Advance Selling with Freebies and Limited Production Capacity

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Abstract

We consider a two-period pricing model in which a seller offers freebies along with the product when making advance sales, and production is constrained by capacity. The seller can offer freebies to increase both market base and customer's valuation toward the product in advance. The customers strategically determine whether to purchase the product in advance and gain freebies when their valuation on the product is uncertain, or delay their purchase decision until the regular selling period. We characterize the optimal pricing, quality level of the freebie and production quantity decisions that maximize the expected profits of the seller over the two periods.

Keywords: Production; supply chain management; pricing; advance selling; freebies.

1. Introduction

Advance selling has become a prevalent sales tool for firms selling products and services. In recent years, several industries have utilized various means of adopting advance selling, such as in electronics, books, music, and entertainment. With the use of advance selling, offering freebies are more appealing in advance selling periods. For instance, Sony Singapore offered a SmartBand SWR 10 for free to customers who pre-ordered Xperia Z2 in late March 2014[1]. In 2016, people

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[1]https://thegeekybeng.wordpress.com/2014/03/25/sony-xperia-z2-pre-order-gets-real-with-dates-and-confirmed-freebies/

who purchased Samsung high-end smartphones such as Galaxy 7 and Galaxy S7 Edge can get a free Gear VR, a mobile virtual reality device developed by Samsung Electronics in collaboration with Oculus[2]. The offer of SmartBand or Gear VR in the pre-order period not only enhances the customer's expectation of purchasing the smartphones by Sony and Samsung, that raises individual utility toward the product, but also helps the seller attract more fashion chasers, which leads to the expansion of the smartphone market (Rudzki and Li, 2007, Banerjee, 2009 and references therein). These instances demonstrate the popularity of freebies when the firms adopt advance selling. Freebies can benefit both sellers and customers who purchase the products in advance. During the pre-order period, customers only see the physical appearance and specifications of the product (e.g., smartphones and other consumer electronics) through commercials or marketing campaigns. They are uncertain whether the product will fit their needs before they receive the product. With freebies, customers become more willing to purchase a product in advance because these freebies raise the utility of buying the product and partially mitigate the uncertainty inherent in placing a pre-order. Freebies also enable sellers to reinforce the exclusiveness of the product and attract more market segments (Montaner et al., 2011; Chu and Keh, 2006).

Freebies and other associated promotional efforts have a considerable effect on optimal pricing and production quantity decisions. To the best of our knowledge, existing literature does not fully cover decisions that consider the existence of freebies. This research intends to fill this gap by investigating the dynamics of advance selling situations and determining under which conditions a seller would be willing to offer the freebies in advance. We try to explore the reason why the seller offers the freebies in advance under production constraint or the seller simply regards the advance period as a marketing campaign for advertising. If the seller indeed offers freebies, how does the seller set the selling prices in both advance and regular selling periods, the quality level of the freebie, and the production quantity to fulfill the customer demand? Furthermore, how does the production capacity constraint play a role in the determination of the optimal decisions? The outcomes observed in this research can shed light on the potential use of freebies as a strategic instrument under advance selling.

In this study, we consider a two-period advance selling problem in which a seller offers a product with freebies during an advance selling period. During the advance selling period, the seller determines the selling price as well as the quality level of the freebie to attract customers to pre-order products in order to augment the market. Upon pre-ordering, all customers are uncertain

[2] http://www.pocket-lint.com/news/136700-get-a-free-gear-vr-with-your-samsung-galaxy-s7-or-s7-edge

of their individual valuations and make strategic moves based on assumptions of whether or not they should buy the product in advance or wait to make a purchase decision. During the regular selling period, the seller, faced with capacity constraints and an unsure market base, sets the selling price and makes the production quantity decision in response to incoming customers and those who did not buy in advance. For the seller, offering freebies in advance increases the valuation of each customer who is more likely to buy in this earlier stage if the cost of offering freebies is affordable. The offer of the freebies also expands the overall market base in advance. For each customer, the major tradeoff is whether to buy in advance with assurance of gaining the product, or wait until his valuation is realized, which could incur the risk of the supply running out.

We characterize the expected profits of the seller over the two periods and derive the optimal pricing, the quality level of the freebies, and production quantity decisions of the seller. Given that capacity is bounded above and the existence of a high or low level market base, the seller can choose either a high production quantity that will fulfill the demand of each incoming customer with the possibility of leftovers, or a low production quantity that will satisfy the minimum demand but could result in product shortage. The seller also sets the selling price of the regular selling period to balance the supply and demand. When determining the selling price in advance, we find a threshold selling price above which all customers will delay the purchase decision until the regular selling period, which is similar to the case where the seller uses the marketing advertising campaign to expand the market base in advance and only sells the product in the regular period. The seller can also set the selling price in advance equal to the threshold and sell the products with freebies to attract all customers to buy earlier, which guarantees a certain level of profit to the seller during the pre-order period. In this case, the seller will make aggressive pricing and quantity decisions in the regular selling period.

Our model also produces a number of interesting numerical results. We show that a larger capacity induces the seller to charge a higher selling price in advance that pushes all customers to the regular selling period. In this case, the seller does not use freebies as a weapon to stimulate early purchase in advance selling. Limited capacity, however, makes the offer of freebies more attractive to the seller because it not only mitigates the tension of not being able to fulfill the demand of each incoming customer in the regular selling period, but also realizes certain profits during the early stage. The market base also influences the decision of production quantity of the seller in the regular selling period. A high probability of ample market base in the regular selling period endows the seller with more confidence in placing a larger order quantity of products, and the benefit of possibly satisfying more customers outweighs the risk of leftover. The seller's optimal strategies are both affected by the additional value perceived by the customers when freebies are offered and the state of the market base during the advance selling period. When the quality level positively enhances the market base and the extra valuation of each customer, the seller adopts an aggressive strategy by setting a low selling price in advance and choosing a high level of production quantity. With early profit in hand, a seller can produce a higher quantity of products that will satisfy all potential customers in the regular selling period.

The remainder of this paper is organized as follows: Section 2 provides a survey of the relevant literature. Sections 3 and 4 outline our models and derive analytical results. We then describe the results of our numerical study in Section 5. The paper concludes with discussion in Section 6. All proofs are provided in the appendix.

2. Literature Review

Our study is closely related to literature on advance selling as a sales format, pricing strategies under advance selling, and offering freebies as tools. Xie and Shugan (2001) indicate that offering advance selling is beneficial to sellers when consumers are faced with uncertain valuation of products. Sellers can adopt price discount during pre-sales to compensate for the loss of utility (Dana, 1998; Shugan and Xie, 2000; Tang et al., 2004; and Xie and Shugan, 2001). Some sellers provide the option of a refund to eliminate valuation uncertainty and sellers are better off when a refund is offered (Xie and Gerstner, 2007). Another stream of research on valuation uncertainty addresses the provision of options in advance selling. Consumers who purchase an option have priority to choose whether or not to execute them. Gallego and Sahin (2010) show that offering options are more profitable for sellers than providing price discount in advance selling. Balseiro et al. (2011) consider a case in which selling both pre-sales tickets and option of purchasing the tickets for sports events. In addition to considering uncertain valuations of customers, Yu et al. (2014) investigate how the inventory level or production capacity during the period of advance selling affects the pricing decision of a seller. Su (2009) discusses the relationship between uncertain valuation and customer's behavior when a seller designs a refund policy. Zhao et al. (2012) consider consumer inertia under the setting of dynamic pricing and Ye and Sun (2016) focus on strategic consumers under newsvendor setting. Nasiry and Popescu (2012) study whether a seller should adopt advance selling and consider customers' regrets at having made pre-orders. Noparumpa et al. (2015) discuss the scenario under which winemakers face quality uncertainty of the wine and adopt advance selling to reduce quality risk and enhance flexible allocations.

Evidently, advance selling can provide sellers with information regarding demand when they are faced with uncertainty about the market. Boyaci and Özer (2010) study a case in which a manufacturer determines the time and duration of advance selling to enable the manufacturer to obtain demand information during capacity planning. Several studies have modeled inventory management in advance selling under the newsvendor setting (Zhao and Stecke, 2010; Prasad, 2011). Zhao and Stecke (2010) investigate the model with loss averse customers, whereas Prasad (2011) considers risk averse customers. Both studies show that a threshold of price exists in determining whether advance selling should be adopted. Kuthambalayan et al. (2015) study the supply-demand mismatch problem by using advance selling with price discount to update the demand forecast of selling season.

Among advance selling literature, three types of pricing strategies are commonly discussed and used in advance selling studies: dynamic pricing, price commitment, and pre-order price guarantees. In dynamic pricing, a seller sets a pre-order price in advance and resets the price during the regular selling period and customers are not certain whether the product will be cheaper (Zhao and Pang, 2011). For instance, Amazon launched Kindle 2 with a pre-order price of $350, but dropped the price to $299 when it came onto the market (Carnoy, 2009; Li and Zhang, 2013).[3]

Under price commitments, sellers announce both prices of a product for advance and regular selling at the beginning of the pre-order period. Customers can then determine in which period to purchase the product by observing both prices and comparing the utilities in both periods. Price commitment can certainly eliminate a customer's uncertainty over waiting for the next period to purchase a product because of price adjustment (Zhao and Pang, 2011; Zhao et al., 2012). In most advance selling cases with price commitment, the pre-order price of a product is lower than the regular selling price and thus, customers have incentives to make advance purchases. Aviv et al. (2009) study a case in which a seller (e.g., Land's End and Syms) simultaneously announces the regular selling price and reveals the discount price for the next period.

A pre-order price guarantee is a strategy wherein the seller promises to provide refunds to pre-order customers when the pre-order price is higher than the regular selling price. Price guarantees can remove the risk for customers who make an earlier purchase at a higher price, and provide the seller with a more aggressive pricing strategy that will not affect customers' willingness to buy

[3] http://www.cnet.com/news/amazon-drops-price-of-kindle-2-to-299/

in advance. Promoting earlier purchase helps sellers conduce more accurate demand forecasts and capacity planning as well as to reduce excess stock (Levin et al., 2007). Li et al. (2014) study how consumer valuation, market condition, and the consumers classification affect on a retailer's advance selling strategy with offering refund. Price guarantee also increases customer's satisfaction toward the seller. For instance, AT&T provided a price guarantee for customers who purchased iPhone 3GS one month before the release of iPhone 4.[4] However, Zhao and Pang (2011) indicate that price guarantees may hurt the revenue of sellers when the price has to be reduced later.

In addition to the price discounts, offering products with freebies or coupons is also widely used by sellers to promote pre-orders. Complimentary freebies and coupons are considered as non-monetary and monetary promotions, respectively. Banerjee (2009) discusses the effect of offering freebies and price discount on the market base. Palazon and Delgado-Ballester (2009) study the benefits to sellers of these two types of promotions. Bodur and Grohmann (2005) show that selling a product with freebies can positively influence customers' attitude toward the product, increase the frequency of buying the product, or stimulate the probability of purchase from potential consumers. According to PROMO magazine in 2006, promotion by offering complimentary freebies increased rapidly by 12.5% since 2001.[5] Banerjee (2009) also finds that offering free gifts as a means of promotion increased by more than 50% in India. Similarly, providing freebies can strengthen the uniqueness of a product and increase its value (Chu and Keh, 2006). Montaner et al. (2011) show that non-monetary promotion has not only become more prevalent in marketing, but also enhances brand awareness of a company. Their study likewise suggests that sellers are better off offering freebies when product quality is high. Khouja et al. (2013) also conclude that offering a free gift card is an effective strategy to price discounts at the end of selling season.

In advance selling, most research works consider either price discount (Dana, 1998; Nasiry and Popescu, 2012; Noparumpa et al., 2015; Prasad, 2011; Tang et al., 2004; Xie and Shugan, 2001; Zhao and Stecke, 2010), refund (Su, 2009; Xie and Gerstner, 2007) or both (Li et al., 2014) to eliminate the uncertainty toward the product. In our research, offering the freebies is the selling format we take into consideration in advance selling, which may append additional utility for customers and attract some potential customers not in the original market segment. Furthermore, capacity of the product is a major factor affecting the decision of advance selling. Most papers consider overall capacity among both advance selling and regular selling periods (Boyaci and Ozer, 2010; Dana,

[4]http://www.engadget.com/2010/06/10/atandt-offering-price-protection-or-iphone-4-swap-to-recent-ipho/

[5]http://www.chiefmarketer.com/special-reports-chief-marketer/higher-gear-01042006

1998; Gallego and Sahin, 2010; Nasiry and Popescu, 2012; Noparumpa et al., 2015; Su, 2009; Xie and Gerstner,2007; Xie and Shugan, 2001; Yu et al., 2014). In our setting, however, the capacity is only restrained in the regular selling period. Furthermore, unlike existing literature (Dana, 1998; Gallego and Sahin, 2010; Li et al., 2014; Nasiry and Popescu, 2012; Prasad, 2011; Su, 2009; Tang et al., 2004; Xie and Gerstner, 2007; Xie and Shugan, 2001; Yu et al., 2014; Zhao and Stecke, 2010) that considers that the market base (or demand function) is unaffected by advance selling, we investigate how the quality level of freebies affects the pricing decisions of the seller due to the augmented market base. Our goal is to examine the optimal strategies of the seller by advance selling with the offer of freebies.

