DIFFERENTIAL GEOMETRY II: HOMEWORK 8

DUE MAY 5

(1) Prove the Poincaré inequality: there exists a constant $c > 0$ such that for any $f \in C^\infty(D)$ with $f|_{\partial D} = 0$ satisfies

$$\int_D |f|^2 \,\mathrm{d}x\mathrm{d}y \leq c \int_D |Df|^2 \,\mathrm{d}x\mathrm{d}y \ .$$

(2) For $ad - bc = 1$, let

$$\varphi(z) = \frac{az+b}{cz+d} \ .$$

Verify that for any $u : \bar{D} \to \mathbb{R}^N$, $E(u) = E(u \circ \varphi)$.

(3) Given $\Gamma \subset \mathbb{R}^N$, let $u : \bar{D} \to \mathbb{R}^N$ be solution to the Plateau problem given by Douglas. Let

$$S_u = \{p \in D : Du(p) = 0\}$$

be its branch locus.

(a) Show that the branch locus is discrete.
(b) Given a example with non-empty S_u. You are free to choose Γ.