Formulating Cross-sector Horizontal Coalition Strategies for Multi-product Assembly Systems with a Common Component

May 13, 2021

Abstract

This study examines various coalition structures for a multiproduct assembly system with a common component. Under different forms of coalition in the channel, the common component supplier may form partial or grand coalitions with the other suppliers to achieve maximum profit and eliminate supply chain inefficiencies. The optimal pricing decisions of suppliers are characterized and possible coalition structures are proposed. Results document that coalition structures, product demand characteristics, and manufacturing costs profoundly impact optimal wholesale price decisions. In addition, component suppliers are not always worse off even when the remaining suppliers form a partial coalition. Conditions under which all suppliers form a grand coalition are also provided to introduce a fair allocation and a non-empty core. A numerical experiment is conducted to show the influence of model parameters on the profit of different coalition structures. Profit allocation between the suppliers under the multiproduct assembly system is also discussed.

Keywords— *Assembly System, Coalition Strategies, Common Component, Coalition Stability*

1 Introduction

Companies today often use common components to satisfy various product categories because of the global trend of outsourcing and technology advancement. For example, Google supplies the Android operating system to manufacturers of smart phones and tablets. Dell Computer provides both cluster servers, which are required by large companies and organizations, and personal computers, which are required by individual customers, with different types of common components, such as RAMs and hard drives. A123 Systems produces key components of lithium–ion batteries used in the consumer electronic devices produced by Black & Decker, electric grid produced by AES Corporation, and electric vehicles produced by Toyota. The company has been working closely with major tier 1 suppliers in joint product design and manufacturing for developing batteries and battery systems for HEVS, PHEVs, and EVs. The sensors supplied by Texas Instruments are used by various manufacturers of self-driving cars, smart grids, and smartwatches. These different product categories, which are targeted at diverse market segments with distinct customer demands, are connected in a unique way because they are manufactured or assembled with common components.

The unique connection enabled by the use of common components has led to an emerging business model of interfirm coordination and integration in supply chain management through forming cross-sector horizontal partnership among suppliers in multiple industrial sectors. For example, Intel, which is the supplier of microprocessors as the common components, has been known for taking the price leadership role in the supply chain networks for different computing devices, such as desktop PCs and ultrabooks (Sutton 1998; Digital Times 2011). Similar forms of cross-sector horizontal coordination among suppliers connected with common components for various business decisions have existed in Japan for several decades through the informal structure of Keiretsu-networks of independent Japanese subcontractors for various products, such as automobiles, lithium batteries, consumer electronics, food, and pharmaceuticals (Miyashita and Russell 1995).

The recent growing and intensive development of Industry 4.0 technologies, such as Internet of Things (IoT), Big Data Analytics (BDA), Cloud Computing (CC), Edge Computing(EC), and Blockchain (BC), etc., have led to a significant change in information acquisition for each supply chain member and provided further opportunities of interfirm collaboration through forming cross-sector horizontal partnerships (Bortolini et al., 2017; Camarinha-Matos et al., 2017; Manavalan and Jayakrishna, 2019; Lins and Oliveria, 2020; Yadav et al., 2020). Gartner defines Industry 4.0 as “a business-outcome-driven digital transformation approach to generate value from the collaboration of multiple partners in ecosystems across value chains and industries” (Forbes 2018). The Internet Protocol (IP) communication technologies, which enable rapid information interchanging, tangibly alter the collaborations within a supply chain. In the conceptual framework for assessing sustainable supply chain management for Industry 4.0 (Manavalan and Jayakrishna, 2019), “collaboration” is identified as one of the five key enablers with “joint development” and “supplier collaboration” as two key collaboration criteria. In the framework of “factory of future in sustainable supply chain ecosystem with Industry 4.0,” IoT not only enables interconnecting the machines, components, devices, and users within an enterprise but also connects multiple digital lines in different sites of suppliers by leveraging cloud and internet technologies (Manavalan and Jayakrishna, 2019). There is a tangible need to delineate the collaboration and coalition of assembly systems under modern architecture.

The emerging business model, which forms cross-sector horizontal partnerships among suppliers connected by the use of common components, is a new means to create idiosyncratic interfirm linkages that may become a source of “relational rent” and competitive advantage (Dyer and Singh 1998). The incentives of the specific supply chain structures caused by forming cross-sector horizontal partnerships studied in the current work can be attributed to three sources of relational rent and competitive advantages discussed in the existing literature of transaction cost economics and strategic management, namely, the relationship-specific assets, interfirm knowledge sharing, and complementary resources/capabilities (Williamson 1983; Dyer and Singh 1998; Clark and Fujimoto 1991; Dyer 1996). In addition, knowledge sharing and transfer between and among suppliers on the product and pricing information may facilitate the design, production, and marketing processes; accordingly, the competitive positions of members in the supply chain network enhance (Grant 1996). Moreover, the adoption of the common component requires the suppliers in different sectors to develop complementary resources and capabilities, such as patents and proprietary technologies, which may become the key driving factors of returns from partnerships through the current and future collaboration (Oliver 1997).

As motivated by these issues, this research studies the horizontal efficiency of supply chains to show the enhancement effect of firms in a coalition on the overall profitability through price coordination. Specifically, this study analyzes how suppliers can cooperate and collectively determine the pricing decisions to increase bargaining power and reduce production costs by forming a cross-sector horizontal partnership among suppliers. We pose the following research questions: How should the common component supplier form alliance with the other suppliers? How do the suppliers determine the prices optimally under different forms of alliance? Is participating in the alliance with the common component supplier always beneficial? Does any fair profit allocation occur if all suppliers form a grand coalition? A number of existing works have already studied the significant potential benefits due to cooperation among suppliers and their applications in various industries, such as auto parts, railroad, and airlines (see Nagarajan et al. 2019; Yin 2010; Nagarajan and Sosic 2009 and references therein). However, the economic impacts of the horizontal partnership have not received considerable attention in the existing literature of operations and supply chain management. Therefore, the current study aims to take a distinct perspective by considering the coalition structures in a multiproduct assembly system, beginning with a scenario in which three suppliers exist, one of which produces common components that are used to assemble two finished products. Cases in which product demands are independent from one another are discussed. The channel suppliers may form coalitions and formulate pricing decisions collectively. The common component supplier leads the coalition with other suppliers to achieve maximum profit and eliminate supply chain inefficiencies. Moreover, the optimal pricing decisions of suppliers in the channel are characterized and possible coalition structures and profit allocation for supply chains are proposed.

This study obtains several interesting results with economic implications. When a common component supplier forms a partial coalition with one of the remaining suppliers, the optimal wholesale price of the common component is unaffected by the demand characteristics of the product with components offered by the coalition. Furthermore, the partial coalition formed without a common component supplier fails to influence the optimal pricing decisions of all suppliers. Both outcomes are due to the specific role of a common component supplier; thus, similar observations are not revealed under the traditional assembly model setup, in which no common component is considered. When the optimal wholesale price decisions among different forms of coalition are compared, the wholesale price charged by a common component supplier under a partial coalition with each remaining supplier serves as the upper and lower bounds on that set without coalitions formed. Under a partial coalition, a noncooperative supplier with low manufacturing costs can successfully reduce the wholesale prices of a non-common component supplier and increase the wholesale prices of a common component supplier. Under the grand coalition, the total wholesale price of components for each product is lower than that of under other coalition structures. Moreover, the grand coalition decreases the prices of the two consumer products and identifies the conditions under which the main analytical results remain valid.

For the stability of a particular form of inter-firm coalition, two conditions are necessary: (i) all participants receive high profits to maintain a stabilized coalition and (ii) the resulting consumer surplus is not decreased to maintain the legal stability of the coalition with a minimum risk of regulatory inference. For the stability of coalition, this study finds that under a certain condition, a non-cooperative supplier in a two-product assembly system can even obtain higher profits under a partial coalition, in which the remaining suppliers still cooperate, than under a model without cooperation. Therefore, an alliance between any suppliers does not always implicate a negative effect on the remaining suppliers. This mostly occurs when the demand for the assembled product with a component provided by one supplier is less dispersed and the manufacturing cost of the component is higher. This phenomenon significantly affects the profit allocations between suppliers when they form an alliance, which may result in an unstable alliance. Therefore, for a two-product assembly system that uses a common component, a grand coalition in which all suppliers cooperate is not always stable. Results show the conditions in which the core is non-empty in the channel, which provides the economic incentives and the legal ground for the successful execution of the grand coalition. A numerical example is given to provide quantitative insights into profit allocations among suppliers under different conditions. The results guide component suppliers as they form coalitions with other parties in the supply chain.

The remainder of the paper is organized as follows. Section 2 reviews relevant literature. Section 3 describes the model. Section 4 focuses on two-product assembly system. A numerical study is conducted in Section 5. Discussions and conclusions are presented in Section 6. The model with stochastic demand, the analysis of the N-product assembly system, and all the proofs are provided in the Appendix.

2 Literature Review

Rooted in the relational view of competitive advantages (Dyer and Singh, 1998; Lavie, 2006), the emerging business model, which forms cross-sector horizontal partnerships among suppliers connected by the use of common components, is a new means to create idiosyncratic inter-firm linkages that may become a source of "relational rent" and competitive advantage (Dyer and Singh 1998). The incentives of the specific supply chain structures caused by forming cross-sector horizontal partnerships studied in the current work can be attributed to three sources of relational rent and competitive advantages discussed in the existing literature of transaction cost economics and strategic management, namely, the relationship-specific assets, inter-firm knowledge sharing, and complementary resources/capabilities (Williamson 1983; Dyer and Singh 1998; Clark and Fujimoto 1991; Dyer 1996). In addition, knowledge sharing and transfer between and among suppliers on the product and pricing information may facilitate the design, production, and marketing processes; accordingly, the competitive positions of members in the supply chain network enhance (Grant 1996). Moreover, the adoption of the common component requires the suppliers in different sectors to develop complementary resources and capabilities, such as patents and proprietary technologies, which may become the key driving factors of returns from partnerships through the current and future collaboration (Oliver 1997).

Information sharing plays a crucial role in various supply chain strategies. It is shown that decisions during the negotiation and coalition are relied on the information provided (Renna, 2010; Raweenwan and Ferrell, 2018). The recent advance of new technologies has enabled the speedy adoption of horizontal partnerships among suppliers in assembly line systems (Bortolini et al., 2017). As shown in the conceptual framework in Figure 1, several technologies play a key role in enabling cross-sector inter-firm collaboration as the key building blocks of the new form of cross-sector collaboration. Industry 4.0, through driving digital transformation to generate value from the collaboration of multiple partners in ecosystems across value chains and industries (Forbes, 2018), has been identified as a key enabler of supplier collaboration networks (Camarinha-Matos et al., 2017; Manavalan and Jayakrishna, 2019). Industry 4.0 turns the enterprises to concentrate on adapting difficult conditions of competition. The advanced information sharing technologies enable man-machine interaction to strengthen the manufacturing sustainability and establish collaboration among the system aligned activities to enlarge the profitability(Fantini et al., 2020; Kiraz et al., 2020). Increasing operational flexibility with Industry 4.0 enabling technologies in final assembly also leads to better collaborative work and assembly support (Salunkhe and Fast-Berglund, 2020; Longo et al., 2017). Internet of things (IoT) not only enables interconnecting the machines, components, devices, and users within an enterprise but also connects multiple digital lines in different sites of suppliers by leveraging cloud and internet technologies (Manavalan and Jayakrishna, 2019). According to Karnouskos et al. (2019), the interactions and collaboration achieved with cyber-physical production systems can lead to next generation infrastructure and emerging behaviors in autonomous and sophisticated industrial systems. In the interconnected smart factory, horizontal collaboration and partnership ensure machinery, IoT devices and engineering processes work together seamlessly (Porter and Heppelmann, 2014; Yang et al., 2018). In addition, the human-technology symbiosis are benefited from this cyber-physical systems technologies (Pacaux-Lemoine et al. 2017). As a distinguishing element of Industry 4.0, lot-size-one production enables collaboration through a network of facilities in order to manufacture customized orders (with minimal lot sizes) by managing the division of tasks, transportation between the factories, and resolving dependencies among the participating manufacturers in an assembly line system (Kannengiesser at al. 2017; Garzon and Alejandro, 2019; Dhungana et al., 2020). Cloud computing is used to store and analyze the enormous datasets involving Industry 4.0 applications among multiple suppliers sharing and analyzing big data necessary to support horizontal partnerships in a supply chain system. Finally, human-centered automation, including computational, visualizational, and information technologies as well as mechatronic systems for digitalization, robotization, decision-aid, and systems maintenance and integration (Jerman et al., 2020; Akash et al., 2019a; Akash et al., 2019b), enables human-machine cooperation for designing and operating effective and socially sustainable assembly line systems (Kovacs et al., 2018; Romero et al., 2020; Pinzone et al., 2020).

One critical element in assembly 4.0 implementation is the integration of human factors in designing and operating assembly line systems (Eynard and Cherfi, 2020; Romero et al., 2020). As discussed in Pacaux-Lemoine et al. (2017), with the adoption of Industry 4.0, machine capabilities have increased in such a way that human control of the process have evolved from simple to highly complicated. Today's intelligent manufacturing systems have become so autonomous that humans are sometimes unaware of the processes running (Mattsson et al., 2020), whereas they still need to intervene to update the production plan or to reconfigure the process when a machine breaks down, or to assist process-intelligent entities for safety or quality of work (Glock et al., 2017; Cohen et al., 2018; Taylor et al., 2020). As a result, the implementation of cyber-physical systems requires the integrated capabilities on controlling machines, assembly lines, factories, and supply chains, as well as on human information processing (Waschull et al., 2020). In a transforming process towards cyber-physical

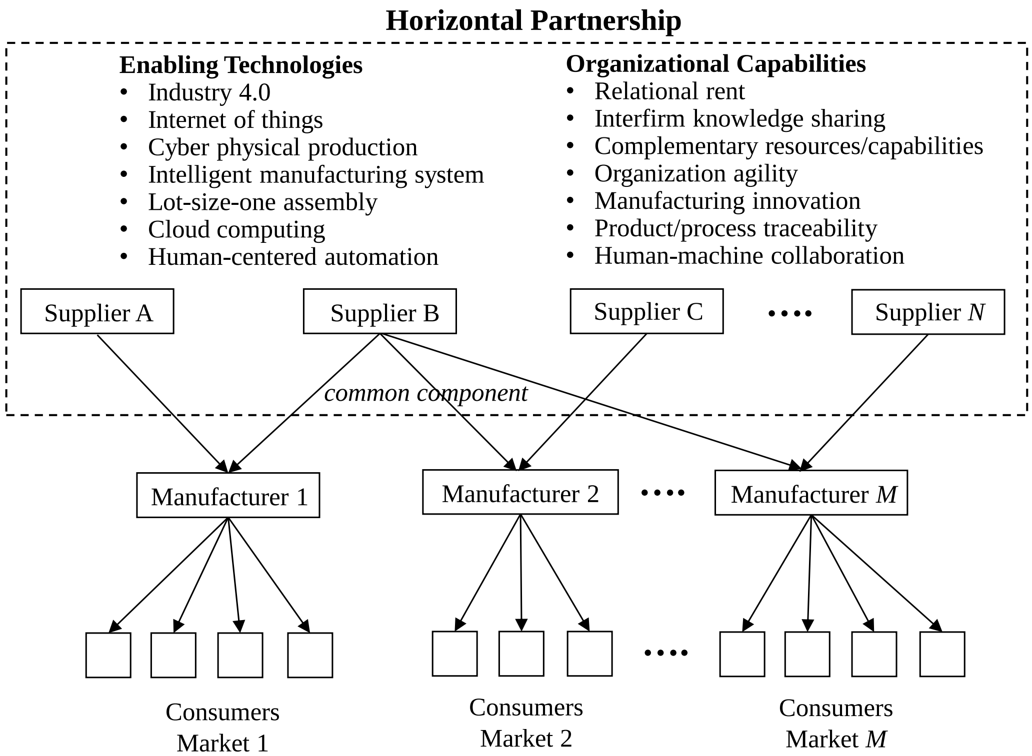

Figure 1: Conceptual Framework.

systems, new human-centric performance indicators and guidelines need to be developed for managers and engineers to formulate their strategy for managing human resources, to improve their awareness on the role of workers, and to detect possible misalignment between the high-level strategies and the operational interventions in assembly line systems (Fantini et al., 2020; Rauch et al., 2020; Fletcher et al., 2020; Pinzone et al., 2020). Human-centered automation technologies, such as motion analysis system (Bortolini et al., 2020), intelligent assistant decision-supporting systems as context aware knowledge-based tools (Belkadi et al., 2020; El Mouayni et al., 2020), digital assistance in knowledge-based maintenance (Kovacs et al., 2018), and social collaboration platforms (Lithoxoidou et al., 2020), also play a key role in facilitating assembly 4.0 implementation. To successfully transform from traditional to smart factory systems, it is also necessary for today's organizations to develop human infrastructure for employees with new job profiles in such areas as mechatronics, robotics, smart system design, IoT design, systems supervision and maintenance, programming, and data analytics, as well as with new competencies such as technical skills/literacy, flexibility adaption, continuous learning, critical and analytical thinking, innovation and creativity, and other soft skills (Jerman et al., 2020; Ansari et al., 2020).