3. Model Description

We consider a seller selling a product to end customers where the selling horizon is divided into two periods, advance selling period followed by regular selling period. Customers are strategic and decide when and whether to buy the product in the selling horizon to maximize their individual utility of purchase. To attract more sales, the seller offers a freebie during the advance selling period. This freebie not only enhances the market base in the advance selling period but also the valuation of each customer toward the product. Throughout the selling horizon, the seller determines the price in each period, the quality level of the freebie, and the quantity that will be offered during the regular selling period to maximize her total expected profit from the two periods. Our primary interest is to characterize the seller's optimal pricing and production decisions and to evaluate the impact of freebie in the multi-period advance selling setting. In the following, we describe the model in detail.

3.1 The Seller

Upon selling the product, the seller faces a unit production cost $c > 0$. During the advance selling period, customers who pre-order the product will obtain a freebie offered by the seller. In addition to determining the selling price $p_1 (> c)$, the seller also determines the quality level of the freebies, $e \in [0, \bar{e}]$, where $\bar{e}$ is some positive constant. We assume the quality level e is fully observable to all customers. The quality e will positively influence the market base during the advance selling period, denoted by $N_1(e)$, that is, $N_1(e)$ increases in e, but the marginal increase is decreasing and thus, $N_1(e)$ is an increasing concave function. The seller can choose a high level of e to expand the market base during the period of advance selling but will incur higher cost $c_g(e)$. We assume $c_g(e)$ is an increasing convex function and $c_g(e) = 0$ as $e = 0$.

In the regular selling period, both customers who did not buy in the previous period and another group of customers N_2 arrive. The size of the new cohort of customers N_2 can be of either high (H) or low (L) level. Let N_{2H} and N_{2L} be, respectively, the market base of high and low level with probability q and $1 - q$ where $q \in [0, 1]$. Faced with the customers, the seller offers the product solely and determines both selling price of product p_2 and production quantity in the regular selling period Q, before information regarding whether $N_2 = N_{2H}$ (with probability q) or $N_2 = N_{2L}$ (with probability $1-q$) is known. Note that quantity Q is constrained by the production capacity of the seller during the regular selling period $T > 0$. In our model setting, we assume the seller only incurs a capacity constraint T in the regular selling period to reflect the fact that all customers obtain the products at the end of the regular selling period and the seller can always satisfy all pre-order customers during the advance selling period since the seller has ample time to respond. During the regular selling period, however, the production quantity has been determined prior to the arrivals of the customers and thus the seller may not meet all market demand and excess demand will be lost.

3.2 The Customers

Faced with two purchase periods, customers strategically choose the alternative that will maximize their expected utility. During the advance selling period, customers are uncertain of their individual valuation v (Xie and Shugan, 2001, and Shugan and Xie, 2004). Therefore, all customers follow the same prior distribution with cumulative density function $F(\cdot)$, with finite support $[0, \bar{v}]$, and probability density function $f(\cdot)$. Let the expectation of customer's valuation be $\tilde{v} = \int_0^{\bar{v}} v f(v)dv$. This uncertainty of v will be resolved at the beginning of the second period. We model the customer's valuation in such a way to show that most customers only see the new smartphone and its specifications at the announcement campaign, but are not sure of their personal needs when they pre-order the phones. More examples can be found in the wine industry as well as among concerts and festivals (Yu et al., 2015). A customer's net utility of buying the product is $\tilde{v} + \delta(e) - p_1$ where p_1 is the selling price of the advance selling period and $\delta(e)$ is the customer's extra valuation of obtaining the freebie during the advance selling period and is an increasing function of the quality, e. During the regular selling period, each customer knows his individual valuation v and obtains net utility $v - p_2$ while buying the product. In both periods, a customer receives no utility in the case of no purchase.

The timing of events is as follows (Figure 1). During the advance selling period, the seller determines selling price p_1 and quality e. Then N_1 customers arrive and each customer decides whether to buy in the current period or wait until the regular selling period. In the regular selling period, each customer's private valuation is realized and the seller determines selling price p_2 and production quantity Q. N_2 (either N_{2H} or N_{2L}) customers arrive and the overall market base is the sum of N_2 and the customers who do not buy in advance. Customers then decide whether or not to buy based on each individual's private valuation. If the number of customers who buy exceeds the supply, each customer obtains the product with equal probability. In our two-period model setting, the tradeoff of each customer is to evaluate and choose to buy the product in either period so as to maximize individual utility. On the other hand, in addition to setting the prices in two periods, the seller determines whether it is worthwhile to offer freebies and if so, how to set the quality level of the freebie in advance.

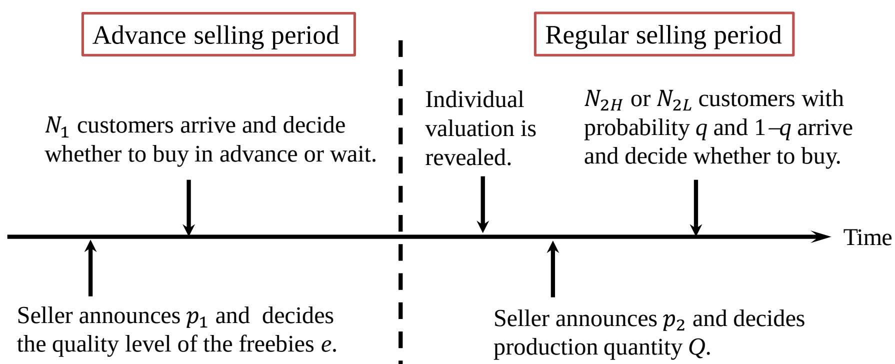

Figure 1: Sequence of events.

Throughout the paper, we assume that the cumulative density function of customers' valuations, $F(\cdot)$, has an increasing failure rate, i.e., $\frac{f(x)}{\bar{F}(x)}$ is increasing in x, and its density function $f(\cdot)$ is twice differentiable where $\bar{F}(x) = 1 - F(x)$. The assumption covers various distributions such as uniform, normal, exponential, Weibull, and gamma, as well as their truncated versions.

4. Formulation and Analysis

In this section, we formulate the problems of the sellers and customers, and discuss the dynamics in each period. Following backward induction, we start from the seller's problem in the regular selling period.

4.1 Regular Selling Period

Optimal production quantity decision We first consider the optimal production quantity decision of the seller, Q. In the regular selling period, the seller knows N_1 customers arrived during the previous period and let D_1 of them buy the product where $0 \leq D_1 \leq N_1$. That is, $N_1 - D_1$ from the previous period remains in the market. Another cohort of customers N_2 (either N_{2H} with probability q or N_{2L} with probability $1-q$) arrives. Among $N_1 - D_1 + N_2$ customers who hope to buy the product in the regular selling period, a customer will buy the product only if his private valuation v is larger than the selling price p_2. From the seller's point of view, the probability that a customer buys the product is $\bar{F}(p_2)$. Hence, the demand during the regular selling period D_2 can be defined as follows:

$$D_2 = \begin{cases} D_{2H} = N_H \bar{F}(p_2) = (N_1 - D_1 + N_{2H})\bar{F}(p_2), & \text{with probability } q \\ D_{2L} = N_L \bar{F}(p_2) = (N_1 - D_1 + N_{2L})\bar{F}(p_2), & \text{with probability } 1-q \end{cases} \tag{1}$$

Given selling price p_2 and capacity T, the seller determines the optimal production quantity Q^* to maximize her expected profit π_s, which is the expected revenue minus the production cost:

$$Q^* = \arg\max_{Q \leq T} \pi_s = \arg\max_{Q \leq T} \{p_2 E[\min\{Q, D_2, T\}] - c \min\{Q, T\}\} \tag{2}$$

Lemma 1. *During the regular selling period, if $D_{2H} < T$, the optimal production quantity Q^* can either be D_{2H} or D_{2L}. If $D_{2L} < T \leq D_{2H}$, the optimal production quantity Q^* can either be T or D_{2L}. If $T \leq D_{2L}$, the optimal production quantity is T.*

Optimal selling price decision We consider the seller's optimal selling price decision during the regular selling period. Based on Lemma 1, the seller's problem can be divided into three cases depending on the level of capacity T.

When $D_{2H} < T$, all demands can be satisfied in the regular selling period. If the seller's optimal quantity $Q^* = D_{2H}$, then seller's profit $\pi_{2H} = (qp_2 - c)D_{2H} + (1-q)p_2 D_{2L}$. If $Q^* = D_{2L}$, the seller's profit is $\pi_{2L} = (p_2 - c)D_{2L}$. Let $p^*_{2H} = \arg\max_{p_2}\{\pi_{2H}\}$ and $p^*_{2L} = \arg\max_{p_2}\{\pi_{2L}\}$. The seller compares $\pi_{2H}(p^*_{2H})$ and $\pi_{2L}(p^*_{2L})$, and chooses the one that yields the maximum profit.

Lemma 2. *When $D_{2H} < T$, if $c \geq \frac{N_1 - D_1 + N_{2L} + (N_{2H} - N_{2L})q}{N_1 - D_1 + N_{2H}}\bar{v}$, the seller sets the production quantity $Q^* = D_{2L}$ and the optimal selling price $p^*_2 = p^*_{2L}$ where p^*_{2L} solves $p^*_{2L} = \frac{\bar{F}(p^*_{2L})}{f(p^*_{2L})} + c$.*

When capacity T is high, a high product cost c induces a lower production quantity in the regular selling period. It is because the seller is uncertain about the demand in this period and being too optimistic on the demand may hurt the seller since the profit margin is squeezed by a higher production cost. Furthermore, a lower q enhances the result as the seller would conservatively determine the production quantity to be sold in the regular selling period.

On the other hand, when the capacity is limited, the optimal production quantity Q^* is constrained by the capacity and two cases are discussed. When $D_{2L} < T \le D_{2H}$, the seller considers whether to determine the production quantity at the level of T or D_{2L}, depending on whether the seller chooses a high or a low production quantity in the regular selling period. In the former case, the seller's profit is $\tilde{\pi}_{2H} = qp_2T + (1-q)p_2D_{2L} - cT$ and let $\tilde{p}^*_{2H}$ be the optimal selling price that maximizes $\tilde{\pi}_{2H}$. For the case where optimal quantity is set at D_{2L}, the seller's profit is equal to $\pi_{2L} = (p_2 - c)D_{2L}$, which is equivalent to the case where $D_{2H} < T$ and the seller's optimal production quantity is $Q^* = D_{2L}$. Moreover, p^*_{2L} is the optimal solution to π_{2L}. For $D_{2L} < T \le D_{2H}$, the seller also chooses the one that benefits her the most.

Finally, when $T \le D_{2L}$, the seller's optimal production quantity is T. In this case, the seller's profit is $\tilde{\pi}_2 = qp_2 \min\{T, D_{2H}\} + (1-q)p_2 \min\{T, D_{2L}\} - cT = (qp_2 + (1-q)p_2 - c)T$. Define $\tilde{p}^*_{2L}$ as the optimal price that maximizes $\tilde{\pi}_2$.

The following lemma summarizes the optimal selling price and production quantity decisions during the regular selling period.

Lemma 3. *With limited capacity, i.e., $T \le D_{2H}$, then*
(a) If $\max\{\frac{1-q}{q}(N_1 - D_1 + N_{2L})f(\bar{v})\bar{v}, D_{2L}\} < T \le D_{2H}$, the seller sets the production quantity $Q^* = D_{2L}$ and the optimal selling price $p^*_2 = p^*_{2L}$ where p^*_{2L} solves $p^*_{2L} = \frac{\bar{F}(p^*_{2L})}{f(p^*_{2L})} + c$, and
(b) If $T \le D_{2L}$, the seller sets the production quantity equal to the capacity, $Q^* = T$ and the optimal selling price $p^*_2 = \tilde{p}^*_{2L}$ where $\tilde{p}^*_{2L} = F^{-1}(1 - \frac{T}{N_1 - D_1 + N_{2L}})$.

