(C-1)} \\
\frac{\partial^2 J_{\mathrm{N}}(p_1, p_2, x, y, t)}{\partial p_1^2} &= \lambda\left[-q\beta f_1(p_1) - 2(1-q)f_1(p_1) - (1-q)f_1'(p_1)(p_1 - c_1)\right] \\
&\quad +\lambda(1-q)f_1'(p_1)\overline{F}_2(p_2)(p_2 - c_2). \qquad \text{(C-2)}
\end{aligned}$$

Note that claims (i) and (iii) directly follow from (C-1) since $p_1 \geq c_1$ and $p_2 \geq c_2$. To show claim (ii), note from (C-1) and (C-2),

$$\frac{\partial^2 J_{\mathrm{N}}(p_1,p_2,x,y,t)}{\partial p_1^2}\Bigg|_{\frac{\partial J_{\mathrm{N}}(p_1,p_2,x,y,t)}{\partial p_1}=0} = \lambda\left[-q\beta f_1(p_1) - 2(1-q)f_1(p_1) - f_1'(p_1)\frac{q\beta\overline{F}_1(p_1)+(1-q)\overline{F}_1(p_1)}{f_1(p_1)}\right] \qquad \text{(C-3)}$$

Note that if $f'(p_1) \geq 0$, then all three terms in the brackets of (C-3) are negative. Now, consider the case that $f'(p_1) < 0$. First, note that $\beta \in (0, 1)$ and $q < 1/2$ (Assumption 2), we have

$$\frac{q\beta\overline{F}_1(p_1) + (1-q)\overline{F}_1(p_1)}{f_1(p_1)} < \frac{\overline{F}_1(p_1)}{f_1(p_1)}, \text{ and} - 2(1-q)f_1(p_1) \leq -f_1(p_1)$$

As a result, given $f'(p_1) < 0$,

$$-f_1'(p_1)\frac{q\beta\overline{F}_1(p_1) + (1-q)\overline{F}_1(p_1)}{f_1(p_1)} - 2(1-q)f_1(p_1) < -f_1'(p_1)\frac{\overline{F}_1(p_1)}{f_1(p_1)} - f_1(p_1). \qquad \text{(C-4)}$$

Now, observe that $-f_1'(p_1)\frac{\overline{F}_1(p_1)}{f_1(p_1)} - f_1(p_1) \leq 0$ (because F_1 is IFR, and thus, $f_1^2(\cdot) + f_1'(\cdot)\overline{F}_1(\cdot) \geq 0$). Hence, all the three terms in the brackets of (C-3) add up to a negative number, concluding the proof of claim (ii).

By the strict unimodality of $J_{\text{N}}(p_1, p_2, x, y, t)$ in $p_1 \in [c_1, b_1)$ for a given $p_2 \in [c_2, b_2)$, let $p_1^*(p_2)$ be the unique optimal value of p_1 at a given p_2. Define the induced function $J_{\text{N}}^*(p_2, x, y, t) := J_{\text{N}}(p_1^*(p_2), p_2, x, y, t)$. We now show that $J_{\text{N}}^*(p_2, x, y, t)$ is strictly unimodal in p_2 for $p_2 \in [c_2, b_2)$.

We prove the unimodality of $J_{\text{N}}^*(p_2, x, y, t)$ by showing (iv) $\left.\frac{dJ_{\text{N}}^*(p_2,x,y,t)}{dp_2}\right|_{p_2=c_2} \geq 0$, (v) $\frac{d^2 J_{\text{N}}^*(p_2,x,y,t)}{dp_2^2} < 0$ whenever $\frac{dJ_{\text{N}}^*(p_2,x,y,t)}{dp_2} = 0$, and (vi) $\left.\frac{dJ_{\text{N}}^*(p_2,x,y,t)}{dp_2}\right|_{p_2=b_2} \leq 0$.

