RIEMANN SURFACE
HOMEWORK 8

DUE: TUESDAY, MAY 10

In this homework set, M is always a genus two Riemann surface.

(1) How many Weiestrass points could M have?

(2) Prove that M admits a meromorphic function with only one pole of order 2.

(3) Suppose that $h : M \to \hat{\mathbb{C}}$ is a holomorphic map of degree 2. Prove that the branch points of h are exactly the Weiestrass points of M. (Hint: You may compose it with an automorphism of $\hat{\mathbb{C}}$ to get another meromorphic function on M.)

(4) Suppose that P_0 and P_1 are two distinct Weierstrass points on M. Prove that there exists a meromorphic function f such that $(f) = P_0^2 P_1^{-2}$. (Hint: Suppose that you can find a function g with only one pole of order 2 at P_1, what can you say about the function $g - g(P_0)$?)

(5) This is a continuation of part (iii). Let P_0 be the base point for the Abel–Jacobi map, $\varphi : M \to J(M) = \mathbb{C}^2/L(M)$. Prove that $\varphi(P_1)$ has order 2. Namely, $\varphi(P_1) + \varphi(P_1) = [0]$.