The implementation of horizontal partnerships in the era of Industry 4.0 also requires organizations to acquire and/or develop new capabilities. As discussed in Fettig et al. (2018) and Sony (2020), organization agility is a key enabling capability as the intelligent network of manufacturing cells, production lines, and other aspects of product manufacturing and selling within an organization will help to quickly respond to customer demands. The horizontal partnership, as a cross-company and company cross-linking within the value chain, will make the entire supply chain responsive to meet customer needs. The cross-sector integration through intelligent technologies and digitalization in different phases of a product's life cycle will help to develop innovative products as per the customer needs (Ennis et al., 2018; Huxtable and Schaefer, 2016). In addition, manufacturing innovation with simultaneous persuasion of both exploration and exploitation of orientations for developing innovation capabilities may reduce the complexities in adopting new technologies (Fischer et al., 2010). As discussed in Gupta et al. (2020), exploration focuses on the innovation and new technologies, whereas exploitation focuses on the update of existing resources such as employees and machines. Furthermore, product and process traceability play a key role as assembly tasks are monitored with sensorized and connected devices to detect in real-time any possible error or non-compliance. Worker activity is continuously monitored to track the assembled components and the task duration to a complete product traceability during the entire assembly process (Bortolini et al. 2017). Finally, the capability for human-machine collaboration is necessary for improving productivity, safety, and engagement in intelligent manufacturing systems (Kovacs et al., 2018; Eynard et al., 2020). According to Hoyer et al. (2020), the complexity of Industry 4.0 technologies and their interactions call for the development of knowledge-based, human-centered approaches to help employees to build and to work in complex environments with new skills, job profiles and competencies (Kovacs et al., 2018;

Jerman et al., 2020; Ansari et al., 2020).

A number of works have studied the behavior of the assembly systems. Song and Zipkin (2003), Bernstein and Decroix (2006), and Atan et al. (2017) provided a detailed survey of such literature. Notably, cyber-physical architecture transforms the assembly manufacturing system into Industry 4.0 (Jazdi 2014; Lee et al. 2015; Manavalan and Jayakrishna 2019). The rapid information sharing among the systems raises new topics in the supply chain. Some studies assumed that demand is characterized by a linear inverse demand function (e.g., Corbett and Karmarkar 2001; Carr and Karmarkar 2005; Majumder and Srinivasan 2008; Nagarajan and Sosic 2009; Yin 2010; David 2015; Nagarajan et al. 2019). Among these works, Corbett and Karmarkar (2001) considered entry decisions in a two-echelon supply chain and Majumder and Srinivasan (2008) focused on network structures in supply chains. Carr and Karmarkar (2005) discussed the prevailing competition in a multi-tier assembly system. David (2015) described the competition and coordination in a two-channel supply chain. Nagarajan et al. (2019) studied the farsighted stable alliance structures between suppliers in the assembly system.

Another stream of works focused on uncertain demand but price insensitive. Wang and Gerchak (2003) considered capacity games between the suppliers and the assembler. They studied the two established contracts according to the manner in which the terms of each contract are set. Gerchak and Wang (2004) considered two supply chain contracts between the assembler and the suppliers and showed how supply chain coordination can be achieved. Bernstein et al. (2007) studied a multi-echelon system in which the assembler moved several assembly jobs to the sub-assemblers and characterized the optimal pricing and the capacity decisions. Fang et al. (2008) analyzed the procurement strategies in an assembly-to-order system when price is a function of delivery lead-time. Zhang et al. (2008) analyzed the optimal stocking quantity of two product configurations and found the possible contracts to coordinate the supply chain. Furthermore, Wang (2006) and Jiang and Wang (2010) assumed that demand is price sensitive and considered joint pricing-production decisions and supplier competitions in the assembly systems, respectively. More recent studies related to assembly systems in the supply chain include Chen and Hall (2007) for supply chain scheduling in assembly systems, Norde et al. (2016) for the issue of incentives for global planning and manufacturing, and Zhang and Huang (2010) for configuration of platform products.

New pricing policies, supply chain cooperations, and new frameworks have also been discussed. Chen and Xiao (2017) described the pricing and replenishment policies under different retailer behaviors. Radhi and Zhang (2018) studied the pricing policy for a dual-channel retailer with same- or cross-channel return. Wei et al. (2019) investigated the manufacturer's and retailer's integration strategies with complementary products. Fang and Cho (2020) studied the effects of different cooperative approaches in managing social responsibility of suppliers. Fu et al. (2020) investigated the pricing and production decisions of the assembly systems with capacity constraints. Leitao et al. (2016) discussed the collaboration of the cyber-physical production for the modern assembly systems.

The model of the current paper is mostly relevant to the research that focuses on a multi-product assembly system with common components (Baker et al. 1986; Gerchak et al. 1988; Cattani 1995; Eynan and Rosenblatt 1996; Eynan 1996; Bernstein et al. 2007; Xiao et al. 2010). Other works relevant to the current study examined the scenarios in which component suppliers form a coalition, including the stability of such a coalition structure in assembly systems. Research by Granot and Sosic (2005) demonstrated the influence of coalition degrees from three retailers on their profits. Granot and Yin (2008) discussed two contracting arrangements depending on whether the suppliers are Stackelberg leaders who provide the wholesale prices of components. Nagarajan and Sosic (2009) studied three forms of competition in assembly systems and Nagarajan and Bassok (2008) considered the idea that suppliers may form coalitions and then subsequently negotiate with the assembler on profit allocation based on Nash bargaining. Yin (2010) analyzed alliance structures in an assembly system with complementary suppliers and He and Yin (2015) discussed supplier and retailer competitions in an assembly system. Huang et al. (2016) considered alliance formations under the setting of one downstream firm and any suppliers when supply risks exist. Li et al. (2018) described the impacts of power structures in a decentralized assembly system. Li and Chen (2020) discussed the coalition of N complementary suppliers scenario. Nagarajan et al. (2019) studied alliance in assembly systems with competing suppliers. Table 1 summarizes the literature on multi-product assembly system with common components.

Table 1: Comparisons of most related literature.

Authors	Year	Demand assumption	Structure of Supply Chain	study focus
Granot and Sosic	2005	demands have substitutability to other demand	multiple suppliers and multiple retailers	alliance formations with the influences of supplier information sharing
Nagarajan and Bassok	2008	a public information to all supply chain members	multiple suppliers and multiple assemblers	the interactions under a negotiation framework
Granot and Yin	2008	stochastic demand	push and pull assembly system	the inefficiency due to horizontal decentralization of suppliers
Nagarajan and Sosic	2009	linear price dependent	multiple complementary suppliers with one assembler	dynamic supplier alliances and coalitions
Yin	2010	deterministic/stochastics price dependent	multiple complementary suppliers with one assembler	the alliance and coalition formation strategies for the complementary suppliers
He and Yin	2015	linear price dependent	decentralized assembly system	competition between suppliers and retailers
Huang et al.	2016	deterministic price dependent	multiple complementary suppliers to one manufacturer	supplier alliance formation strategies under order default risk
Li et al.	2018	random demand with failure rate	two suppliers (main/subcontractor) and one manufacturer	impact of power structures to production and pricing strategy
Nagarajan et al.	2019	linear price dependent	decentralized assembly system	the farsighted stable alliance structures between suppliers
Li and Chen	2020	linear price and quality dependent	multiple complementary suppliers to one manufacturer	the effects of supplier coalitions to whole sale price and quality improvement
This paper		deterministic/stochastic price dependent	multiproduct assembly system with one common component	horizontal coalitions regarding common components and profit allocations of the suppliers

As shown in Table 1, most related research addresses the alliance or coalition formations of the complementary goods assembly systems. Our study is the first to respond to calls for researchers to consider the strategic role of the common component supplier in multi-product decentralized assembly systems that the final markets are non-complementary. This study examines the dynamics of horizontal coalition structures with a common component supplier, and discusses fair profit allocation for each member of the coalition to eliminate supply chain inefficiencies. Besides, this study investigates the demands in both deterministic and stochastic setting where most of the prior research emphasize only one demand setting.

Furthermore, this study characterizes the forms of coalition structures and discusses their managerial insights.

3 Model Description

This study considers a supply chain structure of a common component supplier (Supplier 0) and the other suppliers (Suppliers 1,..., N) for products manufactured by different manufacturers (Manufacturers 1,..., N) which are sold in different market sectors (Markets 1,..., N). Component 0 is common to all products whereas Component $i, i = 1, ...N$ is dedicated to Product i. Without loss of generality, Product i is assumed to have one unit each of Components 0 and i. Each component is offered by one independent component supplier. Let c_i be the unit manufacturing cost of Component i faced by Supplier i. Supplier i sells the component at the wholesale price w_i to N independent manufacturers who, in turn, assemble the products with the associated components and sell to end customers. For a unit of product j, which is sold, Manufacturer j collects p_j that represents the retail price set by the manufacturer. Considering that the products are sold in different markets, retail price of one product charged to end customers are assumed to have no impact on the demand of the other products.

Customer demand for each product is assumed to be deterministic; it decreases linearly with the retail price, which is commonly used in economics and marketing literature and in recent studies related to assembly system alliances (e.g., Nagarajan and Sosic 2009; Yin 2010). Let

$$D_j = a_j - b_j p_j, \text{ for } j = 1, ..., N \tag{1}$$

be the demand function for Product j where a_j represents the baseline sales and the value of $b_j (> 0)$ shows the price sensitivity of end customers. It should be noted that we consider the situation of forming horizontal, cross-sector partnerships where the cross-price effect between products in different market sectors is not significant. Therefore, cross-price elasticity is not considered in our analyses.

The sequence of events is as follows. At the beginning of the period, each supplier simultaneously determines the wholesale price of Component i, w_i. Then, each manufacturer orders the required components from the suppliers, and subsequently assembles and sells the product to end customers at the retail price, $p_j, j = 1, ..., N$. After the selection of retail prices, end customer demand for each product is determined simultaneously and the period ends. All cost parameters and demand functions are assumed to be common knowledge to all parties, who set the prices to maximize their individual profit. This research considers three different scenarios: (a) model without cooperation, (b) partial coalition, and (c) grand coalition. In the next section, a two-product assembly system is analyzed to establish the benchmarks. Figure 2 shows the sequence of events for each scenario under the two-product assembly system.

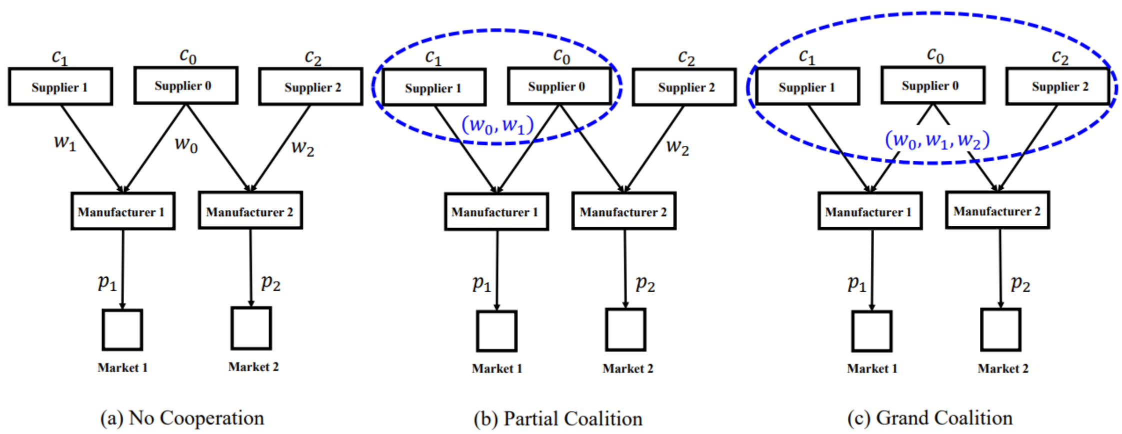

Figure 2: Sequence of events for three scenarios: (a) model without cooperation, (b) partial coalition, and (c) grand coalition.

4 Two-Product Assembly System

An assembly system that produces two products from the three components (denoted by 0, 1, and 2) is considered. Customer demand for Product j is $D_j = a_j - b_j p_j, j = 1, 2$. In the following subsections, a model in which none of the suppliers in the channel cooperate with each other is firstly considered. To ensure that the algebraic verification is amenable to analysis, some of the subsequent theoretical results will use the following assumption: The demand for Product 2 is more dispersed than that for Product 1, i.e., $\frac{a_1}{b_1} \leq \frac{a_2}{b_2}$.

4.1 Model without Cooperation

In this model, each supplier individually chooses a wholesale price, w_i, that maximizes the profits collected from the components sold to the manufacturers. Given the wholesale prices, Manufacturer j determines the retail price, p_j. The current paper uses the backward induction to solve the problem. Given the best response of each manufacturer, the optimal wholesale price of each supplier is obtained, as well as the corresponding profits of both parties. Given the wholesale price, w_i, Manufacturer j sets the retail price, p_j, to maximize profits $(p_j - w_0 - w_j)D_j(p_j)$ for $j = 1, 2$. By solving the manufacturers' retail price problems, the optimal retail price of is:

$$p_j^*(w_0, w_j) = \frac{a_j + b_j(w_0 + w_j)}{2b_j}, \text{ for } j = 1, 2. \tag{2}$$

According to (2), each supplier's problem is to choose w_i to maximize the profit, $\Pi_i^F(w_i)$:

$$\begin{aligned}
\Pi_0^F(w_0) &= (w_0 - c_0)(a_1 - b_1 p_1^*(w_0, w_1) + a_2 - b_2 p_2^*(w_0, w_2)), \\
\Pi_i^F(w_i) &= (w_i - c_i)(a_i - b_i p_i^*(w_0, w_i)), \qquad \text{for } i = 1, 2.
\end{aligned}$$

Let w_i^F be the optimal wholesale price of Supplier i. then for $i = 1, 2$,

$$\begin{aligned}
w_0^F &= \frac{a_1 + a_2 - b_1(c_1 - 2c_0) - b_2(c_2 - 2c_0)}{3(b_1 + b_2)}, \\
w_i^F &= \frac{(4c_i - 2c_0)b_i^2 + (2a_i + b_{3-i}c_{3-i} - 2b_{3-i}c_0 + 3b_{3-i}c_i - a_{3-i})b_i + 3b_{3-i}a_i}{6b_i(b_1 + b_2)}.
\end{aligned} \tag{3}$$

Supplier i's profit, $\Pi_i^F(w_i^F)$, is

$$\begin{aligned}\Pi_0^F(w_0^F) &= \frac{(k_1+k_2)^2}{18(b_1+b_2)},\\ \Pi_i^F(w_i^F) &= \frac{(3k_i(b_1+b_2)-(k_1+k_2)b_i)^2}{72b_i(b_1+b_2)^2}, \text{ where } k_i = a_i - b_i(c_0+c_i). \end{aligned} \tag{4}$$

4.2 Partial Coalition

In certain supply chain environments, the common component supplier may cooperate with other suppliers by choosing the wholesale price collectively to increase the joint profit; this case is denoted as *partial coalition*. This study focuses on the case in which Supplier 0 cooperates with Supplier $i, i = 1, 2$ in the channel, and coalition members collectively determine the wholesale prices of the components. On the basis of the derived optimal retail prices above, the joint profit function of Suppliers 0 and i can be obtained as follows: for $i, j \in \{1, 2\}, i \neq j$:

$$\Pi_{0i}^C(w_0, w_i) = (w_i + w_0 - c_i - c_0)(a_i - b_i p_i^*(w_0, w_i)) + (w_0 - c_0)(a_j - b_j p_j^*(w_0, w_j)). \tag{5}$$

Supplier j's profit function is

$$\Pi_{0i,j}^C(w_j) = (w_j - c_j)(a_j - b_j p_j^*(w_0, w_j)). \tag{6}$$

Let $w_{0i,0}^C$ and $w_{0i,j}^C$, $j = 1, 2$ be the optimal wholesale prices of the common component and Component j, respectively when partial coalition is formed by Suppliers 0 and i. The problems in (5) and (6) are solved simultaneously,

$$\begin{aligned} w_{0i,0}^C &= \frac{a_j + b_j(2c_0 - c_j)}{3b_j},\\ w_{0i,i}^C &= \frac{3a_i - b_i c_0 + 2b_i c_j + 3b_i c_i}{6b_i} - \frac{a_j}{3b_j}, \text{ and } w_{0i,j}^C = \frac{a_j + b_j(2c_j - c_0)}{3b_j}. \end{aligned} \tag{7}$$

Hence, the joint profit of Suppliers 0 and i, Π_{0i}^C, and the profit of Supplier j, $\Pi_{0i,j}^C$, are:

$$\Pi_{0i}^C = \frac{k_i^2}{8b_i} + \frac{k_j^2}{18b_j}, \qquad \text{and} \quad \Pi_{0i,j}^C = \frac{k_j^2}{18b_j}, \qquad \text{for } i, j \in \{1, 2\}, i \neq j. \tag{8}$$

Proposition 1. *Under a partial coalition between Suppliers 0 and i, $w_{0i,0}^C$ is independent from the demand characteristics of Product i, a_i and b_i.*

When Supplier 0 forms a partial coalition with Supplier i, the coalition chooses the wholesale price of the common component as if two independent assembly systems exist: one each for Products i and j. The coalition has total control in determining the overall wholesale price of components for Product i, $w_0 + w_i$; however it only partially controls the second system, in which Supplier j can likewise influence the wholesale price of the component for Product j. This crucial finding can be directly patterned from (2) and (5), indicating that the profit collected from Product i is influenced by $w_0 + w_i$, rather than the individual w_0 or w_i. The demand characteristics of Product i only influence $w_0 + w_i$, which are collectively determined by the coalition. In this sense, w_0 is determined when the coalition competes with Supplier j. Once w_0 has been determined; the coalition balances the profit margin and demand for Product i, and then sets w_i, accordingly. As a result, the optimal common component wholesale price, $w_{0i,0}^C$, is irrelevant to Product i. The above-mentioned observation is due to that Supplier 0 forms the coalition with another supplier in the channel.