Claims (iv) and (vi) are directly from the partial derivative of $J_{\text{N}}(p_1, p_2, x, y, t)$ with respect to p_2 and by utilizing the envelope theorem:

$$\frac{\partial J_{\text{N}}(p_1, p_2, x, y, t)}{\partial p_2} = \lambda(qF_1(c_1) + (1-q)F_1(p_1)) \left[\overline{F}_2(p_2) - f_2(p_2)(p_2 - c_2)\right] \tag{C-5}$$

To conclude that $J_{\text{N}}^*(p_2, x, y, t)$ is strictly unimodal in p_2, it remains to prove claim (v). To that end, we use p_1^* as a shorthand notation for $p_1^*(p_2)$. first note that:

$$\frac{d^2 J_{\text{N}}^*(p_2, x, y, t)}{dp_2^2} = \left.\frac{\partial^2 J_{\text{N}}(p_1, p_2, x, y, t)}{\partial p_2^2}\right|_{p_1=p_1^*} + \frac{dp_1^*}{dp_2} \left.\frac{\partial^2 J_{\text{N}}(p_1, p_2, x, y, t)}{\partial p_2 \partial p_1}\right|_{p_1=p_1^*} \tag{C-6}$$

Using the implicit function theorem, we have

$$\frac{d^2 J_{\text{N}}^*(p_2, x, y, t)}{dp_2^2} = \frac{\left.\frac{\partial^2 J_{\text{N}}(p_1,p_2,x,y,t)}{\partial p_2^2}\right|_{p_1=p_1^*} \left.\frac{\partial^2 J_{\text{N}}(p_1,p_2,x,y,t)}{\partial p_1^2}\right|_{p_1=p_1^*} - \left(\left.\frac{\partial^2 J_{\text{N}}(p_1,p_2,x,y,t)}{\partial p_2 \partial p_1}\right|_{p_1=p_1^*}\right)^2}{\left.\frac{\partial^2 J_{\text{N}}(p_1,p_2,x,y,t)}{\partial p_1^2}\right|_{p_1=p_1^*}} \tag{C-7}$$

Note that the denominator is always negative since $J_{\text{N}}(p_1, p_2, x, y, t)$ is strictly unimodal in p_1 for a given p_2, as we proved in the first part of this lemma. Hence, it suffices to show that the numerator in (C-7) is strictly positive. Note that

$$\begin{aligned}
\frac{\partial^2 J_{\text{N}}(p_1, p_2, x, y, t)}{\partial p_1^2} &= \lambda\left[-q\beta f_1(p_1) - 2(1-q)f_1(p_1) - (1-q)f_1'(p_1)(p_1 - c_1)\right] \\
&\quad + \lambda(1-q)f_1'(p_1)\overline{F}_2(p_2)(p_2 - c_2), && \text{(C-8)} \\
\frac{\partial^2 J_{\text{N}}(p_1, p_2, x, y, t)}{\partial p_2^2} &= \lambda(qF_1(c_1) + (1-q)F_1(p_1)) \left[-2f_2(p_2) - f_2'(p_2)(p_2 - c_2)\right] && \text{(C-9)} \\
\frac{\partial^2 J_{\text{N}}(p_1, p_2, x, y, t)}{\partial p_1 \partial p_2} &= \lambda(1-q)f_1(p_1) \left[\overline{F}_2(p_2) - f_2(p_2)(p_2 - c_2))\right] && \text{(C-10)}
\end{aligned}$$

Using the expressions above, one can check that by (C-5) and (C-10), when $\frac{dJ_{\text{N}}^*(p_2,x,y,t)}{dp_2} = 0$, we obtain $\left.\frac{\partial^2 J_{\text{N}}(p_1,p_2,x,y,t)}{\partial p_1 \partial p_2}\right|_{p_1=p_1^*} = 0$. In addition, one can easily show that $\left.\frac{\partial^2 J_{\text{N}}(p_1,p_2,x,y,t)}{\partial p_2^2}\right|_{p_1=p_1^*} < 0$ by substituting $\overline{F}_2(p_2) - f_2(p_2)(p_2 - c_2) = 0$ (since $\frac{dJ^*_{\mathbf{N}}(p_2,x,y,t)}{dp_2} = 0$) into (C-9) and by the fact that F_2 is IFR. Thus, the numerator in (C-7) is strictly positive, concluding the proof of claim (v).

