RIEMANN SURFACE
HOMEWORK 10

DUE: TUESDAY, MAY 24

(1) Let $U \subset M$ be a open subset such that $\bar{U}$ is compact, and ∂U consists of finitely many analytic arcs. Thus, each point $\zeta_0 \in \partial U$ admits a subharmonic barrier function. Given any continuous function $h(\zeta)$ on ∂U, consider

$$\mathcal{F} = \{v : \text{subharmonic on } U \mid \limsup_{U \ni z \to \zeta} v(z) \leq h(\zeta)\} \ .$$

Denote the upper envelope of $\mathcal{F}$ by $u(z)$. Prove that

$$\lim_{U \ni z \to \zeta} u(z) = h(z) \ .$$

Hint: You can consult [Gameline, p.404–405].

(2) Determine all bipolar Green's functions for the complex plane $\mathbb{C}$.

(3) Determine all bipolar Green's functions for the complex plane $\hat{\mathbb{C}}$.