4.2 Advance Selling Period

Optimal selling price decision Given the best responses in the regular selling period, in the advance selling period the seller determines both the selling price of the product and the quality level of the freebie. Before characterizing the optimal decisions of the seller, we first analyze the customer's best decision. In the advance selling period, all customers are uncertain of their individual valuation and hence, form the expected valuation when determining whether to buy the product in advance. Given selling price p_1, a customer's net utility of buying product, EU_A, is

$$EU_A = \tilde{v} + \delta - p_1 = \int_0^{\bar{v}} vf(v)dv + \delta - p_1. \quad (3)$$

If a customer waits until the regular selling period, his net utility EU_W depends on the selling price offered by the seller and the stock out risk of the product due to the capacity constraint. Based on the analysis in Section 4.1, the selling price during the second period is one of the four prices (i.e., $p^*_{2H}, p^*_{2L}, \tilde{p}^*_{2H}, \tilde{p}^*_{2L}$). If $EU_A \geq EU_W$ and $EU_A \geq 0$, the customer buys the product in advance, whereas if $EU_A < EU_W$ and $EU_W \geq 0$, the consumer buys in the regular selling period.[6]

Consider first the case where $p^*_2 = p^*_{2H}$. The expected utility during the regular selling period is $EU_W = \int_{p^*_2}^{\bar{v}} (v - p^*_2) f(v) dv$ and customers who can afford the product can always obtain it. Hence, a customer buys in advance if

$$EU_A - EU_W = \int_0^{\bar{v}} v f(v) dv + \delta - p_1 - \int_{p^*_2}^{\bar{v}} (v - p^*_2) f(v) dv = p^*_2(1 - F(p^*_2)) + \delta + \int_0^{p^*_2} v f(v) dv - p_1 \geq 0$$

or

$$p_1 \leq p^*_2(1 - F(p^*_2)) + \delta + \int_0^{p^*_2} v f(v) dv = p^U_1 \tag{4}$$

Equation (4) shows that if the selling price in advance is $p^*_1 > p^U_1$ (i.e., $EU_A < EU_W$), all the consumers wait until the second period and the demand in advance $D_1 = 0$; otherwise, consumers will buy in advance and $D_1 = N_1$. Similarly, when $p^*_2 = p^*_{2L}$, a consumer's utility when waiting is

$$EU_W = \frac{qN_L}{N_H} \int_{p^*_2}^{\bar{v}} (v - p^*_2) f(v) dv + (1 - q) \int_{p^*_2}^{\bar{v}} (v - p^*_2) f(v) dv,$$

and $EU_A - EU_W \geq 0$ yields

$$p_1 \leq p^U_1 = \delta + \int_0^{\bar{v}} v f(v) dv + (1 - q + \frac{qN_L}{N_H})(p^*_2 \bar{F}(p^*_2) - \int_{p^*_2}^{\bar{v}} v f(v) dv).$$

Note that EU_W also includes the stock-out risk faced by the customers if $N_2 = N_{2H}$ with probability q and thus the total demand in the regular period $D_2 = D_{2H}$. However, the seller's production quantity is only $Q = D_{2L}$. In this case, each affordable customer buys the product with probability $\frac{D_{2L}}{D_{2H}} = \frac{N_L}{N_H}$.

For cases where $p^*_2 = \tilde{p}^*_{2H}$ and $p^*_2 = \tilde{p}^*_{2L}$, both p^U_1's are equal to

$$\delta + \int_0^{\bar{v}} v f(v) dv + (1 - q + \frac{qT}{N_H \bar{F}(p^*_2)})(p^*_2 \bar{F}(p^*_2) - \int_{p^*_2}^{\bar{v}} v f(v) dv).$$

Based on above discussion, the seller has two options to determine the optimal selling price in advance: (1) set $p^*_1 > p^U_1$ and all consumers will wait until the regular selling period and (2) set $p^*_1 = p^U_1$ and all consumers will purchase during the advance selling period. Upon choosing (1), it

[6]We assume here if a customer is indifferent about buying in advance or in the regular selling period (i.e., $EU_A = EU_W$), he will buy the product in advance.

is similar to the case where the seller initiates a marketing advertising campaign, which extends the market base of the product. The seller, however, only sells the product in the regular period. Compared to (1), the seller adopts advance selling in (2) and uses freebies to not only extend the market base of the product but also increase the valuation of the product from each potential customer. For each option, there are eight possible scenarios considering that p_1^U also depends on the best responses of four p_2^*'s and two levels of production decision Q^* in the regular selling period. After removing trivial cases, the results are summarized in Table 1 .

Case	D_1	Q^*	p_2^*	p_1^U	p_1^*
(a.1)	0	$(N_1 + N_{2H})\bar{F}(p_{2,a1})$	$p_{2,a1}$	$p_{1,a1}$	$p_1^* > p_{1,a1}$
(a.2)	N_1	$N_{2H}\bar{F}(p_{2,a2})$	$p_{2,a2}$	$p_{1,a2}$	$p_1^* = p_{1,a2}$
(b.1)	0	$(N_1 + N_{2L})\bar{F}(p_{2,b1})$	$p_{2,b1}$	$p_{1,b1}$	$p_1^* > p_{1,b1}$
(b.2)	N_1	$N_{2L}\bar{F}(p_{2,b2})$	$p_{2,b2}$	$p_{1,b2}$	$p_1^* = p_{1,b2}$
(c.1)	0	T	$p_{2,c1}$	$p_{1,c1}$	$p_1^* > p_{1,c1}$
(c.2)	N_1	T	$p_{2,c2}$	$p_{1,c2}$	$p_1^* = p_{1,c2}$
(d.1)	0	T	$p_{2,d1}$	$p_{1,d1}$	$p_1^* > p_{1,d1}$
(d.2)	N_1	T	$p_{2,d2}$	$p_{1,d2}$	$p_1^* = p_{1,d2}$

Table 1: All eight possible results (All p_2^* and p_1^U are described in the appendix.)

To facilitate the analysis of the optimal decisions, we assume that the distribution of the customer's valuation follows a uniform distribution over $[0, \bar{v}]$. We have $F(x) = x/\bar{v}$ and $f(x) = 1/\bar{v}$. We first consider the effects of production cost c and the capacity T on the optimal selling price p_2^*.

Proposition 1. (*a*) *With ample capacity T (i.e., $Q^* < T$), the optimal selling price in the regular selling period increases in the product cost and is independent of T; that is $\frac{\partial p_{2,ij}}{\partial c} > 0$ and $\frac{\partial p_{2,ij}}{\partial T} = 0$ for $i = a, b$ and $j = 1, 2$.* (*b*) *With limited capacity T where Q^* is constrained by T (i.e., $Q^* = T$), the optimal selling price is independent of c but may increase or decrease in T; that is, $\frac{\partial p_{2,ij}}{\partial c} = 0$ for $i = c, d$ and $j = 1, 2$, $\frac{\partial p_{2,cj}}{\partial T} > 0$, $\frac{\partial p_{2,dj}}{\partial T} < 0$ for $j = 1, 2$.*

The decision of p_2^* depends on whether or not the capacity T is sufficiently high. If capacity T is high such that the seller's production quantity Q^* is not constrained by T, an increase in the production cost c leads to an increase in p_2^*[7]. This increase in the selling price helps the seller maintain a reasonable profit margin and is not affected by T. On the other hand, if production

[7]In this paper, we use "increasing"/"decreasing" in the weak sense.

quantity is constrained by T, p_2^* is independent of c. In this case, capacity T plays a major role that determines the change in the selling price, depending on whether the seller sets a high or low production quantity. If the seller chooses a high production quantity in the regular selling period ($Q^* = \min\{T, D_{2H}\}$), an increase in T induces an increase in selling price because the risk of leftover inventory increases and thus, the seller needs to raise the price and focus on profit margin. If the seller sets a low production quantity, an increase in T allows the seller to fulfill the minimum demand from customers more easily (i.e., $\min\{T, D_{2L}\}$). The seller simply reduces the selling price, enabling more customers to buy in the regular selling period.

Proposition 2. *Consider the effect of q on the selling price decision in the regular selling period,*
(a) If the seller chooses a high production quantity which is not constrained by T, the corresponding selling price in the regular selling period decreases in q, that is $\frac{\partial p_{2,aj}}{\partial q} < 0$ for $j = 1, 2$,
(b) If the seller chooses a high production quantity which is constrained by T, the corresponding selling price in the regular selling period increases in q, that is $\frac{\partial p_{2,cj}}{\partial q} > 0$ for $j = 1, 2$, and
(c) If the seller chooses a low production quantity, the corresponding selling price in the regular selling period is independent of q, that is $\frac{\partial p_{2,ij}}{\partial q} = 0$ for $i = b, d$ and $j = 1, 2$.

If the seller chooses a high production quantity, she faces the risk of not being able to sell all products if the realized market base is low. An increase in the probability q, however, represents a reduction of such risk. Given a high expected market base, the seller can reduce the price further to penetrate a larger portion of the market, which enhances the seller's profit during the regular selling period. Nevertheless, this price reduction strategy to attract more customers only works when T is sufficiently high. When the production quantity is constrained by T, the seller cannot adjust the selling price downward to induce more sales. Instead, charging a higher price and maintaining a better profit margin will be more beneficial because more high valuation customers are expected to arrive during the second period given the increase in q. Finally, if the seller is conservative and chooses a low production quantity, q does not affect the optimal selling price as the seller's main goal is to fulfill the minimum market base without any leftover.

Proposition 3. *Consider the optimal selling prices in the regular selling period, we have:*
(a) $p_{2,a1} > p_{2,b1}$, $p_{2,a2} > p_{2,b2}$,
(b) $p_{2,a2} > p_{2,a1}$, $p_{2,c2} > p_{2,c1}$, and
(c) $p_{2,b1} = p_{2,b2}$, and $p_{2,d1} > p_{2,d2}$.

Proposition 3(a) shows that when capacity is sufficiently high, the seller choosing a high production quantity will set a high selling price in the regular selling period compared with the case where she chooses a low quantity. When the seller chooses a high production quantity, she faces the uncertainty of not being able to sell all products. Hence, the seller would raise the price to protect the profit margin. Obviously, this price premium balances the risk that the seller faces, and this finding holds regardless of how the seller makes the price decision in advance (i.e., $p_1^* = p_1^U$ or $p_1^* > p_1^U$). Proposition 3(b) discusses how the selling price strategy in advance influences the selling price decision in the regular selling period when the production quantity is high. If the seller sets a high selling price in advance (i.e., $p_1^* > p_1^U$), all customers wait until the second period to make their decision. In this case, the seller does not realize any profit in advance and also needs to carry the risk of low market base because of choosing a high production quantity. The best strategy for the seller is to charge a lower price in the second period to boost the demand. On the other hand, if the selling price in advance is low (i.e., $p_1^* = p_1^U$), the seller will set a higher price in the regular selling period since some profits have been realized during the previous period, which can bear potential risk of leftover in the end.

The results are different if the production quantity decision is low. Proposition 3(c) concludes that if the selling price in advance is high ($p_1^* > p_1^U$), the seller sets a higher selling price in the regular selling period compared with when the selling price in advance is low. Notice that when production quantity is low, the seller knows that she can always sell out all production quantity. Setting a higher price in advance ($p_1^* > p_1^U$) pushes all customers to make purchase decisions in the regular selling period. The seller can then use a high selling price to target high valuation customers and realize higher profit. This price increase definitely improves the margin and overall profits of the seller. In cases where the selling price in advance is low ($p_1^* = p_1^U$), the seller's pricing strategy will be conservative since she already sold the products to a portion of customers in advance, and hence, the price in the second period will be set to match the supply with the demand.

In the following, we switch our focus to the optimal selling price in advance. Based on the earlier analysis, one price threshold p_1^U can be obtained above which all customers wait and make the purchase decision until the second period. Proposition 1 illustrates how the selling price in the regular selling period changes with production cost c and capacity T. We show that the selling price increases in c as long as the production quantity Q^* is not constrained, but is independent of c if otherwise. Furthermore, the change in selling prices in the regular selling period to the capacity T also depends on whether or not production quantity is capacitated. For the setting of the selling price in advance, we observe similar results and conclude with the following proposition.