Proposition 2. *If Suppliers 1 and 2 form a partial coalition, then the optimal wholesale price of each component and the associated profit of each supplier are identical to those of the model without cooperation.*

The proposition is straightforward provided that neither product is substitutable. Thus, the demand for and the retail price of each product are only relevant to the wholesale prices of the assembled components. This finding underscores the advantage of the common component supplier in the electronics, auto-parts, and service industries. Given the advanced technological capabilities and unique components, the common component supplier will not be negatively affected by whether the other suppliers cooperate.

4.3 Grand Coalition

A scenario in which all suppliers in the assembly system cooperate and determine the wholesale prices collectively is considered. Under such a grand coalition, the overall profit of all the suppliers can be described as follows:

$$\begin{aligned}\Pi_{012}^{G}(w_0, w_1, w_2) &= (w_1 + w_0 - c_1 - c_0)(a_1 - b_1 p_1^*(w_0, w_1) \\ &\quad +(w_2 + w_0 - c_2 - c_0)(a_2 - b_2 p_2^*(w_0, w_2)).\end{aligned}$$

Let $w_i^G, i = 0, 1, 2$ be the optimal wholesale price of Supplier i. Solving the above-mentioned problem yields:

$$w_0^G + w_i^G = \frac{a_i + b_i(c_0 + c_i)}{2b_i}, \text{ for } i = 1, 2. \tag{9}$$

Therefore, the resultant overall profit, $\Pi_{012}^{G}(w_0^G, w_1^G, w_2^G)$, is

$$\Pi_{012}^{G}(w_0^G, w_1^G, w_2^G) = \frac{k_1^2}{8b_1} + \frac{k_2^2}{8b_2}. \tag{10}$$

4.4 Comparison

On the basis of the discussions in the previous sections, the pricing decisions of the suppliers in different coalition structures are compared.

Proposition 3. *The wholesale price of the common component under each partial coalition (i.e., Supplier 0 cooperates with Supplier 1 or 2) forms the bounds that under the model without cooperation, w_0^F is between $w_{01,0}^C$ and $w_{02,0}^C$. If $c_1 \geq c_2$, then $w_{02,0}^C \leq w_0^F \leq w_{01,0}^C$.*

Under the model without cooperation, w_0^F is influenced by the manufacturing costs of the two other suppliers and demand characteristics of both products. In determining w_0^F, Supplier 0 must simultaneously balance all determining factors, and thus, set a modest wholesale price. Moreover, in a partial coalition between Suppliers 0 and i, Supplier 0 competes with Supplier j. Accordingly, only the manufacturing cost of Supplier j and the demand function of the product assembled with the components provided by Supplier j are concerned. Given a lower (higher) manufacturing cost of the component provided and a more (less) dispersed customer demand for the product assembled with the components provided by Supplier j, the wholesale price of the common component under the partial coalition with Supplier i becomes higher (lower) and forms the upper (lower) bound of w_0^F.

Proposition 4. *With a partial coalition between Suppliers 0 and 1, if* $c_1 \geq c_2$*, then* $w_{01,2}^C \leq w_2^F$ *and* $w_{01,1}^C \leq w_1^F$*.*

Propositions 3 and 4 indicate that the wholesale price of the common component increases when a partial coalition between the common component and the high-cost component exists in the system, but the wholesale prices of the remaining two components decrease. This rationale indicates that Supplier 0 strengthens bargaining power through an alliance with the supplier with high manufacturing cost. By doing so, Supplier 0 can successfully force the remaining two suppliers to mark down their respective wholesale prices and raise its price to significantly increase the profit margin. This phenomenon is more evident when a manufacturing cost of the non-cooperative supplier, c_2, is lower and the corresponding finished product demand is more dispersed (i.e., higher a_2/b_2). However, the advantage of partial coalition insignificantly benefit Supplier 1 because the profit margin of Supplier 1 decreases, unlike in the model without cooperation. Therefore, Supplier 0 should at least allocate a profit of Π_1^F to Supplier 1 to induce a partial coalition.

Proposition 5. *The optimal total wholesale price of components for each product under a grand coalition is less than that under the other coalition structures. That is, for* $i, j \in$

$\{1,2\}, i \neq j$,

$$w_0^G + w_i^G \leq \min\{w_0^F + w_i^F, w_{0i,0}^C + w_{0i,i}^C\}, \text{ and } w_0^G + w_j^G \leq \min\{w_0^F + w_j^F, w_{0i,0}^C + w_{0i,j}^C\}.$$

The results in Proposition 5 can be attributed to the horizontal decentralization of component suppliers. Consider a scenario in which no supplier cooperate or in which only several suppliers form a partial coalition. The quantity that a supplier sells depends on the retail prices set by the two manufacturers, and on the chosen wholesale prices by other suppliers. In this manner, suppliers are prompted to raise the wholesale price to increase the profit margin considering that any reduction in the wholesale price would decrease profit margin. Moreover, this scenario does not necessarily result in increased sales because of the unanticipated behavior of the other suppliers. However, in a grand coalition, the uncertainty in the quantity sold diminishes: each supplier balances the quantity sold and the profit margin collected from each product, and then sets an appropriate wholesale price. In such a circumstance, a good horizontal efficiency of the supply chain can be achieved. Notably, the proposition also has an interesting implication regarding consumer surplus, which shows that p_j will decrease as $w_0 + w_j$ decreases. With the downward-slopping demand function defined in (1), the consumer surplus in a grand coalition will increase when p_j decreases and D_j increases for each of the two products based on Varian (1992). This finding implies that a grand coalition would lead to not only decreased wholesale prices but also lower consumer prices and higher demands in the two markets. This condition results in increased levels of consumer surplus compared with the other coalition structures.

Proposition 6. $\Pi_i^F \leq \Pi_{0j,i}^C$ *if and only if* $\Pi_j^F \geq \Pi_{0i,j}^C$, *for* $i, j \in \{1,2\}, i \neq j$.

Proposition 6 indicates that Supplier 1 or 2 can possibly obtain higher benefits when the remaining suppliers form a partial coalition, compared with the model without cooperation. Thus, Supplier 1, which serves as non-cooperative supplier, can be better off under a partial coalition between Suppliers 0 and 2 if and only if Supplier 2 is worse off under a partial coalition between Suppliers 0 and 1. Furthermore, consider the case in which Supplier 1 is better off when no party forms a partial coalition. If Supplier 1 can determine the possible coalition structure, Supplier 1 prefers the model without cooperation rather than a partial coalition between Suppliers 0 and 2. Moreover, the coalition formed by Suppliers 0 and 1 benefits Supplier 2. This result is observed given that each supplier's pricing decision under the model without cooperation is influenced by the best responses of the two other suppliers. The profit of each supplier depends on the combination of each party's cost structure and the demand characteristics of both products. However, under a partial coalition, the competition is only between the coalition and the non-cooperative supplier. The coalition competes with the non-cooperative supplier by setting w_0 to collect the maximum possible profit, and then balances the profit margin and the demands to determine the other wholesale price of the coalition. Considering that the final product consists of one unit of each component from the coalition and the non-cooperative supplier in competition with the two parties, the coalition and the non-cooperative supplier possess equal power. Both parties aim to maximize their total profit and subsequently divide the profit equally. However, the model without cooperation does not achieve such equal allocation because the goal of each supplier differs. This inconsistency may benefit Supplier 0 because the said supplier can take a leading role in the profit allocation of the coalition and manipulate self-importance to receive the highest profit in the channel.

4.5 Coalition Stability and Profit Allocation

To understand whether a particular form of coalition is stable in practice, this research studies coalition stability and profit allocation. Previous works (e.g., Nagarajan and Bassok 2008; Granot and Yin 2008; Nagarajan and Sosic 2009; Nagarajan et al. 2019; Gao et al. 2019) addressed the coalition stability problem under the setting that suppliers are farsighted, that is, a coalition decision-making is based not on the immediate effect of an initial deviation but on the "final" outcomes of possible sequences of deviations. This study applies a more commonly used concept, which is the core (Gillies 1959; Kukushkin 2017), to analyze the stability of a grand coalition considering that the notions of the core and the Nash equilibrium used in our non-cooperative analysis are more consistent given that both of them are based on myopic deviations by agents.[1] The concepts of the core and the Nash equilibrium are widely adopted in economics and operations literature and the analysis is simplified in our study.

The core consists of profit allocations that cannot be blocked by any coalition of suppliers. Thus, a core allocation is stable in the sense that no coalition can benefit each of its members by leaving the grand coalition. However, the notion of the core in our model seems unclear because the aggregate profit that a coalition of suppliers can make on its own depends on the coalition structure among the residual suppliers.[2] To circumvent the problem raised by externality, the residual suppliers are supposed to be left unaltered when the members of a coalition decide to leave the grand coalition given that the coalition of the residual suppliers maximizes the aggregate profits of all coalition structures among the residual suppliers. On the basis of these settings, the two-product assembly system can be formulated as a coalition game $g : 2^{\{0,1,2\}} \to \mathbb{R}$ defined by

$$\begin{aligned}
&g(\emptyset) = 0, && g(\{0,1,2\}) = \Pi^G_{012} = \tfrac{k_1^2}{8b_1} + \tfrac{k_2^2}{8b_2},\\
&g(\{0\}) = \Pi^F_0 = \tfrac{(k_1+k_2)^2}{18(b_1+b_2)}, && g(\{1,2\}) = \Pi^F_1 + \Pi^F_2 = \tfrac{[3k_1(b_1+b_2)-(k_1+k_2)b_1]^2}{72b_1(b_1+b_2)^2} + \tfrac{[3k_2(b_1+b_2)-(k_1+k_2)b_2]^2}{72b_2(b_1+b_2)^2},\\
&g(\{1\}) = \Pi^C_{02,1} = \tfrac{k_1^2}{18b_1}, && g(\{0,2\}) = \Pi^C_{02} = \tfrac{k_2^2}{8b_2} + \tfrac{k_1^2}{18b_1},\\
&g(\{2\}) = \Pi^C_{01,2} = \tfrac{k_2^2}{18b_2}, && g(\{0,1\}) = \Pi^C_{01} = \tfrac{k_1^2}{8b_1} + \tfrac{k_2^2}{18b_2},
\end{aligned}$$

and the core of g is the set of payoff vectors:

$$C(g) = \{x \in \mathbb{R}^N : \sum_{i \in N} x_i = g(N); \sum_{i \in S} x_i \geq g(S), \forall S \subseteq N\}.$$

That is, the core is a set of imputations where one cannot find any coalition with a value greater than the sum of its members' payoffs. Thus, no coalition will receive a larger payoff

[1]See Chwe (1994) for more discussions on the myopia of the core and the Nash equilibrium.
[2]For example, Π^F_1 may not be equal to $\Pi^C_{02,1}$.

and leave the grand coalition. In a two-product assembly system, coalition stability is analyzed based on whether a grand or partial coalition structure is preferred by each supplier and on the allocation of profit for each member of the coalition. In earlier observations, Supplier 0 plays an essential role in the coalition, given that a partial coalition without Supplier 0 does not provide any extra benefits. As a result, focus was directed on the strategic role of Supplier 0 in the coalition structure and on the identification of a fair profit allocation to each supplier that ensures the stability of coalition.

In the following result, an easy-to-check necessary and sufficient condition will be provided for the existence of a core allocation that involves only the ratios $\frac{b_1}{b_2}$ and $\frac{k_1}{k_2}$. For example, one can easily derive from condition (c) that the core is non-empty whenever $\frac{b_1}{b_2}$ and $\frac{k_1}{k_2}$ lie in the closed interval $[1, 2]$.

Proposition 7. *For the two-product case, the following conditions are equivalent:*

(a) The coalition game g has a non-empty core.

(b) $\frac{5}{4}(\Pi^C_{02,1} + \Pi^C_{01,2}) \geq \Pi^F_1 + \Pi^F_2$.

(c) $\frac{b_1}{b_2}(\frac{k_1}{k_2} + 1)^2 \geq 4(\frac{k_1}{k_2} - \frac{b_1}{b_2})^2$.

Proposition 7 shows that the core of such coalition game is non-empty if the total profit of Suppliers 1 and 2 under the model without cooperation cannot be too high and $\frac{5}{4}(\Pi^C_{02,1}+\Pi^C_{01,2})$ serves as the upper bound of such total profit (part (b)). Part (c) reveals that the ratios of $\frac{b_1}{b_2}$ and $\frac{k_1}{k_2}$ determine whether the core exists. In the following result, the domain of the core is characterized.

Proposition 8. *Assume that the core is non-empty and let $x = (x_0, x_1, x_2)$ be a core allocation.*

(a) In general,

$$\frac{5}{4}(\Pi^C_{02,1} + \Pi^C_{01,2}) \geq \Pi^F_1 + \Pi^F_2 \geq \Pi^C_{02,1} + \Pi^C_{01,2}, \tag{11}$$

and

$$\frac{9}{4}(\Pi^C_{02,1}+\Pi^C_{01,2})-(\Pi^F_1+\Pi^F_2)\geq x_0\geq \Pi^C_{02,1}+\Pi^C_{01,2},$$

$$\frac{5}{4}\Pi^C_{02,1}\geq x_1\geq \max\{(\Pi^F_1+\Pi^F_2)-\frac{5}{4}\Pi^C_{01,2},\Pi^C_{02,1}\}, \tag{12}$$

$$\frac{5}{4}\Pi^C_{01,2}\geq x_2\geq \max\{(\Pi^F_1+\Pi^F_2)-\frac{5}{4}\Pi^C_{02,1},\Pi^C_{01,2}\}.$$

Moreover, $\Pi^F_1+\Pi^F_2=\Pi^C_{02,1}+\Pi^C_{01,2}$ *if and only if* $\frac{k_1}{k_2}=\frac{b_1}{b_2}$.

(b) The point $p^0=(\Pi^C_{02,1}+\Pi^C_{01,2},\frac{5}{4}\Pi^C_{02,1},\frac{5}{4}\Pi^C_{01,2})$ *is an extreme point of the core. In addition, in case*

$$\Pi^F_1+\Pi^F_2\geq \max\{\frac{5}{4}\Pi^C_{02,1}+\Pi^C_{01,2},\Pi^C_{02,1}+\frac{5}{4}\Pi^C_{01,2}\}, \tag{13}$$

the core region is a triangle with extreme points p^0,

$$p^1=(\frac{9}{4}\Pi^C_{02,1}+\frac{9}{4}\Pi^C_{01,2}-\Pi^F_1-\Pi^F_2,\Pi^F_1+\Pi^F_2-\frac{5}{4}\Pi^C_{01,2},\frac{5}{4}\Pi^C_{01,2}),\ \textit{and}$$

$$p^2=(\frac{9}{4}\Pi^C_{02,1}+\frac{9}{4}\Pi^C_{01,2}-\Pi^F_1-\Pi^F_2,\frac{5}{4}\Pi^C_{02,1},\Pi^F_1+\Pi^F_2-\frac{5}{4}\Pi^C_{02,1}).$$

(c) In case $\Pi^C_{02,1}+\frac{5}{4}\Pi^C_{01,2}>\Pi^F_1+\Pi^F_2\geq\frac{5}{4}\Pi^C_{02,1}+\Pi^C_{01,2}$, *or, symmetrically,* $\frac{5}{4}\Pi^C_{02,1}+\Pi^C_{01,2}>\Pi^F_1+\Pi^F_2\geq\Pi^C_{02,1}+\frac{5}{4}\Pi^C_{01,2}$, *the core region is a quadrilateral.*

(d) In case $\Pi^F_1+\Pi^F_2\leq\min\{\frac{5}{4}\Pi^C_{02,1}+\Pi^C_{01,2},\Pi^C_{02,1}+\frac{5}{4}\Pi^C_{01,2}\}$, *the core region is a Pentagon.*

(e) In case $\Pi^F_1+\Pi^F_2=\frac{5}{4}(\Pi^C_{02,1}+\Pi^C_{01,2})$, *the core is a singleton and* p^0 *is the only core allocation.*

Proposition 8 shows that the core region is determined by the amount $\frac{5}{4}(\Pi^C_{02,1}+\Pi^C_{01,2})-(\Pi^F_1+\Pi^F_2)$, which is the difference between the highest and lowest profits for each firm when the core is employed as the profit allocation rule and (13) holds.