The proof for model P is omitted since it follows a similar sequence of arguments.

Proof of Proposition 1 and 2

Here, we only provide the proof for Proposition 2 as the proof of Proposition 1 follows the same logic.

Proofs of (a): Following from Lemma 1, we obtain that the optimal posted price for both products, $p^*_{\mathbf{N1}}(x,y,t)$ and $p^*_{\mathbf{N2}}(x,y,t)$, are given by the unique pair of p_1 and p_2 that satisfy the first order conditions of $J_{\mathbf{N}}(p_1,p_2,x,y,t)$, that is,

$$\frac{\partial J_{\mathbf{N}}(p^*_{\mathbf{N1}}(x,y,t),p^*_{\mathbf{N2}}(x,y,t),x,y,t)}{\partial p_1} = 0 \text{ and } \frac{\partial J_{\mathbf{N}}(p^*_{\mathbf{N1}}(x,y,t),p^*_{\mathbf{N2}}(x,y,t),x,y,t)}{\partial p_2} = 0. \tag{C-11}$$

For simplification, in the remainder of this proof, we use p^*_1 and p^*_2 as the shorthand notations for $p^*_{\mathbf{N1}}(x,y,t)$ and $p^*_{\mathbf{N2}}(x,y,t)$. Thus, the implicit differentiation of the above identities with respect to q can be obtained:

$$\frac{dp^*_1}{dq}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1^2}\right|_{p_1=p^*_1,p_2=p^*_2} + \frac{dp^*_2}{dq}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1 \partial p_2}\right|_{p_1=p^*_1,p_2=p^*_2} + \left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1 \partial q}\right|_{p_1=p^*_1,p_2=p^*_2} = 0, \tag{C-12}$$

$$\frac{dp^*_1}{dq}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1 \partial p_2}\right|_{p_1=p^*_1,p_2=p^*_2} + \frac{dp^*_2}{dq}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2^2}\right|_{p_1=p^*_1,p_2=p^*_2} + \left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2 \partial q}\right|_{p_1=p^*_1,p_2=p^*_2} = 0. \tag{C-13}$$

We first prove $\frac{dp^*_2}{dq} = 0$. Note that the partial derivative of $J_{\mathbf{N}}(p_1,p_2,x,y,t)$ with respect to p_2 is

$$\frac{\partial J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2} = \lambda(qF_1(c_1) + (1-q)F_1(p_1))\left[\overline{F}_2(p_2) - f_2(p_2)(p_2 - c_2)\right]. \tag{C-14}$$

One can check that $\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1 \partial p_2}\right|_{p_1=p^*_1,p_2=p^*_2} = \left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2 \partial q}\right|_{p_1=p^*_1,p_2=p^*_2} = 0$, and thus, the first and the third terms of (C-13) are both zero. In addition, $\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2^2}\right|_{p_1=p^*_1,p_2=p^*_2} < 0$ as shown in the proof of Lemma 1. As a result, we have $\frac{dp^*_2}{dq} = 0$.

To prove $\frac{dp^*_1}{dq} \geq 0$, note that since $\frac{dp^*_2}{dq} = 0$, (C-12) can be rewritten by:

$$\frac{dp^*_1}{dq} = -\frac{\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1 \partial q}\right|_{p_1=p^*_1,p_2=p^*_2}}{\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1^2}\right|_{p_1=p^*_1,p_2=p^*_2}} \tag{C-15}$$