Proposition 4. *The thresholds of the selling price in advance $p_{1,a1}$, $p_{1,a2}$, $p_{1,b1}$, and $p_{1,b2}$ increase with the production cost c and are independent of T, whereas the thresholds $p_{1,c1}$, $p_{1,c2}$, $p_{1,d1}$, and $p_{1,d2}$ are independent of c and $p_{1,d1}$, and $p_{1,d2}$ decrease in T.*

Our paper focuses mainly on how the freebies offered by sellers in advance influence the customers' purchase behaviors. Offering freebies raises the valuation of the customer, which helps the seller penetrate into more market segments in the earlier selling period. From the customer's point of view, buying the product with offered freebies mitigates the risk of not obtaining the product in the regular selling period. At the same time, customers enjoy higher valuation with purchase in advance. The following proposition discusses the effect of offering freebies in advance and shows that a premium selling price in advance is possible.

Proposition 5. *The optimal selling price in advance may be higher than the selling price in the regular selling period.*

Conventional wisdom suggests that if the seller offers the product in two periods and customers are uncertain of their valuations during the first period, then the selling price during the first period is lower than that in the second period. The price reduction in the first period compensates for the potential risk that the customer's realized valuation would be too low. This conclusion may change when the model setting of advance selling differs depending on capacity constraint and customer valuation in the regular selling period. For example, Xie and Shugan (2001) shows that premium advance price is possible, in particular, when capacity is ample and marginal cost is not too high. Yu et al. (2014), on the other hand, obtain a different result in their ∞-Group model, in which a premium regular selling price leads to a balance between capacity and demand and this result holds when demand is highly predictable.

In our model, upon advance selling, the seller also offers freebies to both boost the demand and raise the extra valuation of the customers who buy the product in advance. Our results show that there exists a threshold below which all customers buy the product. The magnitude of extra valuation $\delta(e)$ affected by the quality e significantly determines whether the seller should adopt a premium selling price strategy in advance. The driving factors that influence the difference between two selling prices are the uncertainties of the customers in advance, risk of stock-out, and the capacity constraint in the regular period. If these negative effects can be compensated by the offer of freebies, which raises the valuation of each customer in advance, then the seller can aggressively raise the selling price in advance to gather earlier profit; otherwise, the offer of freebies does not benefit the seller too much and the result is consistent with the intuition that a lower advance price is set. One extreme case in our model is that if the offer of freebies does not enhance any extra valuation (i.e., $\delta = 0$) to each customer, then a premium advance selling price is never optimal. In fact, we observe a number of practical examples in which the seller adopts different selling price strategies for the two periods when freebies are offered in advance. For example, mobile phone manufacturers provide a microSD card or a USB Type-C cable with the new mobile phone in advance and charge the same price during both periods[8]. On the other hand, record companies often offer a new single, extended play (EP), or other accessories when new albums are released. They charge a premium in advance for diehard fans, but the prices are reduced in the regular selling period[9]. If a seller smartly adopts different pricing strategies for the two periods, she can maintain the margin by offering low-cost freebies without reducing the selling prices during advance selling.

Optimal quality decision In addition to the selling price in advance, the decision of quality level e in advance also has impact on the dynamics of strategies and the customer purchase behavior. The seller can set a higher quality level of the freebies to boost the demand in advance and enhance extra valuation of the customers with the offer of freebies as long as such benefit can cover the associated cost. In the following, we characterize the optimal quality level of the freebies for each scenario. First, the cases where the seller sets $p_1^* = p_1^U$ are considered. In these four cases, all customers buy the product in advance (i.e., $D_1 = N_1$), and hence, the seller's expected profit over two periods is

$$\Pi = \pi_1 + \pi_2^* = (p_1^* - c)N_1(e) - c_g(e) + \pi_2^* \tag{5}$$

where π_2^* represents the seller's optimal profit during the regular period in each case. Since $N_1(e)$ is an increasing concave function and $c_g(e)$ is an increasing convex function in e, the first order condition of equation (5) leads to the optimal quality level for each scenario. Let $e_{i2}, i = a, b, c, d$ be the respective solution to the first order condition. The optimal quality level e^* for case $(i.2)$ is $e^* = \min\{\bar{e}, e_{i2}\}$ where $\bar{e}$ is the upper bound of the quality.

In order to obtain the closed-form of the optimal quality, we further assume the cost of offering

[8] Source: http://www.androidauthority.com/htc-10-thank-gift-now-arriving-customers-pre-ordered-698111/ and http://www.androidauthority.com/t-mobile-shipping-online-orders-lg-g4-may-27-free-128gb-card-early-buyers-611365/.

[9] Jay Chou, a popular musician and singer in Asia, released his album "Jay Chou's Bedtime Stories" in June 2016. The pre-order deluxe edition comes in 3D storybook packaging with an eye mask and is sold at the price \$47.59. The regular edition is sold at the price \$20.59. Source: http://www.yesasia.com/us/jay-chous-bedtime-stories-preorder-deluxe-edition/1050336724-0-0-0-en/info.html.

the freebies $c_g(e) = ke^2$ where $k > 0$ and the market base in advance $N_1(e) = N + \beta e$ where N is the market base when setting $e = 0$ and $\beta \geq 0$ is the augmented coefficient, representing how the market base depends on the quality level. Furthermore, $\delta(e) = \beta_1 e$ where $\beta_1 \geq 0$ is the augmented coefficient that represents how the extra valuation is related to the quality level[10]. The following proposition gives the optimal quality decision and illustrates how k and β affect the optimal quality in advance when $p_1^* = p_1^U$.

Proposition 6. *For the case where the selling price in advance $p_1^* = p_1^U$, if $\beta\beta_1 < k$, the optimal quality level, $e_{i2}^* = \min\{e_{i2}, \bar{e}\}$ where $e_{i2} = \frac{\beta_1 N + (p_{1,i2} - c)\beta}{2k - \beta\beta_1}$ and $p_{1,i2}$ is the optimal selling price in advance for $i = a, b, c, d$. Furthermore, e^* decreases in k, increases in β and in β_1; that is $\frac{\partial e_{i2}^*}{\partial k} < 0$, $\frac{\partial e_{i2}^*}{\partial \beta} > 0$, and $\frac{\partial e_{i2}^*}{\partial \beta_1} > 0$.*

First, consider the effect of k. If the seller chooses $p_1^* = p_1^U$, all customers buy in advance, and the quality level e selected by the seller determines the market base and the extra valuation of the customer for the current period. The seller intends to choose a higher e to enlarge the market base and extra valuation unless the associated cost $c_g(e) = ke^2$ is too high. However, higher k costs the seller more if the same quality level has been chosen, which negatively eliminates the effect of increasing market base and extra valuation. Therefore, the optimal quality is decreasing in k. The effects of β and β_1 are the opposite compared to the effect of k. With this decision, all customers buy in advance and higher β and β_1 obviously strengthen this effect that provides dependable revenue to the seller in advance: a larger market base and a higher valuation toward the product due to the freebie. That is why we find all optimal quality levels, $e_{i2}, i = a, b, c, d$, increase in both β and β_1.

If the selling price in advance is $p_1^* > p_1^U$, all customers will delay their purchase decision until the second period. Depending on the capacity constraint and uncertainty of the demand base during the regular selling period ($N_H = N_1(e) + N_{2H}$ and $N_L = N_1(e) + N_{2L}$), we can also solve for the seller's expected profit over two periods and obtain the optimal quality. Following the same logic of the case where $p_1^* = p_1^U$, the optimal quality e^* for case $(i.1)$ is $e^* = \min\{\bar{e}, \tilde{e}_{i1}, e_{i1}\}$ where $\tilde{e}_{i1}$ is the solution such that the production constraint is satisfied and e_{i1} is the solution to the first-order condition of the seller's profit from two periods, for $i = a, b, c, d$[11]. For $\tilde{e}_{i1}$ (or e_{i1}), in

[10] One may consider the effect of increasing market base to be an endogenous function of consumer utility by simply setting $\beta = 0$ in our model.

[11] Note that $\tilde{e}_{a1}$ solves $(N_1(\tilde{e}_{a1}) + N_{2H})\bar{F}(p_{2,a1}) = T$, $\tilde{e}_{b1}$ solves $(N_1(\tilde{e}_{b1}) + N_{2L})\bar{F}(p_{2,b1}) = T$, $\tilde{e}_{c1}$ solves $D_{2L} = (N_1(\tilde{e}_{c1}) + N_{2L})\bar{F}(p_{2,c1}) = T$. For $\tilde{e}_{d1}$, since the production constraint is automatically satisfied and thus $\tilde{e}_{d1}$ can be

the case of multiple optima, we define $\tilde{e}_{i1}$ (or e_{i1}) to be the smallest.

The effects of model dynamics on the optimal quality for the case of $p_1^* > p_1^U$ are complex but some interesting results can be found compared to the case where $p_1^* = p_1^U$, in particular, the effect of β on the optimal quality. Given that all customers are pushed to the regular selling period, the seller does not need to set the quality level so high to enlarge the market size in advance considering capacity constraint T. If more customers want to buy in the second period, a portion of the customers cannot obtain the product due to the capacity. Hence, higher β does not necessarily induce a higher quality as can be observed earlier. Note from Figure 2 that the optimal quality increases first in β and then decreases accordingly. When β is low, the seller can raise the quality to expand the market base without considering capacity T. As β becomes larger, the seller sets the quality so that the demand in the second period is equal to the capacity. That is why we observe nonmonotonicity pattern of the optimal quality in β. Furthermore, since all customers will buy in the second period, the seller has no incentive to invest in the quality of the freebies to increase extra valuation of customers in advance and hence, the quality is independent of β_1 and decreases in k. Table 2 summarizes the results of optimal decisions over two periods for $\tilde{e}_{i1}$ and e_{ij} where $i = a, b, c, d$ and $j = 1, 2$.

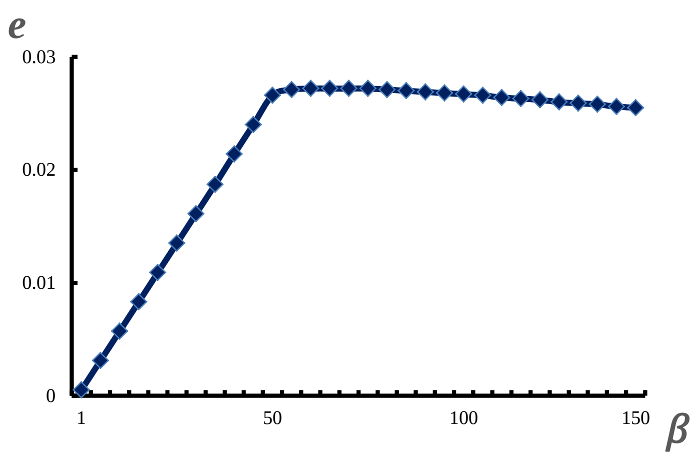

Figure 2: Effect of parameter β on the optimal quality e. Here $q = 0.65, k = 50, \beta_1 = 0.25, c = 0.5, T = 0.8, N_1 = 1, N_{2H} = 1.2$, and $N_{2L} = 0.8$.

Given the complexity of the analysis, in the following proposition, we consider there does not exist capacity constraint, i.e., T is infinitely large and assume $k = \beta = N_{2L} = \bar{v} = 1$ and $c = 0$.

any $e \in [0, \bar{e}]$. Note also that the second-order condition for e_{i1} is satisfied if k is large enough; otherwise e_{i1} does not exist.

Case	D_1	Production	Q^*	p_2^*	p_1^U	p_1^*	e^*	Production Constraint
(a.1)	0	High	$(N_1+N_{2H})\bar{F}(p_{2,a1})$	$p_{2,a1}$	$p_{1,11}$	$p_1^*>p_1^U$	$\min\{\bar{e},e_{a1},\tilde{e}_{a1}\}$	$D_{2H}\le T$
(a.2)	N_1	High	$N_{2H}\bar{F}(p_{2,a2})$	$p_{2,a2}$	$p_{1,a2}$	$p_1^*=p_1^U$	$\min\{\bar{e},e_{a2}\}$	$D_{2H}\le T$
(b.1)	0	Low	$(N_1+N_{2L})\bar{F}(p_{2,b1})$	$p_{2,b1}$	$p_{1,b1}$	$p_1^*>p_1^U$	$\min\{\bar{e},e_{b1},\tilde{e}_{b1}\}$	$D_{2L}\le T$
(b.2)	N_1	Low	$N_{2L}\bar{F}(p_{2,b2})$	$p_{2,b2}$	$p_{1,b2}$	$p_1^*=p_1^U$	$\min\{\bar{e},e_{b2}\}$	$D_{2L}\le T$
(c.1)	0	High	T	$p_{2,c1}$	$p_{1,c1}$	$p_1^*>p_1^U$	$\min\{\bar{e},e_{c1},\tilde{e}_{c1}\}$	$D_{2L}\le T<D_{2H}$
(c.2)	N_1	High	T	$p_{2,c2}$	$p_{1,c2}$	$p_1^*=p_1^U$	$\min\{\bar{e},e_{c2}\}$	$D_{2L}\le T<D_{2H}$
(d.1)	0	Low	T	$p_{2,d1}$	$p_{1,d1}$	$p_1^*>p_1^U$	$\min\{\bar{e},e_{d1},\tilde{e}_{d1}\}$	$D_{2L}>T$
(d.2)	N_1	Low	T	$p_{2,d2}$	$p_{1,d2}$	$p_1^*=p_1^U$	$\min\{\bar{e},e_{d2}\}$	$D_{2L}>T$

Note that $D_{2H}=(N_1-D_1+N_{2H})\bar{F}(p_2^*)$, $D_{2L}=(N_1-D_1+N_{2L})\bar{F}(p_2^*)$.

