Supplementary Material for "Super Nested Arrays: Linear Sparse Arrays with Reduced Mutual Coupling – Part II: High-Order Extensions [1]"

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I. A NUMERICAL EXAMPLE TO DEMONSTRATE THE MECHANICS OF THE PROOF IN SECTION V

Fig. 1 illustrates a numerical example of Section V in [1]. In this example, we consider the 2D representation of a super nested array with $N_1 = 77$, $N_2 = 11$, $Q = 5$. The detailed definition for 2D representations can be found in [2]. Sensors are marked with bullets (•), black squares (■), and black triangles (▲), if they belong to $\mathbb{S}^{(Q-1)} \cap \mathbb{S}^{(Q)}$, $\mathbb{S}^{(Q-1)} \backslash \mathbb{S}^{(Q)}$, and $\mathbb{S}^{(Q)} \backslash \mathbb{S}^{(Q-1)}$, respectively. The table in Fig. 1 lists instances of all the cases and the associated equations for new sensor pairs (n_1', n_2') from the original pair (n_1, n_2). Hence, this example is consistent with Section V in [1].

II. A NUMERICAL EXAMPLE TO DEMONSTRATE THE MECHANICS OF THE PROOF IN SECTION VI

For the proof in Section VI of [1], a numerical example of a super nested array with $N_1 = 72$, $N_2 = 11$, $Q = 5$ is considered in Fig. 2, where some cases are listed along with the associated equations. It can be seen from the numerical values that this example is compatible with Section VI of [1].

REFERENCES

[1] C.-L. Liu and P. P. Vaidyanathan, "Super nested arrays: Linear sparse arrays with reduced mutual coupling – Part II: High-order extensions," *IEEE Trans. Signal Process.*, submitted.

[2] ——, "Super nested arrays: Linear sparse arrays with reduced mutual coupling – Part I: Fundamentals," *IEEE Trans. Signal Process.*, submitted.

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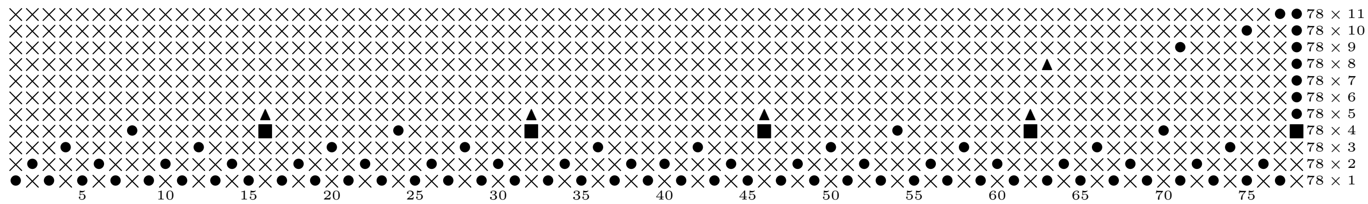
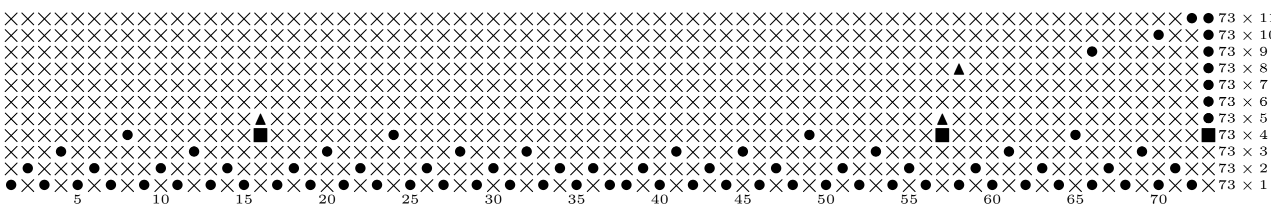