Notably, the core is a set-valued solution. Therefore, a practical question is which core allocation to choose as the profit allocation rule. To obtain insights into the issue, this research analyzes the connections among the core and various single-valued profit allocation methods, including the τ-value (Tijs 1981), the nucleolus (Schmeidler 1969) and the Shapley value (Shapley 1967) in the following.

Proposition 9. *Assume that the core of g is non-empty.*

(a) The τ-value $\tau(g) = (\tau_0, \tau_1, \tau_2)$ of g lies in the core.

(b) In case (13) *holds,*

(i) the τ-value $\tau(g)$ is the center of the core region:

$$\begin{aligned}
\tau_0 &= \frac{11}{6}\Pi^C_{02,1} + \frac{11}{6}\Pi^C_{01,2} - \frac{2}{3}\Pi^F_1 - \frac{2}{3}\Pi^F_2,\\
\tau_1 &= \frac{5}{6}\Pi^C_{02,1} - \frac{5}{12}\Pi^C_{01,2} + \frac{1}{3}\Pi^F_1 + \frac{1}{3}\Pi^F_2,\\
\tau_2 &= \frac{-5}{12}\Pi^C_{02,1} + \frac{5}{6}\Pi^C_{01,2} + \frac{1}{3}\Pi^F_1 + \frac{1}{3}\Pi^F_2
\end{aligned}$$

(ii) the τ-value $\tau(g)$ is equal to the nucleolus of g.

Proposition 9 shows that the core, if non-empty, always contains the τ-value; when (13) holds, the τ-value, as well as the nucleolus, lies in the center of the core region. In contrast to the τ-value, the Shapley value $\phi(g)$ of g may lie outside the core. Proposition 10 explicitly gives the formula for the Shapley value $\phi(g)$ and provide a condition under which the Shapley value belongs to the core and coincides with the τ-value and the nucleolus.

Proposition 10. *The Shapley value of g is the payoff allocation $\phi(g) = (\phi_0, \phi_1, \phi_2)$, where*

$$\begin{aligned}
\phi_0 &= \frac{9}{8}(\Pi^C_{02,1} + \Pi^C_{01,2}) + \frac{1}{3}(\Pi^F_0 - \Pi^F_1 - \Pi^F_2),\\
\phi_1 &= \frac{9}{8}\Pi^C_{02,1} + \frac{1}{6}(\Pi^F_1 + \Pi^F_2 - \Pi^F_0),\\
\phi_2 &= \frac{9}{8}\Pi^C_{01,2} + \frac{1}{6}(\Pi^F_1 + \Pi^F_2 - \Pi^F_0).
\end{aligned}$$

Moreover, if $\frac{k_1}{k_2} = \frac{b_1}{b_2}$, then the Shapley value $\phi(g) = (\frac{9}{8}(\Pi^C_{02,1} + \Pi^C_{01,2}), \frac{9}{8}\Pi^C_{02,1}, \frac{9}{8}\Pi^C_{01,2})$ belongs to the core and coincides with the τ-value and the nucleolus.

5 Numerical Study

On the basis of the aforementioned results, this study conducts several numerical examples to investigate the influence of cost and demand characteristics on the wholesale price decisions and supply chain profits under different coalition structures. According to Bortolini et al. (2017) and Camarinha-Matos et al. (2017), cost and demand factors are two of the major considerations in assembly line design; they are also important in the Industry 4.0 era. The numerical examples thus provide applications scenarios for evaluating different forms of cross-sector horizontal partnerships among suppliers under different coalition settings and decision-making environments.

We first examine the effect of cost and demand characteristics on the wholesale prices and supply chain profits under different coalitions. Without loss of generality, we leverage different levels of b_2 (e.g., $b_2 = 0.75$, 1, or 1.25) to represent different demand characteristics while fixing other demand parameters (e.g., $a_1 = 1$, $a_2 = 1$, $b_1 = 1$). Then, by changing different cost parameters c_0, c_1, and c_2, the wholesale prices decision and the profit under different coalitions are obtained based on the closed-form solutions in Section 4.1 ((2) to (4)), Section 4.2 ((7) and (8)), and Section 4.3 ((9) and (10)), as shown in Tables 2, 3, and 4, respectively.

Table 2: Profit gain of different coalition strategies under different demand characteristics. Here, $a_1 = 1, a_2 = 1, b_1 = 1, c_1 = 0.01$, and $c_2 = 0.01$.

		No Cooperation		Partial Coalition		Grand Coalition		% of Grand Coalition Profit	
b_2	c_0	(w_0, w_1, w_2)	Profit	(w_0, w_1, w_2)	Profit	(w_{01}, w_{02})	Profit	No Cooperation	Partial Coalition
0.75	0.0125	(0.386,0.312,0.3118)	0.1280	(0.4494,0.0618,1.0025)	0.2626	(0.0716,0.5113)	0.2805	45.6233%	93.6196%
	0.015	(0.3876,0.3112,0.311)	0.1274	(0.4511,0.0614,1.0017)	0.2615	(0.0713,0.5125)	0.2793	45.6284%	93.6161%
	0.0175	(0.3893,0.3104,0.3103)	0.1269	(0.4528,0.061,1.0008)	0.2603	(0.0711,0.5138)	0.2781	45.6336%	93.6125%
1	0.0125	(0.3383,0.3358,0.3358)	0.1062	(0.3383,0.1729,1.0025)	0.2256	(0.0531,0.5113)	0.2389	44.4444%	94.4444%
	0.015	(0.34,0.335,0.335)	0.1056	(0.34,0.1725,1.0017)	0.2245	(0.0528,0.5125)	0.2377	44.4444%	94.4444%
	0.0175	(0.3417,0.3342,0.3342)	0.1051	(0.3417,0.1721,1.0008)	0.2233	(0.0525,0.5138)	0.2364	44.4444%	94.4444%
1.25	0.0125	(0.3013,0.3544,0.3547)	0.0966	(0.2717,0.2396,1.0025)	0.2034	(0.042,0.5113)	0.2139	45.1659%	95.0934%
	0.015	(0.303,0.3535,0.3538)	0.0961	(0.2733,0.2392,1.0017)	0.2022	(0.0417,0.5125)	0.2127	45.1701%	95.0970%
	0.0175	(0.3046,0.3527,0.3528)	0.0955	(0.275,0.2388,1.0008)	0.2011	(0.0414,0.5138)	0.2115	45.1742%	95.1006%

From the results above, we observe that (i) increasing the demand parameter b_2 or the cost parameters c_i (i=0,1,2) results in decreasing the supply chain profit in all kinds of coalition structures; (ii) the wholesale price of common component, w_0, contains a negative relationship with the price sensitivity of Product 2 for all coalitions; (iii) the wholesale price of component 1, w_1, contains a positive relationship with the price sensitivity of Product

Table 3: Profit gain of different coalition strategies under different demand characteristics. Here, $a_1 = 1, a_2 = 1, b_1 = 1, c_0 = 0.015$, and $c_2 = 0.01$.

		No Cooperation		Partial Coalition		Grand Coalition		% of Grand Coalition Profit	
b_2	c_1	(w_0, w_1, w_2)	Profit	(w_0, w_1, w_2)	Profit	(w_{01}, w_{02})	Profit	No Cooperation	Partial Coalition
	0.0075	(0.3881,0.3097,0.3108)	0.1123	(0.4511,0.0601,1.0017)	0.2621	(0.0713,0.5113)	0.2799	45.6082%	93.6300%
0.75	0.01	(0.3876,0.3112,0.311)	0.1122	(0.4511,0.0614,1.0017)	0.2615	(0.0713,0.5125)	0.2793	45.6284%	93.6161%
	0.0125	(0.3871,0.3127,0.3113)	0.1121	(0.4511,0.0626,1.0017)	0.2609	(0.0713,0.5138)	0.2787	45.6489%	93.6021%
	0.0075	(0.3404,0.3335,0.3348)	0.1057	(0.34,0.1713,1.0017)	0.2251	(0.0528,0.5113)	0.2383	44.4445%	94.4587%
1	0.01	(0.34,0.335,0.335)	0.1056	(0.34,0.1725,1.0017)	0.2245	(0.0528,0.5125)	0.2377	44.4444%	94.4444%
	0.0125	(0.3396,0.3365,0.3352)	0.1055	(0.34,0.1738,1.0017)	0.2238	(0.0528,0.5138)	0.2370	44.4445%	94.4302%
	0.0075	(0.3033,0.3521,0.3536)	0.1010	(0.2733,0.2379,1.0017)	0.2029	(0.0417,0.5113)	0.2133	45.1862%	95.1110%
1.25	0.01	(0.303,0.3535,0.3538)	0.1009	(0.2733,0.2392,1.0017)	0.2022	(0.0417,0.5125)	0.2127	45.1701%	95.0970%
	0.0125	(0.3026,0.355,0.3539)	0.1008	(0.2733,0.2404,1.0017)	0.2016	(0.0417,0.5138)	0.2121	45.1541%	95.0829%

Table 4: Profit gain of different coalition strategies under different demand characteristics. Here, $a_1 = 1, a_2 = 1, b_1 = 1, c_0 = 0.015$, and $c_1 = 0.01$.

		No Cooperation		Partial Coalition		Grand Coalition		% of Grand Coalition Profit	
b_2	c_2	(w_0, w_1, w_2)	Profit	(w_0, w_1, w_2)	Profit	(w_{01}, w_{02})	Profit	No Cooperation	Partial Coalition
	0.0075	(0.388,0.311,0.3098)	0.1278	(0.4519,0.0606,1)	0.2620	(0.0716,0.5125)	0.2799	44.6059%	94.1379%
0.75	0.01	(0.3876,0.3112,0.311)	0.1274	(0.4511,0.0614,1.0017)	0.2615	(0.0713,0.5125)	0.2793	44.6028%	94.1435%
	0.0125	(0.3873,0.3114,0.3123)	0.1271	(0.4503,0.0622,1.0033)	0.2609	(0.0711,0.5125)	0.2787	44.6090%	94.1322%
	0.0075	(0.3404,0.3348,0.3335)	0.1059	(0.3408,0.1717,1)	0.2250	(0.0531,0.5125)	0.2383	44.4444%	94.4444%
1	0.01	(0.34,0.335,0.335)	0.1056	(0.34,0.1725,1.0017)	0.2245	(0.0528,0.5125)	0.2377	44.4445%	94.4501%
	0.0125	(0.3396,0.3352,0.3365)	0.1054	(0.3392,0.1733,1.0033)	0.2239	(0.0525,0.5125)	0.2370	44.4445%	94.4387%
	0.0075	(0.3034,0.3533,0.352)	0.0963	(0.2742,0.2383,1)	0.2028	(0.042,0.5125)	0.2133	44.5773%	94.7232%
1.25	0.01	(0.303,0.3535,0.3538)	0.0961	(0.2733,0.2392,1.0017)	0.2022	(0.0417,0.5125)	0.2127	44.5801%	94.7289%
	0.0125	(0.3025,0.3538,0.3555)	0.0958	(0.2725,0.24,1.0033)	0.2017	(0.0414,0.5125)	0.2121	44.5745%	94.7175%

2 for all coalitions; (iv) no clear functional relationship of the no-cooperation profit/grand coalition profit or partial coalition profit/grand coalition profit to cost-demand parameters.

We also investigate the influence of demand characteristics on the profit allocation among suppliers. Table 5 shows the effect of the demand parameters (i.e., a_i and b_i, $i = 1, 2$) on the upper and lower bounds of the profit allocated to each supplier based on Proposition 8. In particular, (12) in Proposition 8 specifies the upper and lower bounds of each supplier's profit allocation where $x_i, i = 0, 1, 2$ in (12) represents Supplier i's profit allocation. As a_1 increases, the upper and lower bounds of the profit allocated to Suppliers 0 and 1 (to Supplier 2) increase (decrease). However, the effect of b_1 is entirely the opposite. That is, better demand conditions from Product 1 (i.e., high a_1 and low b_1) enable the corresponding

Table 5: Profit allocated to each supplier under different demand characteristics. Here, $c_0 = 0.015, c_1 = 0.01, c_2 = 0.01, a_2 = 1, b_2 = 1$ and % change is (Upper-Lower)/Lower.

		Supplier 0's profit allocation			Supplier 1's profit allocation			Supplier 2's profit allocation		
a_1	b_1	Upper	Lower	% change	Upper	Lower	% change	Upper	Lower	% change
0.9	0.9	0.1254	0.1003	25.00%	0.0594	0.0343	73.08%	0.0660	0.0409	61.29%
	1	0.1188	0.0953	24.64%	0.0532	0.0297	79.15%	0.0660	0.0425	55.24%
	1.1	0.1129	0.0913	23.68%	0.0481	0.0264	81.72%	0.0660	0.0444	48.67%
1	0.9	0.1393	0.1118	24.64%	0.0737	0.0462	59.64%	0.0660	0.0385	71.59%
	1	0.1320	0.1056	25.00%	0.0660	0.0396	66.67%	0.0660	0.0396	66.67%
	1.1	0.1254	0.1006	24.70%	0.0597	0.0349	71.26%	0.0660	0.0412	60.34%
1.1	0.9	0.1540	0.1245	23.70%	0.0896	0.0601	49.09%	0.0660	0.0365	80.76%
	1	0.1459	0.1170	24.70%	0.0803	0.0513	56.30%	0.0660	0.0371	77.89%
	1.1	0.1386	0.1109	25.00%	0.0726	0.0449	61.76%	0.0660	0.0383	72.41%

suppliers (i.e., Suppliers 0 and 1) to obtain a higher portion of the profit under grand coalition. Similar outcomes can be obtained when one adjusts the demand parameter of Product 2 (i.e., a_2 and b_2). Furthermore, the manufacturing cost, c_1, has similar effect to b_1 and this study skips the demonstration of the effect of c_1 to avoid repetition.

6 Discussions and Conclusions

This study mainly analyzes the formation of horizontal partnership among suppliers in multiple industrial sectors connected using common components as an emerging business model. Compared to traditional assembly systems in which suppliers individually choose the wholesale price, this study discusses how a common component supplier forms a coalition with the other suppliers to reduce supply chain inefficiencies. Several interesting results are obtained. Under a partial coalition, the wholesale price of the common component is independent from the demand characteristics of the product, with the components provided by the cooperative and the common component suppliers. When comparing different forms of coalition, the common component wholesale prices under a partial coalition are found to form the bounds of the wholesale prices under the model without cooperation. Moreover, one supplier is not always worse off when the remaining suppliers form a partial coalition compared with when a model without cooperation is formed, which occurs when the other supplier is worse off as a non-cooperative supplier. To consider coalition stability, this study proposes conditions under which the core is non-empty, such that no other coalition can make each of its member better off. A numerical study is conducted to further analyze the effects of model characteristics on the optimal pricing decisions and the associated profits of the suppliers. The model can be extended to the cases of the stochastic demand and of the N-product assembly system, in which most results and the associated discussions remain valid. This outcome benefits the further study of a complex framework, and provides insightful recommendations for decision makers.

A number of managerial and policy insights and implications can be further derived from our study. For decision makers and managers in the private sector, our study shows the potential opportunities associated with forming horizontal, cross-sector partnerships beyond the conventional vertical control models to explore different forms of coalitions initiated by a common component supplier. While coordinating among partners across multiple sectors has traditionally been a challenging task, the recent advance of smart manufacturing and supply-chain technologies for digitalization, robotization, and systems integration, such as Industry 4.0, IoT, and cloud computing, and mechatronic systems, has enabled today's firms to develop better capabilities for information sharing, flexible manufacturing, and, as a result, better collaborative environments for forming cross-sector partnerships. Enabling necessary changes in the human-centered aspect of organizations for employees with new competencies and job profiles is also of increasing importance for today's organizations in transforming toward future intelligent factory systems. For policy makers in the public sector, our study shows the potential to improve supply chain efficiencies through encouraging or inducing horizontal collaboration among multiple firms in a supply chain. The use of policy instruments, such as financial incentives and subsidies, could be an interesting topic that warrants future research to realize the benefits of horizontal partnerships, especially in the wake of global pandemic and trade wars.

In conclusion, this study not only shows the pricing and the profit impacts of different coalition structures but also identifies the conditions under which a horizontal partnership can be sustained while considering profitability and legality. This study analyzes the effects of different forms of partnerships under the settings of deterministic and stochastic demands and explores the arrangements where a formed partnership is viable (stable). Our analyses demonstrate the potential opportunity for companies that only utilize cross-sector partnerships for joint design and manufacturing decisions, such as A123 Systems, to enhance profitability through collaborated pricing decisions. Our analyses prescribe a mechanism for profit maximization and allocations for companies that also engage in coordinating prices among suppliers, such as Intel. Such analyses will enrich understanding on the theoretical and practical aspects of forming cross-sector horizontal partnership connected by the use of common components as a new form of business model in supply chain management, which can benefit the participating firms and consumers. This research also has some limitations which can be further analyzed in the future. First, the demands of both products are assumed not substituted for each other. This assumption can be relaxed when one considers products are fully or partially substitutes in the same market sector. Besides horizontal competition, this relaxation may enhance competition between the suppliers that changes alliance structure. Moreover, we assume there exists only one common component supplier in the channel. This assumption assists us in analyzing different forms of coalition easily and obtaining the optimal solutions in the closed forms. Considering more complicated scenarios can provide additional insights for practical implications.