Table 2: Optimal decisions over two periods

Also, we consider $\bar{e}$ is high enough and will not be the constraint of the seller's quality decision. Define

$$
\begin{aligned}
A1 &= \frac{16N^2(\beta_1+1)+1+4N}{64}-\frac{(8N+3)^2}{256(1-\beta_1)},\\
A2 &= \frac{(N_{2H}(3+q+8N)-q)^2}{256N_{2H}^2(\beta_1-1)}+\frac{16N^2N_{2H}(1+\beta_1)+16N_{2H}q(N_{2H}-1)+4NN_{2H}(1-q)+N_{2H}+4Nq}{64N_{2H}},\\
A3 &= 6N_{2H}-q+N_{2H}q+16NN_{2H}-16N_{2H}(\beta_1-1)(N-4N_{2H}).
\end{aligned}
$$

We provide the seller's optimal decisions under different conditions in the following proposition.

Proposition 7. *The seller adopts no advance selling with high production quantity (i.e., $p_1^*>p_1^U$ and $Q^*=D_{2H}$) if $\max\{A1,A2\}\ge 0$; adopts advance selling with high production quantity (i.e., $p_1^*=p_1^U$ and $Q^*=D_{2H}$) if $A1<0$ and $A3>0$; adopts advance selling with low production quantity (i.e., $p_1^*=p_1^U$ and $Q^*=D_{2L}$) if $\min\{A2,A3\}<0$. Furthermore, the seller never adopts no advance selling with low production quantity.*

The proposition shows the optimal strategies of the seller regarding whether to adopt advance selling and to choose a high or low production quantity under specific assumptions. If the unit production cost is negligible ($c=0$), the seller sets the quality to a certain level ($e^*=1/8$) under no advance selling. Furthermore, choosing a high production quantity benefits the seller as the seller can use such quantity to fulfill potential high demand without any cost. Compared to the case where the seller adopts advance selling, a low β_1 induces positive $A1$ and $A2$ and thus the optimal strategy for the seller is to adopt no advance selling since offering freebies does not help the seller enhance the valuation of each customer significantly. The seller simply regards the offer of freebies as a marketing campaign and sells the product in the spot market. With an increase of β_1, the effect of freebies on the customer's preference to buy in the advance period is obvious. The magnitude of N_{2H} also plays a pivotal role that determines the seller's production quantity decision. One can expect a high N_{2H} will induce the seller to choose a high production quantity and vice versa.

Based on Proposition 7, we further consider the case where offering freebies does not add additional utility in the advance selling period, i.e., $\beta_1 = 0$, to reflect the fact that the freebies offered by the seller are unrelated items that do not make the product more valuable to the customers. In the following corollary[12], we posit that advance selling always makes the seller better off.

Corollary 1. *The seller always adopts advance selling. Furthermore, when N_{2H} is relatively high (low) compared to N, the seller set a high (low) production quantity.*

Upon adopting advance selling, the seller is able to sell the products at two different prices in two periods. This price differentiation effect is particularly effective when the seller is able to offer the freebies to expand the market base of advance selling period. The best pricing strategy for the seller is to charge a lower price in advance selling when customers are uncertain their valuations and to raise the price in spot when valuations are realized (i.e., $p_1^* < p_2^*$ in cases (a.2) and (b.2)). In addition, market bases N and N_{2H} determine the production quantity in the second period. One can expect that the seller sets a high production quantity with a high N_{2H} relative to N. On the other hand, if N_{2H} is low (close to N_{2L}), the seller focuses more on advance selling period and sets a low quantity in the second period. Notice that the above result is not restricted to the case where $\beta_1 = 0$. Consider the case where offering freebies does not enhance the market base in advance selling $i.e., \beta = 0$. Under this setting, Corollary 1 also applies. In other words, offering freebies strengthens the use of advance selling as the seller benefits from either market expansion (positive β) or additional utility (positive β_1) or both.

Notice that the assumption of no unit production cost (i.e., $c = 0$) significantly drives the result. As we can observe that with no cost of production, the seller can provide as many products as possible and adopt advance selling for market differentiation (charging two different prices in two periods) without worrying any leftover inventory. However, if such cost is incurred, this benefit may

[12]We follow the same assumptions as in Proposition 7: T is infinitely large, $k = \beta = N_{2L} = \bar{v} = 1$ and $c = 0$. Also, $\bar{e}$ is high.

be overcome by the risk of excess inventory and the reduction of profit margin. Under this case, the seller may consider no advance selling. More general cases will be analyzed in the following section.

5. Numerical Study

We conduct a numerical study to gain further managerial insights regarding how the profit of the seller is affected by model characteristics such as production cost c, capacity constraint T, probability of market base being high level q, cost parameter of freebies k and augmented coefficients β and β_1. We consider different sets of parameter combinations including c, k, β, β_1, N_{2L}, and N_{2H}. Depending on whether the seller charges a price above or equal to p_1^U and sets a high or low production quantity, we define four optimal strategies of the seller:

- **Strategy A** (No advance selling and high production quantity): *high* selling price in advance $p_1^* > p_1^U$ and *high* production quantity in the regular selling period $Q^* = \min\{D_{2H}, T\}$.
- **Strategy B** (No advance selling and low production quantity): *high* selling price in advance $p_1^* > p_1^U$ and *low* production quantity in the regular selling period $Q^* = \min\{D_{2L}, T\}$.
- **Strategy C** (Advance selling and high production quantity): *low* selling price in advance $p_1^* = p_1^U$ and *high* production quantity in the regular selling period $Q^* = \min\{D_{2H}, T\}$.
- **Strategy D** (Advance selling and low production quantity): *low* selling price in advance $p_1^* = p_1^U$ and *low* production quantity in the regular selling period $Q^* = \min\{D_{2L}, T\}$.

Figure 3 shows the effects of capacity T and the probability of market base being high level q on the optimal strategies of the seller. Note that the seller makes a production quantity decision before the market base in the regular selling period is realized. Faced with market base uncertainty, the seller can choose high production quantity (e.g., $Q^* = \min\{D_{2H}, T\}$) or low one (e.g., $Q^* = \min\{D_{2L}, T\}$). If the seller is too optimistic about the market demand and chooses the former, then the leftovers could possibly erode profits. When the probability q increases, the seller expects a higher chance that the market base in the regular selling period is high and she is more likely to sell sufficient quantity and realize maximum profit. Hence, with a high probability q, the seller chooses a high production quantity (i.e., Strategies A and C); otherwise, the seller is conservative and she chooses Strategies B and D for low production quantity.

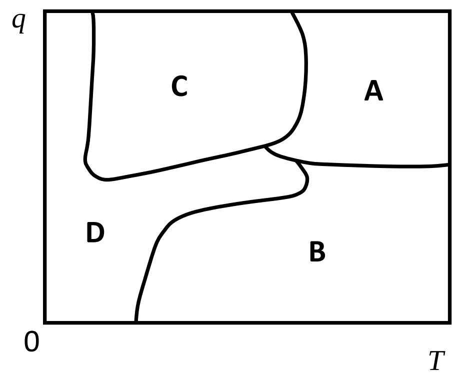

Figure 3: Effects of capacity T and the probability of market base being a high level q on the optimal strategies of the seller. Here $c = 0.5, k = 0.4, \beta = 0.155, \beta_1 = 0.3, N = 1, N_{2H} = 1.2$, and $N_{2L} = 0.8$.

On the other hand, capacity T affects the selling price decision of the seller in advance. If the seller possesses higher capacity in the regular selling period, she does not worry about whether the number of products in the regular selling period can accommodate all buying customers. In this sense, the seller has no incentive to offer freebies and sell the product in advance. Hence, the seller sets a selling price above the threshold (p_1^U) to push all customers to the second period, which is why we observe that the seller adopts Strategies A and B. This circumstance is similar to the case where the seller uses the freebies to expand the market base and sells the product in the second period only given ample capacity. When capacity is limited, the seller's optimal strategy is to set the selling price $p_1^* = p_1^U$ to enhance all customers in advance to buy (i.e., $D_1 = N_1$) and the offer of freebies benefits the seller.

Figure 4 shows how production cost c influences the optimal strategies of the seller. Notice that when production cost c is high, the seller's profit margin over two periods is reduced, and thus, the seller tends to raise the selling price to maintain a reasonable margin. Figure 4 indicates that a high c induces the setting of the selling price in advance higher than p_1^U and all customers will determine whether to buy during the regular selling period (i.e., no advance selling). High production cost also leads to low production quantity decision during the regular selling period because the seller could incur potential profit loss if she is too optimistic of the market base in the second period. Therefore, Strategy B is the best strategy for the seller given high cost c. Moreover, when the cost is sufficiently low, the seller will focus more on augmenting the market share. In this case, the

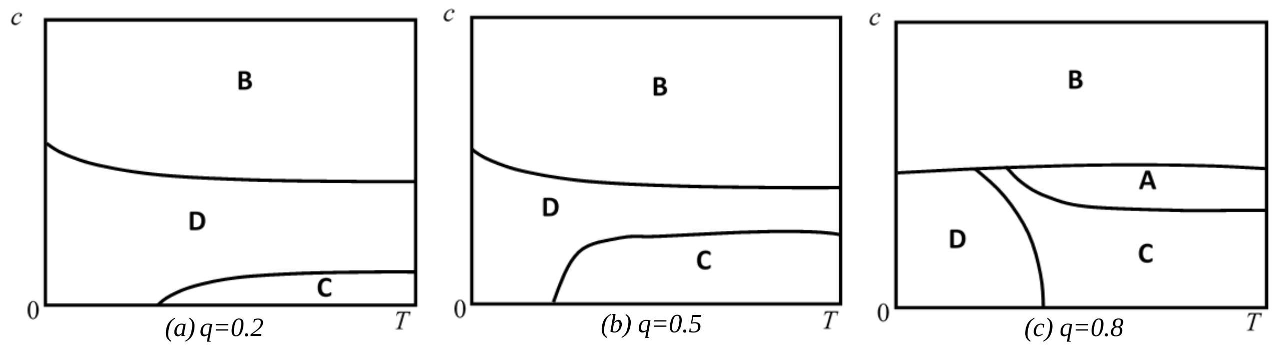

Figure 4: Effects of capacity T and the cost c on the optimal strategies of the seller. Here $k = 0.3, \beta = 0.25, \beta_1 = 0.25, N = 1, N_{2H} = 1.2$, and $N_{2L} = 0.8$.

advance selling price will remain at a relatively low level so that customers who arrive in advance will buy (i.e., Strategies C or D). The price and production quantity decisions in the regular selling period are not solely influenced by the production cost because the seller has already realized some profits in advance. Depending on capacity T and probability q, the seller may choose to have a high (Strategy C) or a low (Strategy D) production quantity to respond to the uncertainty of the market base in the second period. As the probability q increases (Figure 4 (b) and (c)), the effect of production cost is mitigated and the seller aggressively increases the production to accommodate potential demand from both periods.

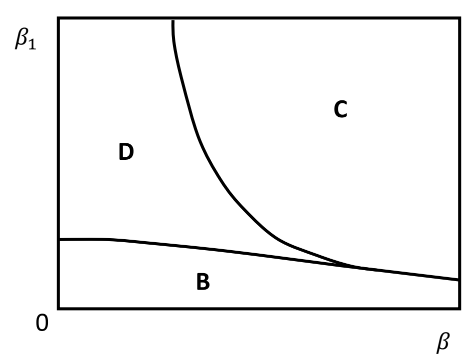

Figure 5: Effects of the parameters β_1 and β on the optimal strategies of the seller. Here $q = 0.5, c = 0.4, k = 0.3, T = 0.8, N = 1, N_{2H} = 1.2$, and $N_{2L} = 0.8$.

