DIFFERENTIAL GEOMETRY I: HOMEWORK 03

DUE SEPTEMBER 30

(1) [W, exercise 14 in ch.1] Is every vector field on the real line, $\mathbb{R}^1$, complete?

(2) For any $n \in \mathbb{N}$, show that $\mathbb{R}^n$ is diffeomorphic to $B_1(0) = \{x \in \mathbb{R}^n : |x| < 1\}$.

(3) Note that for a smooth manifold M, $\Delta M = \{(p, p) \in M \times M : p \in M\}$ is a closed subset in $M \times M$. (In fact, the diagonal map embeds M into $M \times M$.)

   For the map S in the proof of reducing dimension of an injective immersion into the Euclidean space, modify it by constructing a map to S^{N-1}, instead of $\mathbb{R}^N$.

(4) Prove that any n-dimensional manifold can be immersed into $\mathbb{R}^{2n}$. You may use the facts we have proved in class.

(5) Compute the flow of each of the following vector fields on $\mathbb{R}^2$.
   (a) $V = y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y}$.
   (b) $V = \frac{\partial}{\partial x} + (x + 2)y\frac{\partial}{\partial y}$

(6) Prove the Jacobi identity: for any three smooth vector fields U, V, W,

$$[U, [V, W]] + [W, [U, V]] + [V, [W, U]] = 0 \ .$$