References

Akash, K., Polson, P., Reid, T., Jain, N. (2019a), "Improving Human-Machine Collaboration Through Transparency-based Feedback-Part I: Human Trust and Workload Model," *IFAC-PapersOnLine*, **51**(34), 315-321.

Akash, K., Polson, P., Reid, T., Jain, N. (2019b), "Improving Human-Machine Collaboration Through Transparency-based Feedback-Part II: Control Design and Synthesis," *IFAC-*

PapersOnLine, **51**(34), 322-328.

Ansari, F., Hold, P., Khobreh, M. (2020), "A Knowledge-Based Approach for Representing Jobholder Profile toward Optimal Human-Machine Collaboration in Cyber Physcial Production Systems," *CIRP Journal of Manufacturing Science and Technology*, **28**, 87-106.

Atan, Z., Ahmadi, T., Stegehuis, C., de Kok, T., & Adan, I. (2017). Assemble-to-order systems: A review. *European Journal of Operational Research*, **261**(3), 866-879.

Baker, K. R., Magazine, M. J., & Nuttle, H. L. (1986). The effect of commonality on safety stock in a simple inventory model. *Management Science*, **32**(8), 982-988.

Belkadi, F., Dhuieb, M.A., Aguado, J.V., Laroche, F., Bernard, A., and Chinesta, F. (2020), "Intelligent assistant system as a context-aware decision-making support for the workers of the future," *Computers & Industrial Engineering*, **139**, 105732.

Bernstein, F., & DeCroix, G. A. (2004). Decentralized pricing and capacity decisions in a multitier system with modular assembly. *Management Science*, **50**(9), 1293-1308.

Bernstein, F., & DeCroix, G. A. (2006). Inventory policies in a decentralized assembly system. *Operations Research*, **54**(2), 324-336.

Bernstein, F., DeCroix, G. A., & Wang, Y. (2007). Incentives and commonality in a decentralized multiproduct assembly system. *Operations Research*, **55**(4), 630-646.

Bortolini, M., Faccio, M., Gamberi, M., and Pilati, F. (2020), "Motion Analysis System (MAS) for production and ergonomics assessment in the manufacturing processes," *Computers & Industrial Engineering*, **139**, 105485.

Bortolini, M., Ferrari, E., Gamberi, M., Pilati, F., & Faccio, M. (2017). Assembly System Design in the Industry 4.0 Era: A General Framework. *IFAC PapersOnLine*, **50**(1), 5700–5705.

Camarinha-Matos L. M., Fornasiero R., & Afsarmanesh H. (2017). Collaborative Networks as a Core Enabler of Industry 4.0. In: Collaboration in a Data-Rich World. *IFIP Advances in Information and Communication Technology*, **506**, 3-17.

Carr, S. M., & Karmarkar, U. S. (2005). Competition in multiechelon assembly supply chains. *Management Science*, **51**(1), 45-59.

Cattani, K. (1995). Evaluating universal designs for product end of life. Working paper, Stanford University, Stanford, CA 94305.

Chen, Z. L. & Hall, N. G. (2007). Supply chain scheduling: Conflict and cooperation in assembly systems. *Operations Research*, **55**(6), 1072-1089.

Chen, K. & Xiao, T. (2017). Pricing and replenishment policies in a supply chain with competing retailers under different retail behaviors. *Computers & Industrial Engineering*, **103**, 145-157.

Chwe, M. S. (1994). Farsighted coalitional stability. *Journal of Economic Theory*, **63**, 299-325.

Clark, K. B., & Fujimoto, T. (1991). Product Development Performance (Harvard Business School Press, Boston, MA). ClarkProduct Development Performance1991.

Cohen, Y., Golan, M., Singer, G., and Faccio, M. (2018), "Workstation–Operator interaction in 4.0 era: WOI 4.0," *IFAC-PapersOnLine*, **51**(11), 399-404.

Corbett, C., & Karmarkar, U. S. (2001). Competition and structure in serial supply chains with deterministic demand. *Management Science*, **47**(7), 966-978.

David, A., & Adida, E. (2015). Competition and coordination in a two-channel supply chain. *Production and Operations Management*, **24**(8), 1358-1370.

Dhungana, D., Haselböck, A., & Wallner, S. (2020). Generation of Multi-factory Production Plans: Enabling Collaborative Lot-size-one Production. Working Paper.

Digital Times. (2011). Intel downstream partners request CPU price drop. *Digital Times* (September 20, 2011).

Driessen, T.S.H. & Tijs, S.H. (1985). The τ-value, the core and the semiconvex games. *International Journal of Game Theory*, 229-248.

Dyer, J. H. (1996). Specialized supplier networks as a source of competitive advantage: Evidence from the auto industry. *Strategic Management Journal*, **17**, 271-292.

Dyer, J. H., & Singh, H. (1998). The relational view: Cooperative strategy and sources of interorganizational competitive advantage. *Academy of Management Review*, **23**, 660-679.

El Mouayni, I., Etienne, A., Lux, A., Siadat, A., and Dantan, J.-Y. (2020), "A simulation-based approach for time allowances assessment during production system design with consideration of worker's fatigue, learning and reliability," *Computers & Industrial Engineering*, **139**, 105650.

Ennis, C., Barnett, N., De Cesare, S., Lander, R., & Pilkington, A. (2018). A Conceptual Framework for Servitization in Industry 4.0: Distilling Directions for Future Research. *Proceedings of the Advance Services Group Spring Servitization Conference.*

Eynan, A. (1996). The impact of demand's correlation on the effectiveness of component commonality. *International Journal of Production Research*, **34**(6), 1581-1602.

Eynan, A., & Rosenblatt, M. J. (1996). Component commonality effects on inventory costs. *IIE Transactions*, **28**(2), 93-104.

Eynard, B. and Cherfi, Z. (2020), "Digital and organizational transformation of industrial systems," *Computers & Industrial Engineering*, **139**, 106197.

Fang, X., So, K. C., & Wang, Y. (2008). Component procurement strategies in decentralized assemble-to-order systems with time-dependent pricing. *Management Science*, **54**(12), 1997-2011.

Fang, X., & Cho, S. H. (2020). Cooperative approaches to managing social responsibility in a market with externalities. *Manufacturing & Service Operations Management*, forthcoming.

Fantini, P., Pinzone, M., & Taisch, M. (2020). Placing the operator at the centre of Industry 4.0 design: Modelling and assessing human activities within cyber-physical systems. *Computers & Industrial Engineering*, **139**, 105058.

Fettig, K., Gacic, T., Köskal, A., Kuehn, A., & Stuber, F. (2018). Impact of Industry 4.0 on Organizational Structures. *Proceedings of 2018 IEEE International Conference on Engineering Technology and Innovation.*

Fletcher, S.R., Johnson, T., Adlon, T., Larreina, J., Casla, P.,Parigot, L., Alfaro, P.J., Otero, M. (2020), "Adaptive automation assembly: Identifying system requirements for technical efficiency and worker satisfaction," *Computers & Industrial Engineering*, **139**, 105772.

Forbes (2018). Industrie 4.0: Why Openness and Collaboration Make All The Difference. *Forbes* (Jan 24, 2018).

Fu, H., Li, K., & Fu, W. (2020). Investing in suppliers with capacity constraints in a decentralized assembly system. *Computers & Industrial Engineering*, **142**, 106332.

Gao, E., Sowlati, T., & Akhtari, S. (2019). Profit allocation in collaborative bioenergy and biofuel supply chains. *Energy*, **188**, 116013.

Garzon, R., & Alejandro, M. (2019). Autonomous Assembly for Lot-Size-One Production. *Proceedings of IEEE International Conference on Intelligent Robots and Systems*.

Gerchak, Y., Magazine, M. J., & Gamble, A. B. (1988). Component commonality with service level requirements. *Management Science*, **34**(6), 753-760.

Gerchak, Y., & Wang, Y. (2004). Revenue sharing vs. wholesale-price contracts in assembly systems with random demand. *Production and Operations Management*, **13**(1), 23-33.

Gillies, D. B. (1959). Solutions to general non-zero-sum games. In Tucker, A. W.; Luce, R. D.. Contributions to the Theory of Games IV. (Annals of Mathematics Studies 40). Princeton: Princeton University Press.

Glock, C.H., Grosse, E.H., Neumann, W.P., and Sgarbossa, F. (2017), "Human factors in industrial and logistic system design," *Computers & Industrial Engineering*, **111**, 463-466.

Granot, D., & Sosic, G. (2005). Formation of alliances in internet-based supply exchanges. *Management Science*, **51**(1), 92-105.

Granot, D., & Yin, S. (2008). Competition and cooperation in decentralized push and pull assembly systems. *Management Science*, **54**(4), 733-747.

Grant, R. (1996). Prospering in dynamically-competitive environments: Organizational capacity as knowledge integration. *Organization Science*, **7**, 375-387.

Gupta, S., Modgil, S., Gunasekaran, A., & Bag, S. (2020). Dynamic Capabilities and Institutional Theories for Industry 4.0 and Digital Supply Chain. *Supply Chain Forum: An International Journal*, forthcoming.

He, Y., & Yin, S. (2015). Joint selling of complementary components under brand and retail competition. *Manufacturing & Service Operations Management*, **17**(4), 470-479.

Hoyer, C., Gunawan, I., Reaiche, C.H. (2020), "The Implementation of Industry 4.0 - A Systematic Literature Review of the Key Factors," *Systems Research and Behavioral Science*, **37**(4), 557-578.

Huang, X., Boyaci, T., Gumus, M. Ray, S. & Zhang, D. (2016). United we stand or divided we stand? Strategic supplier alliances under order defauly risk. *Management Science*, **62**(5), 1297-1315.

Huxtable, J., & Schaefer, D. (2016). On Servitization of the Manufacturing Industry in the UK. *Procedia CIRP*, **52**(1), 46–51.

Jazdi, N. (2014). Cyber physical systems in the context of Industry 4.0. *In 2014 IEEE international conference on automation, quality and testing, robotics* (pp. 1-4). IEEE.

Jerman, A., Bach, M.P., Aleksic, A. (2020), "Transformation towards Smart Factory System: Examining New Job Profiles and Competencies," *Systems Research and Behavioral Science*, **37**(2), 557-578.

Jiang, L., & Wang, Y. (2010). Supplier competition in decentralized assembly systems with price-sensitive and uncertain demand. *Manufacturing & Service Operations Management*, **12**(1), 93-101.

Kannengiesser, U., Heininger, R., Billy, L., Terpak, P., Neubauer, M., & Stary, C. (2017). Lot-Size One Production. in Stary, C. and Neubauer, M., *S-BPM in the Production Industry: A Stakeholder Approach*, Springer, 69-111.

Karnouskos, S., Ribeiro, L., Leitão, P., Lüder, A., & Vogel-Heuser, B. (2019). Key Directions for Industrial Agent-Based Cyber-Physical Production Systems. *Proceedings of IEEE*

International Conference on Industrial Cyber Physical Systems, 17-22.

Kiraz, A., Canpolat, O., Özkurt, C., & Taşkın, H. (2020). Analysis of the factors affecting the Industry 4.0 tendency with the structural equation model and an application. *Computers & Industrial Engineering*, **150**, 106911.

Kovacs, K., Ansari, F., Geisert, C., Uhlmann, E., Glawar, R., Sihn, W. (2019), "A Process Model for Enhancing Digital Assistance in Knowledge-Based Maintenance," *In Machine Learning for Cyber Physical Systems*, pp87-96. Springer Vieweg, Berlin, Heidelberg

Kukushkin, N. S. (2017). Strong Nash equilibrium in games with common and complementary local utilities. *Journal of Mathematical Economics*, **68**, 1-12.

Lavie, D. (2006). The Competitive Advantage of Interconnected Firms: An Extension of the Resource-Based View. *Academy of Management Review*, **31**, 638–658.

Leitao, P., Karnouskos, S., Ribeiro, L., Lee, J., Strasser, T., & Colombo, A. W. (2016). Smart agents in industrial cyber–physical systems. *Proceedings of the IEEE*, **104**(5), 1086-1101.

Lee, J., Bagheri, B., & Kao, H. A. (2015). *A cyber-physical systems architecture for industry 4.0-based manufacturing systems. Manufacturing letters*, **3**, 18-23.

Li, G., Li, L., Liu, M., & Sethi, S. P. (2018). Impact of power structures in a subcontracting assembly system. *Annals of Operations Research*, 1-24.

Li, T., & Chen, J. (2020). Alliance formation in assembly systems with quality-improvement incentives. *European Journal of Operational Research*, **285**(3), 931-940.

Lins, T., & Oliveira, R. A. R. (2020). Cyber-physical production systems retrofitting in context of industry 4.0. *Computers & Industrial Engineering*, **139**, 106193.

Lithoxoidou, E., Doumpoulakis S., Tsakiris, A., Ziogou, C., Krinidis, S., Paliokas, I.,Ioannidis, D., Votis, K., Voutetakis, S., Elmasllari, E., and Tzovaras, D. (2020), "A novel social gamified collaboration platform enriched with shop-floor data and feedback for the improvement of the productivity, safety and engagement in factories," *Computers & Industrial Engineering*, **139**, 105691.

Longo, F., Nicoletti, L., & Padovano, A. (2017). Smart operators in industry 4.0: A human-centered approach to enhance operators' capabilities and competencies within the new smart factory context. *Computers & Industrial Engineering*, **113**, 144-159.

Majumder, P., & Srinivasan, A. (2008). Leadership and competition in network supply chains. *Management Science*, **54**(6), 1189-1204.

Manavalan, E. & Jayakrishna, K. (2019). A review of Internet of Things (IoT) embedded sustainable supply chain for industry 4.0 requirements. *Computers & Industrial Engineering*, **127**, 925-953.

Mattsson, S., Fast-Berglund, A., Li, D., and Thorvald, P. (2020), "Forming a cognitive automation strategy for Operator 4.0 in complex assembly," *Computers & Industrial Engineering*, **139**, 105360.

Miyashita, K., & Russell, D. (1995). Keiretsu: Inside the hidden Japanese Conglomerates. McGraw-Hill Companies.

Nagarajan, M., & Bassok, Y. (2008). A bargaining framework in supply chains: the assembly problem. *Management Science*, **54**(8), 1482-1496.

Nagarajan, M., & Sosic, G. (2009). Coalition stability in assembly models. *Operations Research*, **57**(1), 131-145.

Nagarajan, M., Sosic, G. & Tong, C. (2019). Dynamic Stable Supplier Coalitions and Invariance in Assembly Systems with Commodity Components. *Operations Research*, **67**(5), 1269-1282.

Norde, H. Ozen, U. & Slikker, M (2016). Setting the right incentives for global planning and operations. *European Journal of Operational Research*, **253**, 441-455.

Oliver, C. (1997). Sustainable competitive advantage: Combining institutional and resource-based views. *Strategic Management Journal*, **18**, 697-714.

Pacaux-Lemoine, M. P., Trentesaux, D., Rey, G. Z., & Millot, P. (2017). Designing intelligent manufacturing systems through Human-Machine Cooperation principles: A human-centered approach. *Computers & Industrial Engineering*, **111**, 581-595.

Pei Breivold, H. (2020). Towards Factories of the Future: Migration of Industrial Legacy Automation Systems in the Cloud Computing and Internet-of-Things Context. *Enterprise Information Systems*, **14**, 542-562.

Pinzone, M., Albè, F., Orlandelli, D., Barletta, I., Berlin, C., Björn, J., and Taisch, M. (2020), "A framework for operative and social sustainability functionalities in Human-Centric Cyber-Physical Production Systems," *Computers & Industrial Engineering*, **139**, 105132.

Porter, M.E., & Heppelmann, J.E. (2014). How Smart, Connected Products Are Transforming Competition. *Harvard Business Review*, **92**(11), 64–88.

Renna, P. (2010). Negotiation policies and coalition tools in e-marketplace environment. *Computers & Industrial Engineering*, **59**(4), 619-629.

Radhi, M. & Zhang, G. (2018). Pricing policies for a dual-channel retailer with cross-channel returns. *Computers & Industrial Engineering*, **119**, 63-75.

Rauch, E., Linder, C., and Dallasega, P. (2020), "Anthropocentric perspective of production before and within Industry 4.0," *Computers & Industrial Engineering*, **139**, 105644.

Raweewan, M., & Ferrell Jr, W. G. (2018). Information sharing in supply chain collaboration. *Computers & Industrial Engineering*, **126**, 269-281.