Finally, we analyze how the offer of freebies in advance influences the seller's optimal strategies. We focus mainly on the effects of how the additional valuation that a customer obtains from freebies in advance depends on the effect level β_1, how the market base depends on the quality level β, and the corresponding cost parameter of offering freebies k. Figure 5 shows the effects of β_1 and β on the seller's optimal strategies. Note that during the advance selling period, all customers are uncertain of their valuations, and hence, they may wait and make purchase decisions during the regular selling period. The offer of freebies in advance enhances the valuation of the customers, inducing them to buy particularly when β_1 is sufficiently high. Furthermore, a high β augments the market base in advance, enabling the seller to realize significant profits when all customers buy. In other words, when both β_1 and β are high, the seller's best strategy is to sell in advance (by setting the selling price $p_1^* = p_1^U$) and all customers choose to pre-order. As the seller has already realized profits in advance (partially due to the large market size from high β), she can take a risk of setting a high production quantity aggressively, although doing which may negatively result in leftover in the second period (Strategy C).

In addition, when both β_1 and β are modest, the seller will not be as aggressive since the profit earned in advance may not cover the leftover in the regular selling period. The seller will still implement advance selling but set a low production quantity to sell the minimum quantity (Strategy D). When both parameters are low, the offer of freebies does not provide enough benefit to the seller and thus the optimal strategy is to set a high selling price in advance and push all customers to make decisions while their valuations are certain. It is like the seller adopts a marketing advertising campaign and use the freebies to enlarge the market base and the regular selling period is the only period to sell the product. The seller also sets a conservative production quantity in the regular selling period (Strategy B). On the other hand, the effect of the cost parameter k is different from that of β or β_1. As the high cost of offering freebies diminishes the effects of inducing customers to buy during the advance period, we observe from Figure 6 that for a given β_1 or β, the increase in k allows the seller to switch from maintaining a high (Strategy C) to a low production quantity (Strategy D) and even to no advance selling (Strategy B).

Remark 1. Similar to Section 4, we also investigate the circumstance under which the offer of the freebies does not have any impact on the market base expansion (i.e., $\beta = 0$) or additional utility of each customer in the advance period (i.e., $\beta_1 = 0$). Based on our numerical results, we find the patterns of strategies basically apply; however, the seller is more likely to adopt no advance selling (Strategies A and B) when $\beta = 0$ or $\beta_1 = 0$. Since offering freebies does not provider significant benefit to the seller either from market expansion or additional valuation of the product, the seller would simply regard advance selling as a marketing campaign and sell the products to all customers

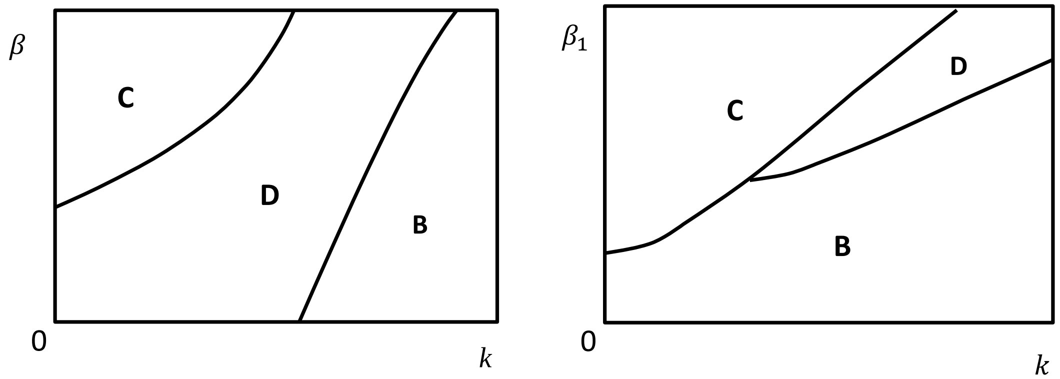

Figure 6: Effects of parameters β, β_1, and k on the optimal strategies of the seller. Here for the left panel, $q = 0.5, \beta_1 = 0.25, c = 0.4, T = 0.5, N = 1, N_{2H} = 1.2$, and $N_{2L} = 0.8$. For the right panel, $q = 0.5, \beta = 0.8, c = 0.5, T = 0.8, N = 1, N_{2H} = 1.2$, and $N_{2L} = 0.8$.

in the regular selling period.

Remark 2. Our results show that the seller is most likely to adopt advance selling when capacity is limited and to adopt no advance selling when capacity is ample. Xie and Shugan (2001) also considers capacity constraint in their model setting and the conclusion is the opposite to ours. The main difference between Xie and Shugan (2001) and ours is that they assume the overall capacity constraint on two periods but we consider this constraint is only on the regular selling period. With limited overall capacity, the seller in Xie and Shugan (2001) can simply sell in spot without worrying about any leftover inventory. In this case, the seller does not use advance selling. With the increase in capacity, this effect diminishes and the seller can adopt advance selling and better price differentiate customers in two periods. In our setting, however, limited capacity in spot incentivizes the seller to adopt advance selling and offering freebies further strengthens such advantage. When the capacity increases, each affordable customer can always purchase and the freebies are viewed as a marketing tool to help push all customers to the spot market (i.e., no advance selling). Therefore, one can expect advance selling is the optimal strategy of the seller when capacity is low in our setting but is never optimal in Xie and Shugan (2001).

6. Conclusion

In this paper, we analyze how the pricing and production quantity strategies of a seller offering freebies in advance selling are influenced by the capacity and additional valuation that freebies provide to customers. The seller offers a product, and the selling horizon is divided into two selling periods, advance and regular. Upon offering freebies in advance, the seller can effectively enhance each customer's valuation toward the product and induce early purchase. Depending on the capacity constraint in the regular selling period and the valuation of freebies enhanced by the customers, the seller can flexibly adjust prices during two periods to maximize expected profits. Customers uncertain of their valuations in advance need to determine strategically whether to buy the product during the early period or to wait until their individual valuation is realized.

We characterize the expected profits of the seller over two periods and optimal pricing and production quantity decisions. Our results show that a threshold selling price exists for advance selling. The seller can set the advance selling price either above the threshold, which pushes all customers to make a purchase decision in the regular selling period, or equal to the threshold so that all customers who arrive will buy the product. The case with advance selling price above the threshold is similar to the scenario where the seller uses marketing advertising to expand the market base and sells the product in the regular selling period only. In the other case, however, the seller offers freebies and follows traditional advance selling strategy. In the regular selling period, the seller determines the selling price and production quantity prior to the realization of the market base. The major tradeoff is balancing the number of incoming customers and leftover inventory given the exogenous capacity that limits the number of products can be sold. Our analysis illustrates how model characteristics, such as production cost, capacity, and probability that the market base in the regular selling period is high, affect the optimal decisions of the seller in both periods.

In our numerical study, the seller adopts one of the four strategies by combining an advance selling price higher than or equal to the threshold price with high or low production quantity of the regular selling period. When the seller has ample capacity, she does not need to be concerned with stockout and thus, the seller tends to set the advance selling price higher to push customers to the regular selling period. In this case, the effect of offering freebies to attract customers is negligible. On the other hand, if the capacity is limited, advance selling benefits the seller more because it ensures all customers to make early purchase, thereby guaranteeing early realized profits. For a high probability of the ample market base in the regular selling period, the seller adopts a more aggressive production strategy without worrying about unsold inventory. Finally, considering the valuation offered by freebies and the effect of market base expansion with efforts exerted by the seller, we find that when both effects are intense, the seller can earn more profit with a low selling price to induce purchase in advance.

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Online Supplement

A. Proofs of Results in Sections 4 and 5

Proofs of Lemma 1

To determine the optimal production quantity, we first remove the capacity constraint T and consider three cases, including (1) $D_{2H} \leq Q$, (2) $D_{2L} \leq Q \leq D_{2H}$, and (3) $Q \leq D_{2L}$.

(1) $D_{2H} \leq Q$

If the production quantity is set above D_{2H}, then the profit of the seller in equation (2) is $\pi_s = qp_2 \min\{Q, D_{2H}\} + (1-q)p_2 \min\{Q, D_{2L}\} - cQ = qp_2 D_{2H} + (1-q)p_2 D_{2L} - cQ$, which decreases in Q. Thus, the optimal production Q^* is D_{2H}.

(2) $D_{2L} \leq Q \leq D_{2H}$

When the production quantity is between D_{2L} and D_{2H}, the seller's profit is $\pi_s = qp_2 \min\{Q, D_{2H}\} + (1-q)p_2 \min\{Q, D_{2L}\} - cQ = (1-q)p_2 D_{2L} + (qp_2 - c)Q$. The first derivative of π_s with respect to Q shows that the optimal quantity $Q^* = D_{2H}$ if $qp_2 - c \geq 0$ and $Q^* = D_{2L}$ otherwise.

(3) $Q \leq D_{2L}$

If the quantity Q is less than D_{2L}, then $\pi_s = qp_2 Q + (1-q)p_2 Q - cQ = (p_2 - c)Q$, which increases in Q. Therefore, the optimal production quantity $Q^* = D_{2L}$.

Combining the three cases, the seller's optimal production quantity Q^* is either D_{2H} or D_{2L}. To determine the optimal production quantity, we observe that when $D_{2H} \leq Q$, the optimal production $Q^* = D_{2H}$. When $D_{2L} \leq Q \leq D_{2H}$, then Q^* can be D_{2L} or D_{2H}. When $Q \leq D_{2L}$, $Q^* = D_{2L}$. In other words, the optimal production quantity Q^* is either D_{2L} or D_{2H}. When the capacity T is included, the result directly follows.

Proofs of Lemma 2

When $D_{2H} < T$, the seller's optimal production quantity Q^* could be either D_{2H} or D_{2L}. First, we consider the case where the seller chooses $Q^* = D_{2H}$. Since $v \in [0, \bar{v}]$ and $F(0) = 0$, $F(\bar{v}) = 1$, we only need to consider $p_2 \in [0, \bar{v}]$. Taking the derivative of the profit π_{2H} with respect to p_2, we have

$$\frac{\partial \pi_{2H}}{\partial p_2} = (N_1 - D_1 + N_{2H}) f(p_2)[q(\frac{\bar{F}(p_2)}{f(p_2)} - p_2) + c] + (1-q)(N_1 - D_1 + N_{2L}) f(p_2)(\frac{\bar{F}(p_2)}{f(p_2)} - p_2). \quad \text{(A-1)}$$

Evaluating at $p_2 = 0$ and $p_2 = \bar{v}$, we have

$$\frac{\partial \pi_{2H}}{\partial p_2}|_{p_2=0} = (N_1 - D_1 + N_{2H})f(0)(q\frac{1}{f(0)} + c) + (1-q)(N_1 - D_1 + N_{2L})f(0)\frac{1}{f(0)} \quad \text{(A-2)}$$

$$\frac{\partial \pi_{2H}}{\partial p_2}|_{p_2=\bar{v}} = (N_1 - D_1 + N_{2H})f(\bar{v})(q(-\bar{v}) + c) + (1-q)(N_1 - D_1 + N_{2L})f(\bar{v})(-\bar{v}) \quad \text{(A-3)}$$

It is obvious that $\frac{\partial \pi_{2H}}{\partial p_2}|_{p_2=0}$ is greater than zero. Since v is IFR and $\frac{\bar{F}(x)}{f(x)} - x$ is strictly decreasing, π_{2H} is also strictly decreasing in p_2. If $c \geq \frac{N_1-D_1+N_{2L}+(N_{2H}-N_{2L})q}{N_1-D_1+N_{2H}}\bar{v}$, then we have $\frac{\partial \pi_{2H}}{\partial p_{2H}}|_{p_2=\bar{v}} \geq 0$ and the optimal solution $p^*_{2H} = \bar{v}$. If $c < \frac{N_1-D_1+N_{2L}+(N_{2H}-N_{2L})q}{N_1-D_1+N_{2H}}\bar{v}$, then $\frac{\partial \pi_{2H}}{\partial p_2}|_{p_2=\bar{v}} < 0$ and solving FOC, we obtain $p^*_{2H} = \frac{\bar{F}(p^*_{2H})}{f(p^*_{2H})} + \frac{cN_H}{qN_H+(1-q)N_L}$, which gives the optimal price.