We first note that the denominator of (C-15) is strictly negative as shown in the proof of Lemma 1. Therefore, it suffices to show that the numerator of (C-15) is non-negative. Taking the partial derivative of (C-1) with respect to q, we have

$$\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1 \partial q} = \lambda\left[f_1(p_1)(p_1-c_1) - (1-\beta)\overline{F}_1(p_1) - f_1(p_1)\overline{F}_2(p_2)(p_2-c_2)\right]. \qquad \text{(C-16)}$$

Since $\left.\frac{\partial J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1}\right|_{p_1=p_1^*,p_2=p_2^*} = 0$, it follows from (C-1) that $p_1 - c_1 = \frac{q\beta\overline{F}_1(p_1)+(1-q)\overline{F}_1(p_1)}{(1-q)f_1(p_1)} + \overline{F}_2(p_2)(p_2-c_2)$. Substituting this expression for $p_1 - c_1$ in (C-16), we obtain

$$\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1 \partial q}\right|_{p_1=p_1^*,p_2=p_2^*} = \frac{\lambda\beta}{1-q}\overline{F}_1(p_1) \geq 0. \qquad \text{(C-17)}$$

where the inequality comes from $q \in (0,1)$. Thus, $\frac{dp_1^*}{dq} \geq 0$.

Proofs of (b): Similarly, the implicit differentiation of the identities in (C-11) with respect to β can also be obtained:

$$\frac{dp_1^*}{d\beta}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1^2}\right|_{p_1=p_1^*,p_2=p_2^*} + \frac{dp_2^*}{d\beta}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial p_2}\right|_{p_1=p_1^*,p_2=p_2^*} + \left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial \beta}\right|_{p_1=p_1^*,p_2=p_2^*} = 0, \qquad \text{(C-18)}$$

$$\frac{dp_1^*}{d\beta}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial p_2}\right|_{p_1=p_1^*,p_2=p_2^*} + \frac{dp_2^*}{d\beta}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2^2}\right|_{p_1=p_1^*,p_2=p_2^*} + \left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2\partial \beta}\right|_{p_1=p_1^*,p_2=p_2^*} = 0. \qquad \text{(C-19)}$$

To prove $\frac{dp_2^*}{d\beta} = 0$, first note from (C-19) that both $\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial p_2}\right|_{p_1=p_1^*,p_2=p_2^*} = \left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2\partial \beta}\right|_{p_1=p_1^*,p_2=p_2^*} = 0$ (Part (a)). Also, $\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2^2}\right|_{p_1=p_1^*,p_2=p_2^*} < 0$ as shown in the proof of Lemma 1. As a result, we have $\frac{dp_2^*}{d\beta} = 0$. To prove $\frac{dp_1^*}{d\beta} \geq 0$, note that given $\frac{dp_2^*}{d\beta} = 0$, (C-18) can be rewritten by:

$$\frac{dp_1^*}{d\beta} = -\frac{\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial \beta}\right|_{p_1=p_1^*,p_2=p_2^*}}{\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1^2}\right|_{p_1=p_1^*,p_2=p_2^*}} \qquad \text{(C-20)}$$

Again, the denominator of (C-20) is strictly negative. Taking the partial derivative of (C-1) with respect to β, we have $\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial \beta} = \lambda q\overline{F}_1(p_1) \geq 0$ which concludes the proof of $\frac{dp_1^*}{d\beta} \geq 0$.

Proof of Proposition 3

In this proof, we show the results of model N in detail. The proof of model P follows a similar sequence of arguments, and thus, omitted. We first show that in model N, the posted price of product 1, $p_{\mathbf{N1}}^*(x,y,t)$, is increasing in $c_1(x,y,t)$ but the posted price of product 2, $p_{\mathbf{N2}}^*(x,y,t)$, is independent of $c_1(x,y,t)$. Then we show that $p_{\mathbf{N1}}^*(x,y,t)$ is decreasing but $p_{\mathbf{N2}}^*(x,y,t)$ is increasing in $c_2(x,y,t)$. Note from Lemma 1, $p_{\mathbf{N1}}^*(x,y,t)$ and $p_{\mathbf{N2}}^*(x,y,t)$ satisfy the first order conditions of $J_{\mathbf{N}}(p_1,p_2,x,y,t)$, that is,

$$\frac{\partial J_{\mathbf{N}}(p_{\mathbf{N1}}^*(x,y,t),p_{\mathbf{N2}}^*(x,y,t),x,y,t)}{\partial p_1} = 0 \text{ and } \frac{\partial J_{\mathbf{N}}(p_{\mathbf{N1}}^*(x,y,t),p_{\mathbf{N2}}^*(x,y,t),x,y,t)}{\partial p_2} = 0. \qquad \text{(C-21)}$$