Romero, D., Stahre, J., & Taisch, M. (2020). The Operator 4.0: Towards socially sustainable factories of the future. *Computers & Industrial Engineering*, **139**, 106128

Salunkhe, O., Fasth Berglund, Å. (2020). Increasing Operational Flexibility Using Industry 4.0 Enabling Technologies in Final Assembly. *IEEE Transactions on Communication Technology*, forthcoming.

Schmeidler, D. (1969). The nucleolus of a characteristic function game. *SIAM Journal on Applied Mathematics*, **17**, 1163-1170.

Shapley, L. S. (1967). On balanced set and cores. *Naval Research Logistics Quarterly*, **14**, 453-460.

Song, J. S., & Zipkin, P. (2003). Supply chain operations: assemble-to-order systems. *Hand-*

books in operations research and management science, 11, 561-596

Sony, M. (2020). Pros and Cons of Implementing Industry 4.0 for the Organizations: A Review and Synthesis of Evidence. *Production & Manufacturing Research*, **8**(1), 244–272.

Sutton, J. (1998). *Technology and Market Structure: Theory and History*. MIT Press. Cambridge, Mass.

Tijs, S.H. (1981). Bounds for the core of a game and the τ-value. *Game Theory and Mathematical Economics*, **198**, 123-132.

Taylor, M.P., Boxall, P., Chen, J.J.J., Xu, X., Liew, A., and Adeniji, A. (2020), "Operator 4.0 or Maker 1.0? Exploring the implications of Industrie 4.0 for innovation, safety and quality of work in small economies and enterprises," *Computers & Industrial Engineering*, **139**, 105486.

Varian, H. R. (1992). *Microeconomic Analysis*. W. W. Norton & Company.

Wang, Y. Z., & Gerchak, Y. (2003). Capacity games in assembly systems with uncertain demand. *Manufacturing & Service Operations Management*, **5**(3), 252-267.

Wang, Y. (2006). Joint pricing-production decisions in supply chains of complementary products with uncertain demand. *Operations Research*, **54**(6), 1110-1127.

Waschull, S., Bokhorst, J.A.C., Molleman, E., and Wortmann, J.C. (2020), "Work design in future industrial production: Transforming towards cyber-physical systems," *Computers & Industrial Engineering*, **139**, 105679.

Wei, J., Wei, J., Zhao, J., & Hou, X. (2019). Integration strategies of two supply chains with complementary products. *International Journal of Production Research*, **57**(7), 1972-1989.

Williamson, O. E. (1983). Credible commitments: Using hostages to support exchange. *American Economic Review*, **73**, 519-535.

Xiao, Y., Chen, J., & Lee, C. Y. (2010). Single-period two product assemble-to-order systems with a common component and uncertain demand patterns. *Production and Operations Management*, **19**(2), 216-232.

Yadav, G., Kumar, A., Luthra, S., Garza-Reyes, J. A., Kumar, V., & Batista, L. (2020). A framework to achieve sustainability in manufacturing organisations of developing economies using industry 4.0 technologies' enablers. *Computers & Industrial Engineering*, **122**, 103280.

Yang, C., Shen, W., & Wang, X. (2018). The Internet of Things in Manufacturing: Key Issues and Potential Applications. *IEEE Systems, Man, and Cybernetics Magazine*, **4**(1), 6–15.

Yin, S. (2010). Alliance formation among perfectly complementary suppliers in a price-sensitive assembly system. *Manufacturing & Service Operations Management*, **12**(3), 527-544.

Zhang, X. & Huang, G. Q. (2010). Game-theoretic approach to simultaneous configuration of platform products and supply chains with one manufacturing firm and multiple cooperative suppliers. *International Journal of Production Economics*, **124**(1), 121-136.

Zhang, X., Ou, J., & Gilbert, S. M. (2008). Coordination of stocking decisions in an assembler-to-order environment. *European Journal of Operational Research*, **189**, 540-558.

A Appendix

All the proofs are put in Section A.1. The case where the demand is stochastic and the case for N-product assembly system are in Sections A.2 and A.3, respectively.

A.1 Proofs

Proof of Proposition 1

The result is directly from equation (7). $\square$

Proof of Proposition 2

Consider partial coalition that includes Suppliers 1 and 2. The joint profit of Suppliers 1 and 2, $\Pi_{12}^C(w_1, w_2)$, and Supplier 0's profit, $\Pi_{12,0}^C(w_0)$, are given by

$$\Pi_{12}^C(w_1, w_2) = (w_1 - c_1)(a_1 - b_1 p_1^*(w_0, w_1)) + (w_2 - c_2)(a_2 - b_2 p_2^*(w_0, w_2)), \quad \text{(A-1)}$$

$$\Pi_{12,0}^C(w_0) = (w_0 - c_0)(a_1 - b_1 p_1^*(w_0, w_1) + a_2 - b_2 p_2^*(w_0, w_2)). \quad \text{(A-2)}$$

Suppliers 1 and 2 set the respective wholesale price based on the first-order conditions of equation (A-1), namely, $\frac{\partial \Pi_{12}^C(w_1,w_2)}{\partial w_1} = 0$ and $\frac{\partial \Pi_{12}^C(w_1,w_2)}{\partial w_2} = 0$. Since the first term of equation (A-1) is independent of w_2 and the second term is independent of w_1 (no substitution), the results of the first-order condition are equivalent to the results in the model without cooperation (equation (3)). In addition, Supplier 0's profit is $\Pi_{12,0}^C(w_{12,0}^C) = \Pi_0^F(w_0^F)$ and the joint profit of Suppliers 1 and 2 is $\Pi_{12}^C(w_{12,1}^C, w_{12,2}^C) = \Pi_1^F(w_1^F) + \Pi_2^F(w_2^F)$ from equation (4). Thus, when Suppliers 1 and 2 form partial coalition, the optimal wholesale prices and the resultant profit of each supplier are the same as those of the model without cooperation. $\square$

Proof of Proposition 3

Define $d_i = a_i + b_i(2c_0 - c_i), i = 1, 2$. Then from equation (3), $w_0^F = (d_1 + d_2)/3(b_1 + b_2)$ and from equation (7), $w_{01,0}^C = d_2/3b_2$ and $w_{02,0}^C = d_1/3b_1$, respectively. Now

$$w_{01,0}^C - w_0^F = \frac{d_2}{3b_2} - \frac{d_1 + d_2}{3(b_1 + b_2)} = \frac{b_1 d_2 - d_1 b_2}{3b_2(b_1 + b_2)}, \text{ and } w_0^F - w_{02,0}^C = \frac{d_1 + d_2}{3(b_1 + b_2)} - \frac{d_1}{3b_1} = \frac{b_1 d_2 - d_1 b_2}{3b_1(b_1 + b_2)}.$$

Observe above that $w_0^F \leq w_{01,0}^C$ if and only if $w_0^F \geq w_{02,0}^C$ because both b_1 and b_2 are positive. The same logic can be applied to the case that $w_0^F \geq w_{01,0}^C$ if and only if $w_0^F \leq w_{02,0}^C$. That is, $w_{01,0}^C$ and $w_{02,0}^C$ form the upper and lower bounds of w_0^F. To show that if $c_1 \geq c_2$, then $w_{02,0}^C \leq w_0^F \leq w_{01,0}^C$, notice that since $\frac{a_1}{b_1} \leq \frac{a_2}{b_2}$ and $c_1 \geq c_2$, it is obtained that$w_{01,0}^C = \frac{d_2}{3b_2} = \frac{a_2+b_2(2c_0-c_2)}{3b_2} \geq \frac{a_1+b_1(2c_0-c_1)}{3b_1} = \frac{d_1}{3b_1} = w_{02,0}^C$. Since $w_{01,0}^C$ and $w_{02,0}^C$ form the upper and lower bounds of w_0^F, which obtain $w_{02,0}^C \leq w_0^F \leq w_{01,0}^C$. □

Proof of Proposition 4

Based on equations (3) and (7), $w_{01,2}^C - w_2^F = -\frac{b_1k_2-b_2k_1}{6b_2(b_1+b_2)}$, and $w_{01,1}^C - w_1^F = -\frac{b_1k_2-b_2k_1}{6b_2(b_1+b_2)} - \frac{k_2}{6b_2} = w_{01,2}^C - w_2^F - \frac{k_2}{6b_2}$. To show the result, first note that both k_i and b_i are positive for $i = 1, 2$. Besides, since $\frac{a_1}{b_1} \leq \frac{a_2}{b_2}$ and $c_1 \geq c_2$, thus, $w_{01,2}^C \leq w_2^F$. The result of $w_{01,1}^C \leq w_1^F$ directly follows due to the fact that $-\frac{k_2}{6b_2} \leq 0$, which completes the proof. □

Proof of Proposition 5

To prove the result, first, it is showed that the optimal wholesale price of each product under a grand coalition is less than that under the model without cooperation, Note from equations (3) and (9), then for $i = 1, 2$, $(w_0^F + w_i^F) - (w_0^G + w_i^G) = \frac{k_1+k_2}{6(b_1+b_2)} \geq 0$.

For the comparison between the grand coalition and the partial coalition, consider Supplier 0 cooperates with Supplier i, then from equations (7) and (9), for $i, j = 1, 2, i \neq j$, $(w_{0i,0}^C + w_{0i,i}^C) - (w_0^G + w_i^G) = 0$, and $(w_{0i,0}^C + w_{0i,j}^C) - (w_0^G + w_j^G) = \frac{k_j}{6b_j} \geq 0$. Notice that if Suppliers 1 and 2 form a partial coalition, based on Proposition 2, the wholesale price decisions are identical to those under the model without cooperation. Thus, the result directly follows, showing the wholesale price of each product under a grand coalition is less than that under the other coalition structures. □

Proof of Proposition 6

Note that based on equations (4) and (8), then for $i = 1, 2$ that $\Pi_i^F - \Pi_{0j,i}^C = \frac{(b_ik_j-b_jk_i)(b_ik_j-5b_jk_i-4b_ik_i)}{72b_i(b_1+b_2)^2}$. To prove the result, it suffices to show that $(\Pi_i^F - \Pi_{0j,i}^C)(\Pi_j^F - \Pi_{0i,j}^C) \leq 0$. That is,

$$\frac{-(b_1k_2 - b_2k_1)^2}{5184b_1b_2(b_1 + b_2)^4}(b_1k_2 - 5b_2k_1 - 4b_1k_1)(b_2k_1 - 5b_1k_2 - 4b_2k_2) \leq 0. \qquad \text{(A-3)}$$

First, it is showed that $(b_1k_2 - 5b_2k_1 - 4b_1k_1) \leq 0$. Notice from equation (3) that the profit margin of supplier 1 under the model without cooperation is non-negative, that is,

$$w_1^F - c_1 = \frac{(4c_1 - 2c_0)b_1^2 + (2a_1 + b_2c_2 - 2b_2c_0 + 3b_2c_1 - a_2)b_1 + 3b_2a_1}{6b_1(b_1 + b_2)} - c_1 \geq 0. \quad \text{(A-4)}$$

Since both b_1 and b_2 are positive, the inequality in equation (A-4) is identical to

$$-b_1b_2c_2 \leq 4c_1b_1^2 - 2c_0b_1^2 + 2a_1b_1 - 2b_1b_2c_0 + 3b_1b_2c_1 - a_2b_1 + 3a_1b_2 - 6b_1(b_1 + b_2)c_1. \quad \text{(A-5)}$$

Thus, $b_1k_2 - 5b_2k_1 - 4b_1k_1 \leq 2b_1b_2c_0 + 8c_1b_1^2 + 2b_1^2c_0 - 2a_1b_1 + 8b_1b_2c_1 - 2a_1b_2 - 6b_1c_1(b_1 + b_2) \leq 0$, where the first inequality is from equation (A-5) and the second inequality is from $k_1 = a_1 - b_1(c_0 + c_1) \geq 0$. Thus, $(b_1k_2 - 5b_2k_1 - 4b_1k_1) \leq 0$. it is followed that the same logic to prove $b_2k_1 - 5b_1k_2 - 4b_2k_2 \leq 0$. The result directly follows from equation (A-3). □

Proof of Proposition 7

Bondareva (1963) and Shapley (1967) independently prove that a coalitional game g has a non-empty core if and only if $g(N) \geq \sum_{S \subseteq N} \lambda_S g(S)$ for each system of non-negative weights $\{\lambda_S : S \subseteq N\}$ satisfying $\sum_{S \subseteq N} \lambda_S \chi_S = \chi_N$, where χ_S is the characteristic vector defined by $i = 0, 1, 2$,

$$\chi_S(i) = \begin{cases} 1, & \text{if } i \in S, \\ 0, & \text{otherwise.} \end{cases}$$

In the light of the Bondareva-Shapley Theorem, it follows that $C(g) \neq \emptyset$ if and only if the following inequalities hold:

(i) $g(\{0, 1, 2\}) \geq g(\{0\}) + g(\{1, 2\})$

(ii) $g(\{0, 1, 2\}) \geq g(\{1\}) + g(\{0, 2\})$

(iii) $g(\{0, 1, 2\}) \geq g(\{2\}) + g(\{0, 1\})$

(iv) $g(\{0, 1, 2\}) \geq g(\{0\}) + g(\{1\}) + g(\{2\})$

(v) $g(\{0,1,2\}) \geq \frac{1}{2}[g(\{0,1\}) + g(\{0,2\}) + g(\{1,2\})]$.

First note that inequalities (i), (ii) and (iii) always hold since the members of a coalition can always act as if they are independent from each other. Moreover, by the arithmetic-geometric mean inequality, then

$$\begin{aligned}\frac{k_1^2}{b_1} + \frac{k_2^2}{b_2} &= \frac{b_2k_1^2 + b_1k_2^2}{b_1b_2} = \frac{(b_2k_1^2 + b_1k_2^2)(b_1 + b_2)}{b_1b_2(b_1 + b_2)} = \frac{b_1b_2k_1^2 + b_1b_2k_2^2 + b_2^2k_1^2 + b_1^2k_2^2}{b_1b_2(b_1 + b_2)} \\ &\geq \frac{b_1b_2k_1^2 + b_1b_2k_2^2 + 2b_1b_2k_1k_2}{b_1b_2(b_1 + b_2)} = \frac{k_1^2 + k_2^2 + 2k_1k_2}{(b_1 + b_2)} = \frac{(k_1 + k_2)^2}{b_1 + b_2},\end{aligned}$$

which immediately leads to inequality (iv). Hence, $C(g) \neq \emptyset$ if and only if

$$\frac{5k_1^2}{b_1} + \frac{5k_2^2}{b_2} \geq \frac{[3k_1(b_1 + b_2) - (k_1 + k_2)b_1]^2}{b_1(b_1 + b_2)^2} + \frac{[3k_2(b_1 + b_2) - (k_1 + k_2)b_2]^2}{b_2(b_1 + b_2)^2}, \tag{A-6}$$

or equivalently, $\frac{5}{4}[\Pi^C_{02,1} + \Pi^C_{01,2}] \geq \Pi^F_1 + \Pi^F_2$. Moreover, since

$$\begin{aligned}&\frac{5k_1^2}{b_1} + \frac{5k_2^2}{b_2} - \frac{[3k_1(b_1 + b_2) - (k_1 + k_2)b_1]^2}{b_1(b_1 + b_2)^2} - \frac{[3k_2(b_1 + b_2) - (k_1 + k_2)b_2]^2}{b_2(b_1 + b_2)^2} \\ &\quad = \frac{(k_1 + k_2)^2b_1b_2 - 4(k_1b_2 - k_2b_1)^2}{b_1b_2(b_1 + b_2)} = \frac{k_2^2b_2^2}{b_1b_2(b_1 + b_2)}\left[\frac{b_1}{b_2}(\frac{k_1}{k_2} + 1)^2 - 4(\frac{k_1}{k_2} - \frac{b_1}{b_2})^2\right],\end{aligned}$$

the inequality (equation (A-6)) holds if and only if $\frac{b_1}{b_2}(\frac{k_1}{k_2} + 1)^2 \geq 4(\frac{k_1}{k_2} - \frac{b_1}{b_2})^2$. □

Proof of Proposition 8

The first inequality of (11) follows from Proposition 7, and the second inequality of (11) follows from the fact that $\Pi^F_1 + \Pi^F_2 - \Pi^C_{02,1} - \Pi^C_{01,2} = \frac{5(b_1k_2 - b_2k_1)^2}{72b_1b_2(b_1+b_2)} \geq 0$, from which one immediately obtains that $\Pi^F_1 + \Pi^F_2 = \Pi^C_{02,1} + \Pi^C_{01,2}$ if and only if $\frac{k_1}{k_2} = \frac{b_1}{b_2}$. Moreover, it is straightforward to check (b), (c), (d) and (e). □

Proof of Proposition 9

The τ-value of g is defined as the unique point $\tau(g) = (\tau_0, \tau_1, \tau_2)$ on the line segment $[a, b]$

such that $\tau_0 + \tau_1 + \tau_2 = g(\{0,1,2\})$, where $b_i = g(\{0,1,2\}) - g(\{0,1,2\}\backslash\{i\})$ and

$$a_i = \max\{g(S) - \sum_{k \in S\backslash\{i\}} b_k | i \in S \subseteq \{0,1,2\}\} \text{ for } i = 0,1,2.$$

Hence,

$$\begin{array}{ll} b_0 = \frac{9}{4}(\Pi^C_{02,1} + \Pi^C_{01,2}) - (\Pi^F_1 + \Pi^F_2) & a_0 = \Pi^C_{02,1} + \Pi^C_{01,2} \\ b_1 = \frac{5}{4}\Pi^C_{02,1} & a_1 = \max\{\Pi^C_{02,1}, \Pi^F_1 + \Pi^F_2 - \frac{5}{4}\Pi^C_{01,2}\} \\ b_2 = \frac{5}{4}\Pi^C_{01,2} & a_2 = \max\{\Pi^C_{01,2}, \Pi^F_1 + \Pi^F_2 - \frac{5}{4}\Pi^C_{02,1}\}. \end{array}$$

One can directly check $\tau(g) \in C(g)$, or apply Corollary 2.5 of Driessen and Tijs (1985), which shows that the τ-value of a three-player game is a core allocation if $b_i \geq a_i$ for $i = 0,1,2$. Moreover, in case (13) holds, then $a_1 = \Pi^F_1 + \Pi^F_2 - \frac{5}{4}\Pi^C_{01,2}$ and $a_2 = \Pi^F_1 + \Pi^F_2 - \frac{5}{4}\Pi^C_{02,1}$, and hence $\tau(g) = (p^0 + p^1 + p^2)/3$, where $p^i, i = 0,1,2$ are given in Proposition 8. Finally, the coincidence of $\tau(g)$ and the nucleolus follows immediately from Theorem 2.3 (iv) of Driessen and Tijs (1985). This completes the proof. □

Proof of Proposition 10

The computation of Shapley value is straightforward. Moreover, assume $\frac{k_1}{k_2} = \frac{b_1}{b_2}$, then it is not difficult to check the following facts:

1. The core is non-empty.

2. g is super-additive and $\Pi^F_1 + \Pi^F_2 = \Pi^C_{02,1} + \Pi^C_{01,2}$.

