COMPLEX ANALYSIS
HOMEWORK 5

DUE: TUESDAY, OCTOBER 20

(1) Evaluate any *three* of [p.161, #3] by the method of residues.

(2) Let $f(z)$ be analytic on $\overline{B(0;1)}$ with $f(0) = 0$, $f'(0) \neq 0$ and $f(z) \neq 0$ for $0 < |z| \leq 1$. Let $\rho = \min_{|z|=1}\{|f(z)|\}$. By assumption, $\rho > 0$. Define an analytic function $g(w)$ on $B(0;\rho)$ by

$$g(w) = \frac{1}{2\pi i}\int_{|z|=1} \frac{z\, f'(z)}{f(z) - w}\mathrm{d}z \ .$$

What is this function $g(w)$ related to $f(z)$? (The proof of the open mapping theorem may help. You can also apply the residue theorem on the integral.)

(3) [p.154, #2]: How many roots of the equation $z^4 - 6z + 3 = 0$ lie in the annulus, $\{z \in \mathbb{C} \mid 1 < |z| < 2\}$?

(4) How many roots of the equation $2z^5 - z^3 + 2z^2 - z + 8 = 0$ lie in the unit disk, $\{z \in \mathbb{C} \mid |z| < 1\}$?

(5) Let λ be a real number greater than 1. Show that the equation $\lambda - z - \exp(-z) = 0$ has exactly one root in the right half plane, $\{z \in \mathbb{C} \mid \operatorname{Re} z > 0\}$. Show that the solution is real. What happens to the solution as $\lambda \to 1$?