For simplification, we use p_1^* and p_2^* as the shorthand notations for $p_{\mathbf{N1}}^*(x,y,t)$ and $p_{\mathbf{N2}}^*(x,y,t)$, respectively. Hence, the implicit differentiation of the above identities with respect to c_1 can be obtained:

$$\frac{dp_1^*}{dc_1}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1^2}\right|_{p_1=p_1^*,p_2=p_2^*}+\frac{dp_2^*}{dc_1}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial p_2}\right|_{p_1=p_1^*,p_2=p_2^*}+\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial c_1}\right|_{p_1=p_1^*,p_2=p_2^*}=0, \tag{C-22}$$

$$\frac{dp_1^*}{dc_1}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial p_2}\right|_{p_1=p_1^*,p_2=p_2^*}+\frac{dp_2^*}{dc_1}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2^2}\right|_{p_1=p_1^*,p_2=p_2^*}+\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2\partial c_1}\right|_{p_1=p_1^*,p_2=p_2^*}=0. \tag{C-23}$$

We first prove $\frac{dp_2^*}{dc_1}=0$. Note that the partial derivative of $J_{\mathbf{N}}(p_1,p_2,x,y,t)$ with respect to p_2 is

$$\frac{\partial J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2}=\lambda(qF_1(c_1)+(1-q)F_1(p_1))\left[\overline{F}_2(p_2)-f_2(p_2)(p_2-c_2)\right]. \tag{C-24}$$

It is not hard to check that $\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial p_2}\right|_{p_1=p_1^*,p_2=p_2^*}=\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2\partial q}\right|_{p_1=p_1^*,p_2=p_2^*}=0$. In addition, $\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2^2}\right|_{p_1=p_1^*,p_2=p_2^*}<0$ from the proof of Lemma 1. Hence, we have $\frac{dp_2^*}{dc_1}=0$.

To prove $\frac{dp_1^*}{dc_1}\geq 0$, note that since $\frac{dp_2^*}{dc_1}=0$, (C-22) can be rewritten by:

$$\frac{dp_1^*}{dc_1}=-\frac{\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial c_1}\right|_{p_1=p_1^*,p_2=p_2^*}}{\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1^2}\right|_{p_1=p_1^*,p_2=p_2^*}} \tag{C-25}$$

The denominator of (C-25) is strictly negative from the proof of Lemma 1. Taking the partial derivative of (C-1) with respect to c_1, we have $\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial c_1}=\lambda(1-q)f_1(p_1)\geq 0$, which concludes the proof of $\frac{dp_1^*}{dc_1}\geq 0$.

To prove $\frac{dp_1^*}{dc_2}\leq 0$ and $\frac{dp_2^*}{dc_2}\geq 0$, note that the implicit differentiation of (C-21) with respect to c_2 can be obtained:

$$\frac{dp_1^*}{dc_2}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1^2}\right|_{p_1=p_1^*,p_2=p_2^*}+\frac{dp_2^*}{dc_2}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial p_2}\right|_{p_1=p_1^*,p_2=p_2^*}+\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial c_2}\right|_{p_1=p_1^*,p_2=p_2^*}=0, \tag{C-26}$$

$$\frac{dp_1^*}{dc_2}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial p_2}\right|_{p_1=p_1^*,p_2=p_2^*}+\frac{dp_2^*}{dc_2}\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2^2}\right|_{p_1=p_1^*,p_2=p_2^*}+\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2\partial c_2}\right|_{p_1=p_1^*,p_2=p_2^*}=0. \tag{C-27}$$

