DIFFERENTIAL TOPOLOGY
HOMEWORK 9

DUE: MONDAY, APRIL 21

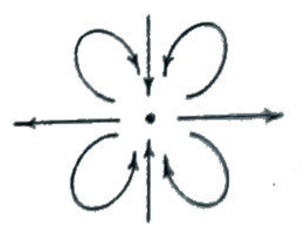
The following exercises are taken from [GP].

(1) Prove that $\chi(X \times Y) = \chi(X) \cdot \chi(Y)$. (*Hint.* Both $\mathbf{Id}_X$ and $\mathbf{Id}_Y$ are homotopic to a Lefschetz map.)

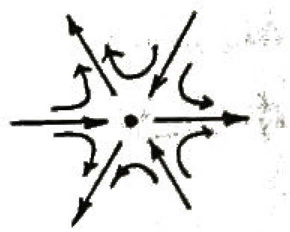
(2) Consider the following maps from $\mathbb{C} \cong \mathbb{R}^2$ into itself. The maps will be described in terms of complex coordinate.
  (a) Check that $z \mapsto z^m + z$ has a fixed point with local Lefschetz number m at the origin, where m is a positive integer.
  (b) Show that for any $c \neq 0$, the homotopic map $z \mapsto z^m + z + c$ is a Lefschetz map, with m fixed points that are all close to zero if c is small.
  (c) Show that the map $z \mapsto \bar{z}^m + z$ has a fixed point of local Lefschetz number $-m$ at the origin.

(3) Let Σ be a compact, oriented surface (without boundary). Suppose that Σ has a *triangulation*. Apply the Poincaré–Hopf theorem to prove that $V - E + F = \chi(\Sigma)$ where V, E, F are the total number of vertices, edges, faces of the triangulation, respectively.

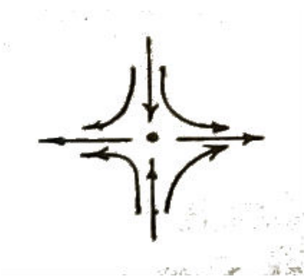
(4) Find out the index of the following vector fields. (No justications needed.)



(a)

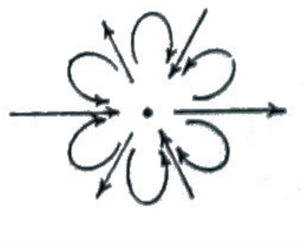


(b)

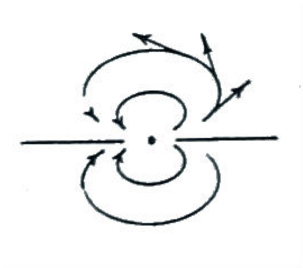


(c)

(d)

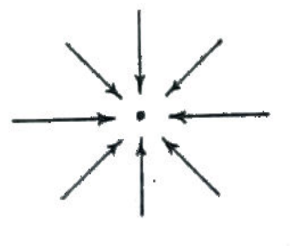


(e)



(f)

(g)



(h)