TOPICS IN DIFFERENTIAL GEOMETRY: HOMEWORK 07

DUE APRIL 14

(1) Suppose that $f : S^2 \to S^3$ is a conformal[1], harmonic map. Prove that its image must belong to a great S^2.

Hint:

- Work on a chart $\mathbb{C} \subset S^2$. Regard f as an $\mathbb{R}^4$-valued function. What does the harmonicity mean for f?
- There exists an $n : S^2 \to \mathbb{R}^4$, which is unit normal of the image of f in $T_{f(p)}S^3$.
- Note that n, $\partial_x f$, $\partial_y f$ and f form a basis for $\mathbb{R}^4$.

(2) Derive the Euler Lagrange equation for the area/volume functional for $f : U \subset \mathbb{R}^2 \to (M, g)$.

$$A(f) = \int_U \left| \frac{\partial f}{\partial x} \wedge \frac{\partial f}{\partial y} \right| \mathrm{d}x\mathrm{d}y \ .$$

[1] It must be an immersion.