Similarly, note that $\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial p_2}\right|_{p_1=p_1^*,p_2=p_2^*}=0$ and $\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2^2}\right|_{p_1=p_1^*,p_2=p_2^*}<0$. Furthermore, from (C-24), $\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_2\partial c_2}\right|_{p_1=p_1^*,p_2=p_2^*}=\lambda(qF_1(c_1)+(1-q)F_1(p_1))f_2(p_2)\geq 0$. As a result, we have from (C-27) that $\frac{dp_2^*}{dc_2}\geq 0$. To prove $\frac{dp_1^*}{dc_2}\leq 0$, note that $\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial p_2}\right|_{p_1=p_1^*,p_2=p_2^*}=0$ and $\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1^2}\right|_{p_1=p_1^*,p_2=p_2^*}<0$. In addition, $\left.\frac{\partial^2 J_{\mathbf{N}}(p_1,p_2,x,y,t)}{\partial p_1\partial c_2}\right|_{p_1=p_1^*,p_2=p_2^*}=-\lambda(1-q)f_1(p_1)\overline{F}_2(p_2)\leq$

0. As a result, we obtain that $\frac{dp_1^*}{dc_2} \leq 0$.

Proof of Proposition 4

Proofs of (a): For this proof, let $p_{s1}^*(\beta)$ and $p_{s2}^*(\beta)$ be the optimal posted prices and $J_s(p_{s1}^*(\beta), p_{s2}^*(\beta), x, y, t, \beta)$ be the retailer's optimal expected total revenue with x and y units in inventory and t periods to go, for $s = P, N$. Define two different bargaining power $\beta_1 < \beta_2$. We will show $J_s(p_{s1}^*(\beta_2), p_{s2}^*(\beta_2), x, y, t, \beta_2) \geq J_s(p_{s1}^*(\beta_1), p_{s2}^*(\beta_1), x, y, t, \beta_1)$. Note that the bargaining power only affects the final price that bargainers pay for the product. More specifically, when β increases, the final price that a bargainer with reservation price r pays ($\beta \min\{p_{s2}^*, r\} + (1-\beta)c_2$ in model P and $\beta \min\{p_{s1}^*, r\} + (1-\beta)c_1$ in model N) also increases. Suppose the retailer with bargaining power β_2 sets the same posted prices as she does with bargaining power β_1. Since an increase in the bargaining power only increases the retailer's revenue from bargainers without reducing the revenue from price-takers (see (6) and (8)), we have $J_s(p_{s1}^*(\beta_1), p_{s2}^*(\beta_1), x, y, t, \beta_2) \geq J_s(p_{s1}^*(\beta_1), p_{s2}^*(\beta_1), x, y, t, \beta_1)$. However, $p_{s1}^*(\beta_1)$ and $p_{s2}^*(\beta_1)$ are the sub-optimal solutions when $\beta = \beta_2$. As a result, we have $J_s(p_{s1}^*(\beta_2), p_{s2}^*(\beta_2), x, y, t, \beta_2) \geq J_s(p_{s1}^*(\beta_1), p_{s2}^*(\beta_1), x, y, t, \beta_2)$, and the result follows.

Proofs of (b): Similar to (a), let $p_{s1}^*(q)$ and $p_{s2}^*(q)$ be the optimal posted prices of both products and $J_s(p_{s1}^*(q), p_{s2}^*(q), x, y, t, q)$ be the retailer's optimal expected total revenue with x and y units in inventory and t periods to go, for $s = P, N$. When $\beta \to 0$, the retailer's optimal expected revenue in both models are given by