3. g is convex.

4. $g(\{0\}) = \Pi^F_0 = \Pi^C_{02,1} + \Pi^C_{01,2}$.

5. $g(\{0,1\}) + g(\{0,2\}) + g(\{1,2\}) = g(\{0,1,2\}) + \sum_{i=0}^{2} g(\{i\})$.

Based on these facts, one can verify that all the conditions of Driessen and Tijs's (1985) Theorem 4.9 hold, implying that the τ-value, the nucleolus and the Shapley of g are equal to one another. Together with the well-known fact that the Shapley value of any convex game is a core allocation, one obtains the desired result. □

A.2 Stochastic Demand Case

This section extends the setting to the case in which the end customer demand is stochastic. In this uncertain demand environment, the newsvendor setting is adopted and the end customer demand is assumed to be fiercely competitive (e.g. Wang and Gerchak 2003; Gerchak and Wang 2004; Bernstein and DeCroix 2004; Fang et al. 2008; Zhang et al. 2008). Thus, the retail price of the product, $p_j, j = 1, 2$, is determined only by the market and not a decision variable of the downstream manufacturer. This setting is common in literature that considers a common component in assembly systems (e.g., Bernstein et al., 2007). The same relationship structure as in the deterministic case is followed, in which Component 0 is the common component and Components 1 and 2 are specific to Products 1 and 2, respectively. Each supplier sets wholesale price w_i to the manufacturer with cost, $c_i, i = 0, 1, 2$. Each manufacturer then sells the product to end customers at the retail price, p_j. End customer demand, D_j, is a random variable for all suppliers and manufacturers, and substitutions do not exist between the two products. D_j is assumed to follow a uniform distribution over the interval $[0, d_j]$. Let $F_j(\cdot)$ be the cumulative density function of D_j and its density function is $f_j(\cdot)$. Let $T_j := p_j - c_0 - c_j$ be the overall channel profit margin of Product j. Without loss of generality, one has the following assumption: $T_1 \leq T_2$.

The sequence of events is as follows. At the beginning of the period, each supplier simultaneously decides the wholesale price of the component, w_j, to maximize respective profits. After observing the wholesale prices, Manufacturer j determines an order quantity, q_j, based on the wholesale price of the product and the end customer demand distribution to maximize the expected profit. Suppliers 0 and j fulfill the order quantity of Manufacturer j before the beginning of the retail period. Finally, the end customer demand is realized, and the unfulfilled demand is lost. Excess inventory has no salvage value. Suppose that all the cost parameters and the demand distributions are common knowledge to all the parties in the channel.

Following the same structure, backward induction is used to solve the problem by characterizing the manufacturer's optimal order quantity given the wholesale price of each supplier. $S_j(q_j)$ is defined as the expected sales of Product j if the order quantity is q_j. One obtains[3] $S_j(q_j) = q_j - \int_0^{q_j} F_j(y)dy$, for $j = 1, 2$. and the expected profit of Manufacturer j is:

$$\Pi_j(q_j) = p_j S_j(q_j) - q_j(w_0 + w_j), \text{ for } j = 1, 2. \tag{A-7}$$

Let q_j^* be the optimal order quantity of Manufacturer j, then

$$q_j^*(w_0, w_j) = F_j^{-1}(1 - \frac{w_0 + w_j}{p_j}) = d_j(1 - \frac{w_0 + w_j}{p_j}), \text{ for } j = 1, 2. \tag{A-8}$$

A.2.1 Model without Cooperation

In the model without cooperation, each supplier chooses wholesale price, w_i, to maximize the supplier's expected profit. Thus, the suppliers' expected profit are:

$$\begin{aligned}
\Pi_0^F(w_0) &= (q_1^*(w_0, w_1) + q_2^*(w_0, w_2))(w_0 - c_0), \\
\Pi_i^F(w_i) &= q_i^*(w_0, w_i)(w_i - c_i), \qquad \text{for } i = 1, 2.
\end{aligned}$$

Let $w_i^F, i = 0, 1, 2$ be the optimal wholesale price of Supplier i. Define that $m_j := p_j/d_j, j = 1, 2$, then:

$$\begin{aligned}
w_0^F &= \frac{m_2(2c_0 + p_1 - c_1) + m_1(2c_0 + p_2 - c_2)}{3(m_1 + m_2)}, \\
w_i^F &= \frac{m_j(4c_i + 2p_i - 2c_0) + m_i(3c_i + 3p_i - 2c_0 - p_j + c_j)}{6(m_1 + m_2)}, \text{ for } i, j \in \{1, 2\}, i \neq j.
\end{aligned}$$

[3]To simplify the model, the salvage value and lost sales costs of the product at the end of the selling period are not explicitly considered. These unconsidered factors can be taken into account by adjusting the retail and wholesale prices. As discussed in more details in Cachon (2002), adding more cost parameters to the model does not yield further managerial insights and only raises computational complexity.

The Supplier i's profit, $\Pi_i^F(w_i^F)$ is

$$\begin{aligned}\Pi_0^F(w_0^F) &= \frac{(m_2T_1+m_1T_2)^2}{9m_1m_2(m_1+m_2)},\\ \Pi_i^F(w_i^F) &= \frac{(m_i(-3T_i+T_j)-2m_jT_i)^2}{36m_i(m_1+m_2)^2},\ \text{for } i,j\in\{1,2\}, i\neq j.\end{aligned}$$

A.2.2 Partial Coalition

Under a partial coalition, Supplier 0 collectively determines the wholesale prices with Supplier i. Given the manufacturer's best response in equation (A-8), for $i,j \in \{1,2\}, i \neq j$, the joint profit function of Suppliers 0 and i can be derived as follows.

$$\Pi_{0i}^C(w_0,w_i) = q_i^*(w_0,w_i)(w_0+w_i-c_0-c_i)+q_j^*(w_0,w_j)(w_0-c_0). \tag{A-9}$$

Similarly, Supplier j's profit function is $\Pi_{0i,j}^C(w_0,w_j) = q_j^*(w_0,w_j)(w_j-c_j)$. Let $w_{0i,0}^C$ and $w_{0i,j}^C, j=1,2$ be the optimal wholesale prices of Component 0 and Component j, respectively under a partial coalition between Suppliers 0 and i,

$$\begin{aligned}w_{0i,0}^C &= \frac{2c_0+p_j-c_j}{3},\\ w_{0i,i}^C &= \frac{3c_i+3p_i-c_0-2p_j+2c_j}{6},\ \text{and } w_{0i,j}^C = \frac{2c_j+p_j-c_0}{3},\ \text{for } i,j\in\{1,2\}, i\neq j.\end{aligned}$$

The joint profit of Suppliers 0 and i, $\Pi_{0i}^C(w_{0i}^C,w_i^C)$, and the profit of Supplier j, $\Pi_{0i,j}^C(w_{0i,j}^C)$, are $\Pi_{0i}^C(w_{0i}^C,w_{0i,i}^C) = \frac{9m_iT_i^2+4m_jT_j^2}{36m_1m_2}$ and $\Pi_{0i,j}^C(w_{0i,j}^C) = \frac{T_j^2}{9m_j}$.

Proposition 11. *Under the stochastic case, the optimal wholesale price of the common component, $w_{0i,0}^C$, is independent of p_i, c_i, and d_i under partial coalition.*

Propositions 1 and 11 indicate that under a partial coalition, the common component supplier should not be concerned about the cost structures of the supplier that it cooperates with and the resultant product demand distribution characteristics. The propositions identify the intrinsic rationales that a partial coalition formed by Suppliers 0 and i only has a fractional influence on the wholesale price decision, $w^C_{0i,j}$, which is the main driver, that $w^C_{0i,0}$ only depends on the manufacturing cost of the common component and Component j and the retail price, p_j whether or not the demand is uncertain.

A.2.3 Grand Coalition

Similarly, a grand coalition is considered, in which all three suppliers determine the wholesale prices collectively. Given the manufacturer's best response in equation (A-8), the joint profit function of all suppliers under the stochastic model can be derived $\Pi^G_{012}(w_0, w_1, w_2) = q^*_1(w_0, w_1)(w_0 + w_1 - c_0 - c_1) + q^*_2(w_0, w_2)(w_0 + w_2 - c_0 - c_2)$. Let $w^G_i, i = 0, 1, 2$ be the optimal wholesale price of each supplier under a grand coalition and Π^G_{012} be the optimal overall profit, one can observe $w^G_0 + w^G_i = \frac{c_0+c_i+p_i}{2}$ and $\Pi^G_{012} = \frac{m_2T_1^2+m_1T_2^2}{4m_1m_2}$.

A.2.4 Comparison

In this subsection, three models are compared when the end customer demand is stochastic. Based on the following proposition, all results in the deterministic case can be applied to the stochastic case.

Proposition 12. *Under the stochastic case, Propositions 2 to 6 remain valid.*

Proposition 12 attests that all main insights from the deterministic case remain valid even when the end customer demand is stochastic. In the deterministic case and from equations (1) and (2), the optimal end customer demand can be obtained as follows $D^*_j(w_0, w_j) = \frac{a_j/b_j-w_0-w_j}{2/b_j}$ for $j = 1, 2$. Similarly, in the stochastic case, the optimal order quantity can be written as $q^*_j(w_0, w_j) = \frac{p_j-w_0-w_j}{p_j/d_j} = \frac{p_j-w_0-w_j}{m_j}$, for $j = 1, 2$.

As a consequence, under different forms of coalition, most conclusions and implications on wholesale prices and social welfare are valid for both deterministic and stochastic cases. $k_i = a_i - b_i(c_0 + c_i)$, in the deterministic case, and $T_i = p_i - c_0 - c_i$, in the stochastic case, play an essential role in deriving results. Note that k_i represents the largest demand that a supply chain can extract, and T_i stands for the largest profit margin in the channel in the two demand environments. Given the cost structure of each component, the relative magnitude of k_i's and T_i's in each case leads to a result that benefits the analysis of the coalition stability of both cases. In addition, under the deterministic case, the retail price set by the manufacturer determines the demand of each product. Under the stochastic case, however, the demand is a random variable that follows a probability distribution and the retail price of the manufacturer is given. These variants between the two cases determine the drivers that affect the aforementioned results. The assumption of $c_1 \geq c_2$ highly influences the results under the deterministic case but such assumption is unnecessary under the stochastic case.

Similar to the deterministic case, to address the coalition formation problem among suppliers $\{0, 1, 2\}$ under the stochastic case, consider the corresponding coalition game $g^S : 2^{\{0,1,2\}} \rightarrow \mathbb{R}$ given by

$$
\begin{array}{ll}
g^S(\emptyset) = 0 & g^S(\{0,1,2\}) = \frac{T_1^2}{4m_1} + \frac{T_2^2}{4m_2} \\
g^S(\{0\}) = \frac{(m_2T_1+m_1T_2)^2}{9m_1m_2(m_1+m_2)} & g^S(\{1,2\}) = \frac{[3T_1(m_1+m_2)-(m_1T_2+m_2T_1)]^2}{36m_1(m_1+m_2)^2} + \frac{[3T_2(m_1+m_2)-(m_1T_2+m_2T_1)]^2}{36m_2(m_1+m_2)^2} \\
g^S(\{1\}) = \frac{T_1^2}{9m_1} & g^S(\{0,2\}) = \frac{T_2^2}{4m_2} + \frac{T_1^2}{9m_1} \\
g^S(\{2\}) = \frac{T_2^2}{9m_2} & g^S(\{0,1\}) = \frac{T_1^2}{4m_1} + \frac{T_2^2}{9m_2}
\end{array}
$$

Note that $g^S(\{1,2\}) \geq g^S(\{1\}) + g^S(\{2\})$:

$$
\begin{aligned}
&g^S(\{1,2\}) - g^S(\{1\}) - g^S(\{2\}) \\
&= \frac{(T_1 - T_2)(5T_1m_1 + 4T_1m_2 - T_2m_1)}{36(m_1 + m_2)^2} + \frac{(T_2 - T_1)(5T_2m_2 + 4T_2m_1 - T_1m_2)}{36(m_1 + m_2)^2} \\
&= \frac{5(T_1 - T_2)^2}{36(m_1 + m_2)} \geq 0.
\end{aligned}
$$

The next proposition provides the sufficient and necessary conditions for the core to be non-empty.

Proposition 13. *The coalition game g^S has a non-empty core if and only if $\frac{5}{4}(\Pi^C_{02,1}+\Pi^C_{01,2}) \geq \Pi^F_1 + \Pi^F_2$.*

Proofs of Propositions 11 to 13.

Proof of Proposition 11

The result is directly from equation (A-10). □

Proof of Proposition 12

Similar to the deterministic case, to prove Proposition 2, the joint profit of Suppliers 1 and 2, $\Pi_{12}^C(w_1, w_2)$, and Supplier 0's profit, $\Pi_{12,0}^C(w_0)$, under the partial coalition between Suppliers 1 and 2 are $\Pi_{12}^C(w_1, w_2) = \frac{p_1-w_0-w_1}{m_1}(w_1-c_1)+\frac{p_1-w_0-w_2}{m_2}(w_2-c_2)$ and $\Pi_{12,0}^C(w_0) = (\frac{p_1-w_0-w_1}{m_1} + \frac{p_1-w_0-w_2}{m_2})(w_0 - c_0)$. Following the same logic and based on the first-order conditions, one can obtain that the optimal wholesale price under partial coalition are identical to the one under the model without cooperation. One can obtain that $\Pi_{12,0}^C(w_{12,0}^C) = \Pi_0^F(w_0^F)$ and $\Pi_{12}^C(w_{12,1}^C, w_{12,2}^C) = \Pi_1^F(w_1^F) + \Pi_2^F(w_2^F)$. Proposition 2 is still valid under the stochastic case.

To prove Proposition 3, note that $T_i = p_i - c_0 - c_i$ and $m_i = p_i/d_i > 0$, with the assumption of $T_2 \geq T_1$, it can be showed that $w_{01,0}^C - w_0^F = \frac{m_2[(p_2-c_2)-(p_1-c_1)]}{3(m_1+m_2)} = \frac{m_2(T_2-T_1)}{3(m_1+m_2)} \geq 0$ and $w_0^F - w_{02,0}^C = \frac{m_1[(p_2-c_2)-(p_1-c_1)]}{3(m_1+m_2)} = \frac{m_1(T_2-T_1)}{3(m_1+m_2)} \geq 0$, which mean $w_{02,0}^C \leq w_0^F \leq w_{01,0}^C$.

