

TOPICS IN DIFFERENTIAL GEOMETRY: HOMEWORK 12

DUE MAY 19

- (1) Let Σ be a minimal submanifold in (M, g) . Suppose that V is a Killing vector field on M . Prove by checking the Jacobi operator directly that V^\perp is a Jacobi field on Σ .
- (2) Denote by $\bar{\nabla}$ the Levi-Civita connection of the standard metric on S^n . Let $p = (0, \dots, 0, 1) \in S^n$. For any $v \in T_p S^n$ and any skew-symmetric endomorphism \bar{h} on $T_p S^n$, show that there exists a Killing vector field V on S^n such that $V|_p = v$ and $\bar{\nabla}V|_p = \bar{h}$.
- (3) Let S_r^n be the sphere of radius r . Consider $\Sigma = S_{\frac{1}{\sqrt{2}}}^n \times S_{\frac{1}{\sqrt{2}}}^n$ as a submanifold in S_1^{2n+1} . Show that it is minimal, and compute $|A|^2$.