Same logic applies to the case where the seller chooses $Q^* = D_{2L}$. Taking the derivative of π_{2L} with respect to p_2, we have

$$\frac{\partial \pi_{2L}}{\partial p_2} = (N_1 + N_2 - D_1)(\bar{F}(p_2) + (p_2 - c)(-f(p_2)))$$
$$= (N_1 + N_2 - D_1)f(p_2)(\frac{\bar{F}(p_2)}{f(p_2)} - p_2 + c)$$

Evaluating at $p_2 = 0$ and $p_2 = \bar{v}$, we have

$$\frac{\partial \pi_{2L}}{\partial p_2}|_{p_2=0} = (N_1 + N_2 - D_1)(1 - (0-c)f(0)) > 0,$$
$$\frac{\partial \pi_{2L}}{\partial p_2}|_{p_2=\bar{v}} = (N_1 + N_2 - D_1)(0 - (\bar{v}-c)f(\bar{v})) < 0$$

Since $\frac{\bar{F}(x)}{f(x)} - x$ is strictly decreasing, solving the FOC leads to the unique optimal solution, $p^*_{2L} = \frac{\bar{F}(p_{2L})}{f(p_{2L})} + c$.

The seller compares (p^*_{2H}, D_{2H}) and (p^*_{2L}, D_{2L}) and chooses the one which gives her the maximum profit. If $c \geq \frac{N_1-D_1+N_{2L}+(N_{2H}-N_{2L})q}{N_1-D_1+N_{2H}}\bar{v}$, $\pi_{2H} = 0$ with the selling price in the regular selling period $p^*_{2H} = \bar{v}$, while $\pi_{2L} > 0$ with the selling price in the regular selling period $p^*_{2L} = \frac{\bar{F}(p_{2L})}{f(p_{2L})} + c$. Therefore, the seller will choose $(p^*_2, Q^*) = (p^*_{2L}, D_{2L})$. Otherwise, the seller just chooses the one which benefits her the most.

Proofs of Lemma 3

We divide the analysis into two cases: (a) $D_{2L} < T \leq D_{2H}$ and (b) $D_{2L} \leq T$.

(a) When $D_{2L} < T \leq D_{2H}$, the seller decides the production quantity at the level of T or D_{2L}. Note that if the seller chooses to produce D_{2L}, the profit is equal to π_{2L} and the optimal selling price in the regular selling period is p^*_{2L}. If the seller chooses to produce T, then the profit is $\tilde{\pi}_{2H} = qp_2T + (1-q)p_2D_{2L} - cT$. Taking the derivative of $\tilde{\pi}_{2H}$ with respect to p_2, we have

$$\frac{\partial \tilde{\pi}_{2H}}{\partial p_2} = qT + (1-q)(N_1 - D_1 + N_{2L})f(p_2)(\frac{\bar{F}(p_2)}{f(p_2)} - p_2). \quad \text{(A-4)}$$

Evaluating at $p_2 = 0$ and $p_2 = \bar{v}$, we have

$$\frac{\partial \tilde{\pi}_{2H}}{\partial p_2}|_{p_2=0} = qT + (1-q)(N_1 - D_1 + N_{2L})f(0)\frac{1}{f(0)} > 0, \quad \text{(A-5)}$$

$$\frac{\partial \tilde{\pi}_{2H}}{\partial p_2}|_{p_2=\bar{v}} = qT + (1-q)(N_1 - D_1 + N_{2L})f(\bar{v})(-\bar{v}). \quad \text{(A-6)}$$

Since v is IFR, $\frac{\bar{F}(x)}{f(x)} - x$ is strictly decreasing. If the capacity constraint $T \geq \frac{1-q}{q}(N_1 - D_1 + N_{2L})f(\bar{v})\bar{v}$, then $\frac{\partial \tilde{\pi}_{2H}}{\partial p_2}|_{p_2=\bar{v}} \geq 0$ and the optimal selling price in the regular selling period $\tilde{p}^*_{2H} = \bar{v}$. If $T < \frac{1-q}{q}(N_1 - D_1 + N_{2L})f(\bar{v})\bar{v}$, then $\frac{\partial \tilde{\pi}_{2H}}{\partial p_2}|_{p_2=\bar{v}} < 0$. Thus, the optimal selling price in the regular selling period $\tilde{p}^*_{2H}$ is the solution of FOC, $\tilde{p}^*_{2H} = \frac{\bar{F}(\tilde{p}^*_{2H})}{f(\tilde{p}^*_{2H})} + \frac{qT}{(1-q)(N_1-D_1+N_{2L})f(\tilde{p}^*_{2H})}$. Hence, if $\max\{\frac{1-q}{q}(N_1 - D_1 + N_{2L})f(\bar{v})\bar{v}, D_{2L}\} < T \leq D_{2H}$, the seller sets a low production quantity $Q^* = D_{2L}$ and the optimal regular selling price $p^*_2 = p^*_{2L}$.

Similarly, the seller compares $(\tilde{p}^*_{2H}, T)$ and (p^*_{2L}, D_{2L}) and chooses the one which gives her the maximum profit. It is not difficult to show that the sellers chooses $(p^*_2, Q^*) = (p^*_{2L}, D_{2L})$ if $T \geq \frac{1-q}{q}(N_1 - D_1 + N_{2L})f(\bar{v})\bar{v}$, since the profit of the seller by setting the production quantity equal to T is zero. Otherwise, the seller chooses the pair that results in the maximum profit.

(b) For the case that $D_{2L} \geq T$, all the production quantity $Q^* = T$ would be sold out. Therefore, it is optimal for the seller to set the selling price in the regular selling period that makes the demand $D_{2L} = (N_1 - D_1 + N_{2L})\bar{F}(p_2) = T$. That is, $\tilde{p}^*_{2L} = F^{-1}(1 - \frac{T}{N_1-D_1+N_{2L}})$.

All p^*_2 and p^U_1 solutions for Table 1.

$$p_{2,a1} = \frac{\bar{F}(p_{2,a1})}{f(p_{2,a1})} + \frac{c(N_1 + N_{2H})}{N_1 + qN_{2H} + (1-q)N_{2L}} \quad \text{(A-7)}$$

$$p_{1,a1} = \delta + p_{2,a1}\bar{F}(p_{2,a1}) + \int_0^{p_{2,a1}} vf(v)dv \quad \text{(A-8)}$$

$$p_{2,a2} = \frac{\bar{F}(p_{2.a2})}{f(p_{2,a2})} + \frac{c(N_{2H})}{qN_{2H} + (1-q)N_{2L}} \quad \text{(A-9)}$$

$$p_{1,a2} = \delta + p_{2,a2}\bar{F}(p_{2,a2}) + \int_0^{p_{2,a1}} vf(v)dv \quad \text{(A-10)}$$

$$p_{2,b1} = \frac{\bar{F}(p_{2,b1})}{f(p_{2,b1})} + c \quad \text{(A-11)}$$

$$p_{1,b1} = \delta + \int_0^{\bar{v}} vf(v)dv + (1 - q + \frac{q(N_1 + N_{2L})}{N_1 + N_{2H}})(p_{2,b1}\bar{F}(p_{2,b1}) - \int_{p_{2,b1}}^{\bar{v}} vf(v)dv) \quad \text{(A-12)}$$

$$p_{2,b2} = \frac{\bar{F}(p_{2,b2})}{f(p_{2,b2})} + c \tag{A-13}$$

$$p_{1,b2} = \delta + \int_0^{\bar{v}} vf(v)dv + (1 - q + \frac{qN_{2L}}{N_{2H}})(p_{2,b2}\bar{F}(p_{2,b2}) - \int_{p_{2,b2}}^{\bar{v}} vf(v)dv) \tag{A-14}$$

$$p_{2,c1} = \frac{\bar{F}(p_{2,c1})}{f(p_{2,c1})} + \frac{qT}{(1-q)(N_1 + N_{2L})f(p_{2,c1})} \tag{A-15}$$

$$\begin{aligned} p_{1,c1} = {} & \delta + \int_0^{\bar{v}} vf(v)dv \\ & + (1 - q + \frac{qT}{(N_1 + N_{2H})\bar{F}(p_{2,c1})})(p_{2,c1}\bar{F}(p_{2,c1}) - \int_{p_{2,c1}}^{\bar{v}} vf(v)dv) \end{aligned} \tag{A-16}$$

$$p_{2,c2} = \frac{\bar{F}(p_{2,c2})}{f(p_{2,c2})} + \frac{qT}{(1-q)N_{2L}f(p_{2,c2})} \tag{A-17}$$

$$\begin{aligned} p_{1,c2} = {} & \delta + \int_0^{\bar{v}} vf(v)dv \\ & + (1 - q + \frac{qT}{N_{2H}\bar{F}(p_{2,c2})})(p_{2,c2}\bar{F}(p_{2,c2}) - \int_{p_{2,c2}}^{\bar{v}} vf(v)dv) \end{aligned} \tag{A-18}$$

$$p_{2,d1} = F^{-1}(1 - \frac{T}{N_1 + N_{2L}}) \tag{A-19}$$

$$\begin{aligned} p_{1,d1} = {} & \delta + \int_0^{\bar{v}} vf(v)dv \\ & + (1 - q + \frac{qT}{(N_1 + N_{2H})\bar{F}(p_{2,d1})})(p_{2,d1}\bar{F}(p_{2,d1}) - \int_{p_{2,d1}}^{\bar{v}} vf(v)dv) \end{aligned} \tag{A-20}$$

$$p_{2,d2} = F^{-1}(1 - \frac{T}{N_{2L}}) \tag{A-21}$$

$$p_{1,d2} = \delta + \int_0^{\bar{v}} vf(v)dv + (1 - q + \frac{qT}{N_{2H}\bar{F}(p_{2,d2})})(p_{2,d2}\bar{F}(p_{2,d2}) - \int_{p_{2,d2}}^{\bar{v}} vf(v)dv). \tag{A-22}$$

Proofs of Proposition 1

Taking the derivative of each optimal price in the regular selling period with respect to c, we have

$$\begin{aligned} \frac{\partial p_{2,a1}}{\partial c} &= \frac{N_1 + N_{2H}}{N_1 + qN_{2H} + (1-q)N_{2L}} > 0, \quad \frac{\partial p_{2,a2}}{\partial c} = \frac{cN_{2H}}{qN_{2H} + (1-q)N_{2L}} > 0, \\ \frac{\partial p_{2,b1}}{\partial c} &= \frac{1}{2} > 0, \quad \frac{\partial p_{2,b2}}{\partial c} = \frac{1}{2} > 0, \end{aligned}$$

and it is not difficult to show that $\frac{\partial p_{2,a1}}{\partial T} = \frac{\partial p_{2,a2}}{\partial T} = \frac{\partial p_{2,b1}}{\partial T} = \frac{\partial p_{2,d1}}{\partial T} = 0$. Also, we have

$$\frac{\partial p_{2,c1}}{\partial c} = \frac{\partial p_{2,c2}}{\partial c} = \frac{\partial p_{2,d1}}{\partial c} = \frac{\partial p_{2,c1}}{\partial c} = 0,$$

and

$$\frac{\partial p_{2,c1}}{\partial T} = \frac{q\bar{v}}{2(1-q)(N_1+N_{2L})} > 0, \quad \frac{\partial p_{2,c2}}{\partial T} = \frac{q\bar{v}}{2(1-q)N_{2L}} > 0,$$
$$\frac{\partial p_{2,d1}}{\partial T} = \frac{-\bar{v}}{N_1+N_{2L}} < 0, \quad \frac{\partial p_{2,d2}}{\partial T} = \frac{-\bar{v}}{N_{2L}} < 0.$$

Proof of Proposition 2

(a) If the seller chooses a high production quantity which is not constrained by T, the corresponding selling price in the regular selling period is either $p_{2,a1}$ or $p_{2,a2}$. Taking the derivative of each price with respect to q, we have

$$\frac{p_{2,a1}}{\partial q} = \frac{c(N_{2L}-N_{2H})}{2(N_1+qN_{2H}+(1-q)N_{2L})^2} < 0 \quad \text{and} \quad \frac{\partial p_{2,a2}}{\partial q} = \frac{c(N_{2L}-N_{2H})}{2(qN_{2H}+(1-q)N_{2L})^2} < 0.$$

Thus, the selling price in the regular selling period decreases in q.

(b) If the seller chooses a high production quantity which is constrained by T, the corresponding selling in the regular selling period is either $p_{2,c1}$ or $p_{2,c2}$. Take the derivative of each price with respect to q and have

$$\frac{\partial p_{2,c1}}{\partial q} = \frac{\bar{v}T}{2(1-q)^2(N_1+N_{2L})} > 0 \quad \text{and} \quad \frac{\partial p_{2,c2}}{\partial q} = \frac{\bar{v}T}{2(1-q)^2N_{2L}} > 0.$$

Thus, in this case the selling price in the regular selling period increases in q.