$$\begin{aligned} J_{\mathbf{N}}(p_{\mathbf{N1}}^*(q), p_{\mathbf{N2}}^*(q), x, y, t, q) &= \lambda[(1-q)\overline{F}_1(p_{\mathbf{N1}}^*(q))(p_{\mathbf{N1}}^*(q) - c_1) \\ &\quad +[qF_1(c_1) + (1-q)F_1(p_{\mathbf{N1}}^*(q))]\overline{F}_2(p_{\mathbf{N2}}^*(q))(p_{\mathbf{N2}}^*(q) - c_2)] + V_{\mathbf{N}}(x, y, t-1) \\ J_{\mathbf{P}}(p_{\mathbf{P1}}^*(q), p_{\mathbf{P2}}^*(q), x, y, t, q) &= \lambda[\overline{F}_1(p_{\mathbf{P1}}^*(q))(p_{\mathbf{P1}}^*(q) - c_1) + (1-q)F_1(p_{\mathbf{P1}}^*(q))\overline{F}_2(p_{\mathbf{P2}}^*(q))(p_{\mathbf{P2}}^*(q) - c_2)] \\ &\quad +V_{\mathbf{P}}(x, y, t-1). \end{aligned} \tag{C-28}$$

Define two fractions of bargainers, $q_1 < q_2$. We will show

$$J_s(p_{s1}^*(q_1), p_{s2}^*(q_1), x, y, t, q_1) \geq J_s(p_{s1}^*(q_2), p_{s2}^*(q_2), x, y, t, q_2),$$

for $s = P, N$. Note from (C-28) that when q increases, by charging the same posted prices, $J_s(p_{s1}^*(q), p_{s2}^*(q), x, y, t, q)$ will decrease due to the facts that $p_{si}^*(q) \geq c_i$ for $s = P, N$ and $i = 1, 2$. Suppose now the retailer with fraction q_1 sets the same posted prices as she does with the fraction q_2. We obtain that $J_s(p_{s1}^*(q_2), p_{s2}^*(q_2), x, y, t, q_1) \geq J_s(p_{s1}^*(q_2), p_{s2}^*(q_2), x, y, t, q_2)$. However, $p_{s1}^*(q_2)$ and $p_{s2}^*(q_2)$ are not the optimal solutions when $q = q_1$, we obtain that $J_s(p_{s1}^*(q_1), p_{s2}^*(q_1), x, y, t, q_1) \geq$

$J_s(p_{s1}^*(q_2), p_{s2}^*(q_2), x, y, t, q_1)$. Thus, we obtain

$$J_s(p_{s1}^*(q_1), p_{s2}^*(q_1), x, y, t, q_1) \geq J_s(p_{s1}^*(q_2), p_{s2}^*(q_2), x, y, t, q_2).$$

Proof of Proposition 5

For model P, recall from Lemma 1 that the definitions $p_1^*(p_2) = \arg\max_{p_1}\{J_{\mathbf{P}}(p_1, p_2, x, y, t)\}$ and $J_{\mathbf{P}}^*(p_2, x, y, t) := J_{\mathbf{P}}(p_1^*(p_2), p_2, x, y, t)$. Also, we showed that $J_{\mathbf{P}}(p_1, p_2, x, y, t)$ is strictly unimodal in p_1 for a given p_2 and $J_{\mathbf{P}}^*(p_2, x, y, t)$ is strictly unimodal in p_2. By the envelope theorem, we have $\frac{dJ_{\mathbf{P}}^*(p_2,x,y,t)}{dp_2} = \frac{\partial J_{\mathbf{P}}(p_1,p_2,x,y,t)}{\partial p_2}\Big|_{p_1=p_1^*(p_2)}$. Substituting the partial derivative of $J_{\mathbf{P}}(p_1, p_2, x, y, t)$ with respect to p_2, we obtain:

$$\frac{dJ_{\mathbf{P}}^*(p_2, x, y, t)}{dp_2} = \lambda F_1(p_1^*(p_2)) \left[q\beta\overline{F}_2(p_2) + (1-q)(\overline{F}_2(p_2) - f_2(p_2)(p_2 - c_2))\right]. \tag{C-29}$$