To prove Proposition 4, when Suppliers 0 and 1 form a partial coalition, with the assumption of $T_2 \geq T_1$, it can be showed that $w_1^F - w_{01,1}^C = \frac{m_2[(p_2-c_2)-(p_1-c_1)]}{6(m_1+m_2)} + \frac{p_2-c_2-c_0}{6} = \frac{m_2(T_2-T_1)}{6(m_1+m_2)} + \frac{T_2}{6} \geq 0$, and $w_2^F - w_{01,2}^C = \frac{m_2[(p_2-c_2)-(p_1-c_1)]}{6(m_1+m_2)} = \frac{m_2(T_2-T_1)}{6(m_1+m_2)} \geq 0$. The results directly follow. To prove Proposition 5 holds under the stochastic case, given that $p_i - c_0 - c_i \geq 0$, then for $i, j \in \{1, 2\}, i \neq j$,

$$
\begin{aligned}
(w_0^F + w_i^F) - (w_0^G + w_i^G) &= \frac{m_2(p_1 - c_0 - c_1) + m_2(p_2 - c_0 - c_2)}{6(m_2 + m_1)} \geq 0 \\
(w_{0i,0}^C + w_{0i,i}^C) - (w_0^G + w_i^G) &= 0, \\
(w_{0i,0}^C + w_{0i,j}^C) - (w_0^G + w_j^G) &= \frac{p_j - c_0 - c_j}{6} \geq 0, \\
(w_0^F + w_j^F) - (w_0^G + w_j^G) &= \frac{m_2(p_1 - c_0 - c_1) + m_1(p_2 - c_0 - c_2)}{6(m_2 + m_1)} \geq 0
\end{aligned}
$$

To prove Proposition 6 holds under the stochastic case, then for $i, j \in \{1, 2\}, i \neq j$, $\Pi_i^F - \Pi_{0j,i}^C = \frac{(m_i(T_j-3T_i)-2T_im_j)^2}{36m_i(m_1+m_2)^2} - \frac{T_i^2}{9m_i} = \frac{(T_j-T_i)(m_i(T_j-T_i)-4T_i(m_1+m_2))}{36(m_1+m_2)^2}$. To prove the result, it suffices to show that $(\Pi_i^F - \Pi_{0j,i}^C)(\Pi_j^F - \Pi_{0i,j}^C) = \frac{-(T_j-T_i)^2[m_j(T_i-T_j)-4T_j(m_i+m_j)][m_i(T_j-T_i)-4T_i(m_i+m_j)]}{1296(m_i+m_j)^4} \leq 0$.

Since $w_i^F - c_i \geq 0$, one can show that $w_i^F - c_i = \frac{c_i m_j + 3c_i(m_i+m_j) + m_i p_i + 2p_i(m_i+m_j) - 2c_0(m_i+m_j) - m_i p_j + m_i c_j}{6(m_i+m_j)} - c_i \geq 0$. Therefore, $m_i(T_j - T_i) - 4T_i(m_i + m_j) \leq 0$. Follow the same logic, with the fact that $w_j^F - c_j \geq 0$, then $m_j(T_i - T_j) - 4T_j(m_i + m_j) \leq 0$. Hence, one can conclude that $(\Pi_i^F - \Pi_{0j,i}^C)(\Pi_j^F - \Pi_{0i,j}^C) \leq 0$ and the results follow. □

Proof of Proposition 13

By the Bondareva-Shapley Theorem recalled in the proof of Proposition 7, it follows that g^S has a non-empty core if and only if the following inequalities hold:

(i) $g^S(\{0,1,2\}) \geq g^S(\{0\}) + g^S(\{1,2\})$

(ii) $g^S(\{0,1,2\}) \geq g^S(\{1\}) + g(\{0,2\})$

(iii) $g^S(\{0,1,2\}) \geq g^S(\{2\}) + g^S(\{0,1\})$

(iv) $g^S(\{0,1,2\}) \geq g^S(\{0\}) + g^S(\{1\}) + g^S(\{2\})$

(v) $g^S(\{0,1,2\}) \geq \frac{1}{2}[g^S(\{0,1\}) + g^S(\{0,2\}) + g^S(\{1,2\})]$.

Clearly, inequalities (i), (ii) and (iii) always hold since the members of a coalition can always act as if they are independent from each other. Moreover, since $g^S(\{1,2\}) \geq g^S(\{1\}) + g^S(\{2\})$, inequality (iv) holds as well. This implies that the core of g^S is non-empty if and only if $\frac{5T_1^2}{m_1} + \frac{5T_2^2}{m_2} \geq \frac{[3T_1(m_1+m_2)-(m_1T_2+m_2T_1)]^2}{m_1(m_1+m_2)^2} + \frac{[3T_2(m_1+m_2)-(m_1T_2+m_2T_1)]^2}{m_2(m_1+m_2)^2}$, or equivalently, $\frac{5}{4}(\Pi_{02,1}^C + \Pi_{01,2}^C) \geq \Pi_1^F + \Pi_2^F$. □

A.3 N-Product Assembly System

In this section, the model is extended to a general and realistic case with more than two ($N > 2$) products where there exist one common component supplier (denoted by Supplier 0) and N remaining suppliers (denoted by Supplier i, $i = 1, 2, ..., N$). Manufacturer j's profit function remains to be $(p_j - w_0 - w_j)D_j, j = 1, 2, ..., N$ where D_j is defined in (1). Therefore, the optimal retail price of Manufacturer j, $p_j^*(w_0, w_j)$, is equivalent to the outcome in (2). In what follows, three circumstances are considered as well, in which suppliers determine their individual wholesale price separately, form a partial coalition, and form a grand coalition.

A.3.1 Model without Cooperation

Under the model without cooperation, suppliers individually determine the wholesale price. Supplier i's profit is $\Pi_i^F = (w_i - c_i)(a_i - b_i p_i^*(w_0, w_i))$, for $i = 1, 2, ..., N$. Supplier 0's profit is $\Pi_0^F = (w_0 - c_0)\sum_{i=1}^N (a_i - b_i p_i^*(w_0, w_i))$. Solving each supplier's profit function, the optimal wholesale price of common component supplier, w_0^F, and of Supplier i, $w_i^F, i = 1, 2, ..., N$ as well as the profit of each supplier can be obtained:

$$
\begin{aligned}
w_0^F &= \frac{\sum_{i=1}^N (a_i - b_i(c_i - 2c_0))}{3\sum_{i=1}^N b_i}, \\
w_i^F &= \frac{\sum_{j=1}^N b_j(3a_i + b_i(3c_i + c_j - 2c_0)) - b_i \sum_{j=1}^N a_j}{6b_i \sum_{j=1}^N b_j}, \qquad \text{for } i = 1, 2, ..., N, \\
\Pi_0^F &= \frac{1}{18} \cdot \frac{(\sum_{i=1}^N (a_i - b_i(c_0 + c_i)))^2}{\sum_{i=1}^N b_i}, \\
\Pi_i^F &= \frac{(\sum_{j=1}^N b_j(3a_i - b_i(3c_i + 2c_0 - c_j)) - b_i \sum_{j=1}^N a_j)^2}{72 b_i (\sum_{j=1}^N b_j)^2} \qquad \text{for } i = 1, 2, ..., N. \quad \text{(A-10)}
\end{aligned}
$$

A.3.2 Partial Coalition

Under a partial coalition, part of the suppliers form an alliance and collectively determine the wholesale prices. Here, use $\mathfrak{D}$ to denote the set of partial coalition and the suppliers in $\mathfrak{D}$ form an alliance with Supplier 0 to determine the wholesale prices collectively. For example, if $N = 5$ and $\mathfrak{D} = \{1, 2, 4\}$, then the common component supplier forms an alliance with Suppliers 1, 2, and 4, while Suppliers 3 and 5 formulate pricing decisions separately. Therefore, the joint profit function of the suppliers in $\mathfrak{D}$ is $\Pi_{0i,i\in\mathfrak{D}}^C = \sum_{i\in\mathfrak{D}}(w_i + w_0 - c_i - c_0)(a_i - b_i p_i^*(w_0, w_i)) + \sum_{i\notin\mathfrak{D}}(w_0 - c_0)(a_i - b_i p_i^*(w_0, w_i))$ and the supplier who does not participate in the alliance obtains a profit $\Pi_{i,i\notin\mathfrak{D}}^C = (w_i - c_i)(a_i - b_i p_i^*(w_0, w_i))$. $w_{0i,0}^C$ is defined as the optimal wholesale price of Supplier 0. Furthermore, $w_{i,i\in\mathfrak{D}}^C$ and $w_{i,i\notin\mathfrak{D}}^C$ are the optimal wholesale prices of Supplier i, if the supplier is in the alliance and not in the alliance, respectively. We obtain:

$$
\begin{aligned}
w^C_{0i,0} &= \frac{\sum_{i\notin\mathfrak{D}}(a_i + b_i(2c_0 - c_i))}{3\sum_{i\notin\mathfrak{D}} b_i},\\
w^C_{i,i\in\mathfrak{D}} &= \frac{\sum_{j\notin\mathfrak{D}} b_j(3a_i - b_i(c_0 - 2c_j - 3c_i)) - 2b_i\sum_{j\notin\mathfrak{D}} a_j}{6b_i\sum_{j\notin\mathfrak{D}} b_j},\\
w^C_{i,i\notin\mathfrak{D}} &= \frac{\sum_{j\notin\mathfrak{D}} b_j(3a_i - b_i(2c_0 - c_j - 3c_i)) - b_i\sum_{j\notin\mathfrak{D}} a_j}{6b_i\sum_{j\notin\mathfrak{D}} b_j},\\
\Pi^C_{0i,i\in\mathfrak{D}} &= \frac{(\sum_{j\notin\mathfrak{D}}(a_j - b_j(c_0 + c_j)))^2}{18\sum_{j\notin\mathfrak{D}} b_j} + \sum_{j\in\mathfrak{D}} \frac{(a_j - b_j(c_0 + c_j))^2}{8b_j}\\
\Pi^C_{i,i\notin\mathfrak{D}} &= \frac{(\sum_{j\notin\mathfrak{D}} b_j(3a_i + b_i(c_j - 3c_i - 2c_0)) - b_i\sum_{j\notin\mathfrak{D}} a_j)^2}{72b_i(\sum_{j\notin\mathfrak{D}} b_j)^2}.
\end{aligned}
\tag{A-11}
$$

A.3.3 Grand Coalition

Under a grand coalition, all suppliers collectively make pricing decisions. The overall profit is $\Pi^G = \sum_{i=1}^N (w_i + w_0 - c_i - c_0)(a_i - b_i p_i^*(w_0, w_i))$. After some algebra, the optimal wholesale price of Supplier i, $w_i^G, i = 0, 1, 2, ..., N$, is equal to that in equation (11). To wit, the number of suppliers in the assembly system does not impact the optimal wholesale price decisions of the suppliers in the grand coalition. Then the profit $\Pi^G = \sum_{i=1}^N \frac{k_i^2}{8b_i}$.

A comparison of the optimal wholesale prices in each circumstance from (A-10) and (A-11) leads to the following result.

Proposition 14. *Propositions 1, 2, and 5 remain valid for the N-product assembly case.*

The proposition suggests that the major analytical results and insights previously derived in the two-product assembly system can be extended to a general case in which N products are in the system. The next questions to answer are whether the grand coalition is stable and what fair allocation can be arranged. Note that the corresponding coalition game

$g^N : 2^{\{0,1,\ldots,n\}} \to \mathbb{R}$ for the N-product case is defined by

$$g(C) = \begin{cases} \sum_{i\in C} \frac{(3k_i \sum_{j\in C} b_j - b_i \sum_{j\in C} k_j)^2}{72 b_i (\sum_{j\in C} b_j)^2}, & \text{if } 0 \notin C, \\ \frac{(\sum_{j\notin C} k_j)^2}{18 \sum_{j\notin C} b_j} + \sum_{0\neq j\in C} \frac{k_j^2}{8b_j}, & \text{if } 0 \in C. \\ 0, & \text{if } C = \emptyset. \end{cases}$$

Note that under the N-product assembly system, the description of the domain of the core is complex and messy. Hence, in the following proposition, this study shows the condition under which the stability of grand coalition can be maintained and one profit allocation lies in such core.

Proof of Proposition 14

To prove that Proposition 1 is still valid, from equation (A-11) that $w^C_{0i,0} = \frac{\sum_{i\notin \mathfrak{D}}(a_i + b_i(2c_0 - c_i))}{3\sum_{i\notin \mathfrak{D}} b_i}$, which clearly shows that the optimal wholesale price of the common component is independent from the demand characteristics of product i within the cooperation.

To prove that Proposition 2 can be extended to the general case, first define $\mathbb{K}$ as the set of partial coalition under which the common component supplier is excluded. Therefore, the joint profit function of the suppliers in $\mathbb{K}$ is $\Pi^C_{\mathbb{K}, i\in\mathbb{K}} = \sum_{i\in\mathbb{K}}(w_i - c_i)(a_i - b_i p_i^*(w_0, w_i))$; the profit function of the suppliers not in the cooperation are $\Pi^C_{\mathbb{K}, j\notin\mathbb{K}} = (w_j - c_j)(a_j - b_j p_j^*(w_0, w_j)), \forall j \notin \mathbb{K}$; the profit function for common component supplier is $\Pi^C_{\mathbb{K},0} = \sum_{k\in\forall i,\forall j}(w_0 - c_0)(a_k - b_k p_k^*(w_0, w_k))$. Accordingly, one can get the optimal whole sale price for each supplier.

$$\begin{aligned} w^C_{\mathbb{K},0} &= \frac{\sum_{i=1}^N (a_i - b_i(c_i - 2c_0))}{3\sum_{i=1}^N b_i}, \\ w^C_{\mathbb{K},i} &= \frac{\sum_{j=1}^N b_j(3a_i + b_i(3c_i + c_j - 2c_0)) - b_i \sum_{j=1}^N a_j}{6b_i \sum_{j=1}^N b_j}, \quad \text{for } \forall i. \end{aligned} \tag{A-12}$$

The above optimal wholesale prices are identical to the ones under the model without cooperation. To extend Proposition 5 to N-product case, the total profit function is $\Pi^G = (w_0 - c_0)\sum_{i=1}^N (a_i - b_i p_i^*(w_0, w_i)) + \sum_{i=1}^N (w_i - c_i)(a_i - b_i p_i^*(w_0, w_i))$. By some algebra, one obtains that $w_0^G + w_i^G = \frac{(a_i + b_i(c_0+c_i))}{2b_i}$. Therefore,

$$\begin{aligned}
(w_0^F + w_i^F) - (w_0^G + w_i^G) &= \frac{\sum_{j=1}^N (a_j - b_j(c_0+c_j))}{6\sum_{j=1}^N b_j} \geq 0, \\
(w_{0i,0}^C + w_{i,i\in\mathbb{D}}^C) - (w_0^G + w_i^G) &= 0, \\
(w_{0i,0}^C + w_{i,j\notin\mathbb{D}}^C) - (w_0^G + w_j^G) &= \frac{\sum_{k\notin\mathbb{D}} (a_k - b_k(c_0+c_k))}{6\sum_{k\notin\mathbb{D}} b_k} \geq 0,
\end{aligned} \tag{A-13}$$

where the inequalities above come from the fact that $a_i - b_i(c_0+c_i) \geq 0$ for all i. □

Proposition 15. *The profit allocation* $(y_0, y_1, \ldots, y_n) = (\sum_{i=1}^n \frac{k_i^2}{18b_i}, \frac{5k_1^2}{72b_1}, \ldots, \frac{5k_n^2}{72b_n})$ *lies in the core of* g*, and thus can maintain the stability of grand coalition such that every supplier will be better off compared to any other forms of coalition if for* $i = 1, \ldots, n$*,*

$$\frac{5k_i^2}{72b_i} \geq \frac{(3k_i \sum_{j\in C} b_j - b_i \sum_{j\in C} k_j)^2}{72b_i(\sum_{j\in C} b_j)^2}. \tag{A-14}$$

Proof of Proposition 15

In the proof of Proposition 7, $\frac{k_1^2}{b_1} + \frac{k_2^2}{b_2} \geq \frac{(k_1+k_2)^2}{b_1+b_2}$. This inequality can be inductively generalized as follows:

$$\sum_{i\in C} \frac{k_i^2}{b_i} \geq \frac{(\sum_{i\in C} k_i)^2}{\sum_{i\in C} b_i} \text{ for all } C \subseteq \{1, \ldots, n\}. \tag{A-15}$$

It suffices to show that $\sum_{i\in C} y_i \geq g(C)$ for each $C \subseteq \{0, 1, \ldots, n\}$. Consider two cases.

Case I. $0 \notin C \subseteq \{0, 1, \ldots, n\}$.

By (A-14), $\sum_{i\in C} y_i = \sum_{i\in C} \frac{5k_i^2}{72b_i} \geq \sum_{i\in C} \frac{(3k_i \sum_{j\in C} b_j - b_i \sum_{j\in C} k_j)^2}{72b_i(\sum_{j\in C} b_j)^2} = g(C)$.

Case II. $0 \in C \subseteq \{0, 1, \ldots, n\}$. The inequality (A-15) implies

$$\begin{aligned}
\sum_{i\in C} y_i - g(C) &= \sum_{i=1}^n \frac{k_i^2}{18b_i} + \sum_{0\neq i\in C} \frac{5k_i^2}{72b_i} - \frac{(\sum_{j\notin C} k_j)^2}{18\sum_{j\notin C} b_j} - \sum_{0\neq j\in C} \frac{k_j^2}{8b_j} \\
&= \sum_{j\notin C} \frac{k_i^2}{18b_i} - \frac{(\sum_{j\notin C} k_j)^2}{18\sum_{j\notin C} b_j} \geq 0.
\end{aligned}$$

□