(c) If the seller chooses a low production quantity, the selling price in the regular selling period could be $p_{2,b1}$, $p_{2,b2}$, $p_{2,d1}$ or $p_{2,d2}$. Taking the derivative of each price with respect to q, we have $\frac{\partial p_{2,b1}}{\partial q} = \frac{\partial p_{2,b2}}{\partial q} = \frac{\partial p_{2,d1}}{\partial q} = \frac{\partial p_{2,d2}}{\partial q} = 0$. Thus, the selling price in the regular selling period with a low production quantity decision is independent of q.

Proofs of Proposition 3

Note the optimal selling prices and threshold selling prices are shown in equations (A-7) to (A-22).

(a) Consider the case that the capacity is high enough. If the selling price in advance p_1^* is above p_1^U, the selling price in the regular selling period will be either $p_{2,a1}$ or $p_{2,b1}$, depending on the seller's production quantity. Compare these two prices and we have

$$p_{2,a1} - p_{2,b1} = \frac{c}{2}\left(\frac{N_1+N_{2H}}{N_1+qN_{2H}+(1-q)N_{2L}} - 1\right) > 0.$$

On the other hand, if the selling price in advance p_1^* is equal to p_1^U, the selling price in the regular selling period will be wither $p_{2,a2}$ or $p_{2,b2}$, also depending on the production quantity. Compare these two prices and we have

$$p_{2,a2} - p_{2,b2} = \frac{c}{2}\left(\frac{N_{2H}}{qN_{2H}+(1-q)N_{2L}} - 1\right) > 0.$$

(b) Now consider the case that the seller chooses a high production quantity. We compare the selling prices in the regular selling period:

$$\begin{aligned} p_{2,a2} - p_{2,a1} &= \frac{c}{2}\left(\frac{N_1(N_{2H} - \mu_2)}{(N_1 + \mu_2)\mu_2}\right) > 0 \\ p_{2,c2} - p_{2,c1} &= \frac{q\bar{v}T}{2(1-q)N_{2L}} - \frac{q\bar{v}T}{2(1-q)(N_1 + N_{2L})} > 0, \end{aligned}$$

where $\mu_2 = qN_{2H} + (1-q)N_{2L}$.

(c) If the seller chooses a low production quantity which is not constrained by T, it is not difficult to show that $p_{2,b1} = p_{2,b2}$. However, if the low production quantity is constrained by T, we have

$$p_{2,d1} - p_{2,d2} = \frac{\bar{v}TN_1}{(N_1 + N_{2L})N_1} > 0$$

Proofs of Proposition 4

For the case where the capacity T is high enough such that the production quantity is not constrained, the optimal selling price can be $p_{1,a1}, p_{1,a2}, p_{1,b1}$, or $p_{1,b1}$. Taking the derivative of the thresholds of the selling price in advance with respect to c, we have

$$\begin{aligned} \frac{\partial p_{1,a1}}{\partial c} &= \frac{(\bar{v}(N_1 + qN_{2H} + (1-q)N_{2L}) - c(N_1 + N_{2H}))(N_1 + N_{2H})}{4\bar{v}(N_1 + qN_{2H} + (1-q)N_{2L})^2} > 0 \\ \frac{\partial p_{1,a2}}{\partial c} &= \frac{(\bar{v}(qN_{2H} + (1-q)N_{2L}) - cN_{2H})N_{2H}}{4\bar{v}(qN_{2H} + (1-q)N_{2L})^2} > 0 \\ \frac{\partial p_{1,b1}}{\partial c} &= \frac{2(\bar{v} - c)(N_1 + (1-q)N_{2H} + qN_{2L})}{8\bar{v}(N_1 + N_{2H})} > 0 \\ \frac{\partial p_{1,b2}}{\partial c} &= \frac{2(\bar{v} - c)((1-q)N_{2H} + qN_{2L})}{8\bar{v}N_{2H}} > 0. \end{aligned}$$

From Lemma 2, we learned that there is a constraint, $c < (1-q)\frac{N_1 - D_1 + N_{2L}}{N_1 - D_1 + N_{2H}}\bar{v} + q\bar{v}$, for cases (a.1) and (a.2). Therefore, $\bar{v}(N_1 + qN_{2H} + (1-q)N_{2L}) - c(N_1 + N_{2H} > 0$ and $\bar{v}(qN_{2H} + (1-q)N_{2L}) - cN_{2H} > 0$.

Similarly, for the case that the production quantity is constrained by capacity T, we obtain the optimal selling price can be $p_{1,c1}, p_{1,c2}, p_{1,d1}$, or $p_{1,d2}$. Take the derivative of the thresholds of the selling price in advance with respect to c and we have $\frac{\partial p_{1,c1}}{\partial c} = \frac{\partial p_{1,c2}}{\partial c} = \frac{\partial p_{1,d1}}{\partial c} = \frac{\partial p_{1,d2}}{\partial c} = 0$.

Now take the derivative of the thresholds with respect to T. It is not difficult to show that $\frac{\partial p_{1,a1}}{\partial T} = \frac{\partial p_{1,a2}}{\partial T} = \frac{\partial p_{1,b1}}{\partial T} = \frac{\partial p_{1,b2}}{\partial T} = 0$. Also we can obtain

$$\begin{aligned} \frac{\partial p_{1,d1}}{\partial T} &= -\frac{\bar{v}T(N_1 + qN_{2L} + (1-q)N_{2H})}{(N_1 + N_{2H})(N_1 + N_{2L})^2} < 0 \\ \frac{\partial p_{1,d2}}{\partial T} &= -\frac{\bar{v}T(qN_{2L} + (1-q)N_{2H})}{N_{2H}N_{2L}^2} < 0. \end{aligned}$$

Proofs of Proposition 5

Note the optimal selling prices and threshold selling prices are shown in equations (A-7) to (A-22). Let $p'_{1,i}$ be the threshold of the selling price in advance without freebies, $i = a1, a2, b1, b2, c1, c2, d1, d2$. In the following we compare the threshold of the selling price in advance without freebies (p'_1) and the selling price in the regular selling period (p^*_2):

$$
\begin{aligned}
p_{2,a1} - p'_{1,a1} &= \frac{(c(N_1 + N_{2H}) - \bar{v}(N_1 + qN_{2H} + (1-q)N_{2L}))^2}{8\bar{v}(N_1 + qN_{2H} + (1-q)N_{2L})^2} - \delta \\
p_{2,a2} - p'_{1,a2} &= \frac{(cN_{2H} - \bar{v}(qN_{2H} + (1-q)N_{2L}))^2}{8\bar{v}(qN_{2H} + (1-q)N_{2L})^2} - \delta \\
p_{2,b1} - p'_{1,b1} &= \frac{c}{2} + \frac{(\bar{v} - c)^2(N_1 + (1-q)N_{2H} + qN_{2L})}{8\bar{v}(N_1 + N_{2H})} - \delta \\
p_{2,b2} - p'_{1,b2} &= \frac{c}{2} + \frac{(\bar{v} - c)^2((1-q)N_{2H} + qN_{2L})}{8\bar{v}N_{2H}} - \delta \\
p_{2,c1} - p'_{1,c1} &= \frac{q\bar{v}T}{2(1-q)(N_1 + N_{2L})} + \frac{\bar{v}(qT - (1-q)(N_1 + N_{2L}))^2}{8(1-q)(N_1 + N_{2L})^2} \\
&\quad - \frac{\bar{v}(qT - (1-q)(N_1 + N_{2L}))qT}{4(1-q)(N_1 + N_{2H})(N_1 + N_{2L})} - \delta \\
p_{2,c2} - p'_{1,c2} &= \frac{q\bar{v}T}{2(1-q)N_{2L}} + \frac{\bar{v}(qT - (1-q)N_{2L})^2}{8(1-q)N_{2L}^2} - \frac{\bar{v}(qT - (1-q)N_{2L})qT}{4(1-q)N_{2H}N_{2L}} - \delta \\
p_{2,d1} - p'_{1,d1} &= \frac{\bar{v}}{2}(1 - \frac{T}{N_1 + N_{2L}} \frac{N_1 + qN_{2L} + (1-q)N_{2H}T}{(N_1 + N_{2L})(N_1 + N_{2L})} \\
&\quad + \frac{T}{N_1 + N_{2L}} \frac{2(N_1 + N_{2H})(N_1 + N_{2L})}{(N_1 + N_{2L})(N_1 + N_{2L})}) - \delta \\
p_{2,d2} - p'_{1,d2} &= \frac{\bar{v}}{2}\left(1 - \frac{(qN_{2L} + (1-q)N_{2H})T^2 - 2N_{2H}N_{2L}T}{N_{2L}N_{2L}^2}\right) - \delta
\end{aligned}
$$

Note that the magnitude of δ significantly influences the sign of above differences. In other words, if δ is too low, $p_{2,c1} - p'_{1,c1}$ may be negative. If $\delta = 0$, it is not difficult to show that all above differences, i.e., $p_2 - p'_1$ are all non-negative (Note that the results of $p_{2,c1} - p'_{1,c1} > 0$ and $p_{2,c1} - p'_{1,c1} > 0$ are due to the fact that the seller would choose the selling price in the regular selling period as $p_{2,c1}$ or $p_{2,c2}$ only if $T < \frac{(1-q)N_{2L}}{q}$ (Lemma 3)). In other words, if the extra valuation $\delta = 0$, a premium advance selling price is never optimal.

Proofs of Proposition 6

With the assumption that the cost of offering the freebies $c_g(e) = ke^2$, the market base in advance $N_1(e) = N + \beta e$ and extra valuation $\delta(e) = \beta_1 e$, based on the first-order condition, we have the solution of the quality level e_{i2}, $i = a, b, c, d$ given that the second-order condition is satisfied from the fact that $\beta\beta_1 < k$. We obtain $e_{i2} = \frac{\beta_1 N + (p_{1,i2} - c)\beta}{2k - \beta\beta_1}$ where $i = a, b, c, d$.

For the case where the selling price in advance $p^*_1 > p^U_1$, taking the derivative of the corresponding quality with respect to β, k, and β_1, we have $\frac{\partial e_{i2}}{\partial \beta} > 0$, $\frac{\partial e_{i2}}{\partial k} < 0$, and $\frac{\partial e_{i2}}{\partial \beta_1} > 0$ for $i = a, b, c, d$. Since

$e_{i2}^* = \min\{e_{i2}, \bar{e}\}$ and $\bar{e}$ is a constant and the results directly follow.

Proofs of Proposition 7

Without capacity constraint, we focus on the cases (a.1), (a.2), (b.1), and (b.2). Assume further $k = N_{2L} = \beta = \bar{v} = 1$, $c = 0$, and let

$$\begin{aligned} A1 &= 16(N + 4\bar{e} + 4N^2(1+\beta_1))(1-\beta_1) - (8N+3)^2, \\ A2 &= \frac{(N_{2H}(3+q+8N)-q)^2}{256N_{2H}^2(\beta_1-1)} + \frac{\bar{e} - q + N^2 + N_{2H}q + N^2\beta_1}{4} + \frac{N(N_{2H} - N_{2H}q + q)}{16N_{2H}}, \\ A3 &= 6N_{2H} - q + N_{2H}q + 16NN_{2H} - 16N_{2H}(\beta_1 - 1)(N - 4N_{2H}). \end{aligned}$$

The results directly follow by simple algebra.

Proofs of Corollary 1

Without capacity constraint, we focus on the cases (a.1), (a.2), (b.1), and (b.2). Assume $k = N_{2L} = \beta = \bar{v} = 1$, and $\beta_1 = c = 0$, we obtain by simple algebra that

$$\Pi_{a2} > \Pi_{a1}, \text{ and } \Pi_{b2} > \Pi_{b1}, \tag{A-23}$$

where Π_{ij} corresponds to the seller's profit over two periods for case $(i.j)$ for $i = a, b$ and $j = 1, 2$. That is, adopting advance selling dominates no advance selling.

Then we compare cases (a.2) and (b.2). we have $e_{a2}^* < e_{b2}^*$, $p_{1,a2} < p_{1,b2}$, $p_{2,a2} = p_{2,b2}$, and

$$\Pi_{a2} - \Pi_{b2} = \frac{-q(N_{2H}-1)(6N_{2H} - q + N_{2H}q - 64N_{2H}^2 + 32NN_{2H})}{256N_{2H}^2}. \tag{A-24}$$

Since $N_{2H} > N_{2L} = 1$, when N_{2H} is high (low), $\Pi_{a2} - \Pi_{b2}$ is more likely to be positive (negative) and setting a high (low) production quantity makes the seller better off.