In addition, in model N, based on the first order condition of $J_{\mathbf{N}}(p_1, p_2, x, y, t)$, the optimal posted price of product 2 satisfies the following identity (observe from (C-24)):

$$\overline{F}_2(p_{\mathbf{N2}}^*) - f_2(p_{\mathbf{N2}}^*)(p_{\mathbf{N2}}^* - c_2) = 0. \tag{C-30}$$

Using both (C-29) and (C-30), we can write:

$$\frac{dJ_{\mathbf{P}}^*(p_2, x, y, t)}{dp_2}\Big|_{p_2=p_{\mathbf{N2}}^*} = \lambda q\beta F_1(p_1^*(p_{\mathbf{N2}}^*))\overline{F}_2(p_{\mathbf{N2}}^*) \geq 0. \tag{C-31}$$

Therefore, $J_{\mathbf{P}}^*(p_2, x, y, t)$ is non-decreasing at $p_2 = p_{\mathbf{N2}}^*$. Note that since $J_{\mathbf{P}}^*(p_2, x, y, t)$ is strictly unimodal in p_2, the result follows that the optimizer of $J_{\mathbf{P}}^*(p_2, x, y, t)$, which is given by $p_{\mathbf{P2}}^*$, is at least as large as $p_{\mathbf{N2}}^*$, which concludes the proof.

Proof of Proposition 6

When F_i is uniform over $[0, b_i]$, $i = 1, 2$, we can solve the systems of equations based on the first order conditions of $J_s(p_1, p_2, x, y, t)$ for both models by substituting $F_i(x) = \frac{x}{b_i}$, $f_i(x) = \frac{1}{b_i}$, and $f'(x) = 0$ into (C-1) and (C-5). We can find that in model P, $p_{\mathbf{P1}}^*(x, y, t)$ and $p_{\mathbf{P2}}^*(x, y, t)$ are

$$\begin{aligned}
p_{\mathbf{P1}}^*(x, y, t) &= \frac{q^2(\beta-1)^2(b_2-c_2)^2}{4b_2(q\beta - 2q + 2)} \\
&\quad + \frac{q[2\beta((b_2-c_2)^2 + b_2(b_1+c_1)) - 2((b_2-c_2)^2 + 2b_2(b_1+c_1))] + ((b_2-c_2)^2 + 4b_2(b_1+c_1))}{4b_2(q\beta - 2q + 2)} \\
p_{\mathbf{P2}}^*(x, y, t) &= \frac{(1-q+q\beta)b_2 + (1-q)c_2}{q\beta - 2q + 2}.
\end{aligned} \tag{C-32}$$

In model N, $p^*_{\mathbf{N1}}(x,y,t)$ and $p^*_{\mathbf{N2}}(x,y,t)$ are given by

$$\begin{aligned} p^*_{\mathbf{N1}}(x,y,t) &= -\frac{4b_1b_2(q-q\beta-1)-b_2(1-q)(b_2-2c_2)-c_2^2(1-q)-4c_1b_2(1-q)}{4b_2(q\beta-2q+2)}, \\ p^*_{\mathbf{N2}}(x,y,t) &= \frac{b_2+c_2}{2}. \end{aligned} \tag{C-33}$$

Using (C-32) and (C-33) and taking the derivatives with respect to c_1 and c_2, respectively, we obtain:

$$\begin{aligned} \frac{d}{dc_1}(p^*_{\mathbf{P1}}(x,y,t)-p^*_{\mathbf{N1}}(x,y,t)) &= \frac{q\beta}{2(q\beta-2q+2)} \geq 0, \text{ and} \\ \frac{d}{dc_2}(p^*_{\mathbf{P2}}(x,y,t)-p^*_{\mathbf{N2}}(x,y,t)) &= -\frac{q\beta}{2(q\beta-2q+2)} \leq 0, \end{aligned}$$

where the inequalities come from $q \in (0,1)$ and $\beta \in (0,1)$.