Case	$n_1 \in \mathbb{S}^{(Q-1)}\backslash\mathbb{S}^{(Q)}$	$n_2 \in \mathbb{S}^{(Q-1)}$	$n_1' \in \mathbb{S}^{(Q)}$	$n_2' \in \mathbb{S}^{(Q)}$	Remark
1-2	$78\times3+16$	$78\times2+20$	$78\times3+8$	$78\times2+12$	(10): $\ell_1=0, q=3, \ell_2=2$
1-2	$78\times3+16$	$78\times1+10$	$78\times3+24$	$78\times1+18$	(11): $\ell_1=0, q=2, \ell_2=2$
1-3b	$78\times3+32$	$78\times2+4$	$78\times4+16$	$78\times3-12$	(13): $\ell_1=1, q=3, \ell_2=0$
2	$78\times3+32$	$78\times3+8$	$78\times0+31$	$78\times0+7$	(15): $\ell_1=1, \ell_2=0$
4-2	$78\times3+16$	$78\times2-18$	$78\times3+8$	$78\times2-26$	(16): $\ell_1=0, q=2, \ell_2=4$
4-2	$78\times3+16$	$78\times3-28$	$78\times3+24$	$78\times3-20$	(17): $\ell_1=0, q=3, \ell_2=3$
4-3(b)i	$78\times3+32$	$78\times1-35$	$78\times3+24$	$78\times0+35$	$\ell_1=1, q=1, \ell_2=17$
4-3(b)ii	$78\times3+32$	$78\times3-36$	$78\times2+4$	$78\times1+14$	(24): $\ell_1=1, q=3, \ell_2=4, R=10, P=1$
4-3(b)iii	$78\times3+32$	$78\times2-38$	78×5	$78\times3+8$	(27): $\ell_1=1, q=2, \ell_2=9, R=8, P=3$
7-1	$78\times3+32$	78×8	$78\times4+32$	78×9	(29): $\ell_1=1, \ell_2=8$
7-1	$78\times3+16$	78×11	$78\times0+1$	$78\times8-15$	(30): $\ell_1=0, \ell_2=11$
9	$78\times3+16$	$78\times9-7$	$78\times0+23$	78×6	(32): $\ell_1=0, q=3$
13-2	$78\times4-16$	$78\times1-17$	$78\times4-8$	$78\times1-9$	(34): $\ell_1=0, q=1, \ell_2=8$
13-2	$78\times4-16$	$78\times1-1$	$78\times4-24$	$78\times1-9$	(35): $\ell_1=0, q=1, \ell_2=0$
13-3c	$78\times4-32$	$78\times3-4$	$78\times0+51$	$78\times0+1$	(37): $\ell_1=1, q=3, \ell_2=0$
13-3d	$78\times4-32$	$78\times2-6$	$78\times3-20$	$78\times1+6$	(38): $\ell_1=1, q=2, \ell_2=1$
19	78×4	$78\times1+18$	$78\times4+16$	$78\times1+34$	(40): $q=2, \ell_2=4$
19	78×4	$78\times2+20$	$78\times4-8$	$78\times2+12$	(41): $q=3, \ell_2=2$
20	78×4	$78\times3+8$	$78\times1-1$	$78\times0+7$	(43): $\ell_2=0$
25-1	78×4	78×10	78×5	78×11	$q=10$
25-2	78×4	78×11	$78\times1-15$	$78\times8-15$	(45): $q=11$
27	78×4	$78\times9-7$	$78\times0+7$	78×5	(47): $q=3$

Fig. 1. An example to demonstrate some cases in Section V in [1]. Notations, equation numbers, and indices follow from [1]. Here $N_1=77$, $N_2=11$ and $Q=5$. Bullets (●) denote sensors in $\mathbb{S}^{(Q)}\cap\mathbb{S}^{(Q-1)}$, black squares (■) represent sensors in $\mathbb{S}^{(Q-1)}\backslash\mathbb{S}^{(Q)}$, whereas black triangles (▲) show $\mathbb{S}^{(Q)}\backslash\mathbb{S}^{(Q-1)}$. The table on the bottom provides numerical values of n_1, n_2, n_1', and n_2' for different cases, as discussed in Section V in [1].



Case	$n_1 \in \mathbb{S}^{(Q-1)} \backslash \mathbb{S}^{(Q)}$	$n_2 \in \mathbb{S}^{(Q-1)}$	$n_1' \in \mathbb{S}^{(Q)}$	$n_2' \in \mathbb{S}^{(Q)}$	Remark
1-1	$73 \times 3 + 16$	$73 \times 2 + 20$	$73 \times 3 + 8$	$73 \times 2 + 12$	(48): $q = 3$
1-1	$73 \times 3 + 16$	$73 \times 1 + 10$	$73 \times 3 + 24$	$73 \times 1 + 18$	(49): $q = 2$
1-2	$73 \times 3 + 16$	$73 \times 1 + 36$	$73 \times 2 + 28$	$73 \times 0 + 48$	(50): $q = 2$
4-2	$73 \times 3 + 16$	$73 \times 3 - 32$	$73 \times 1 + 2$	$73 \times 0 + 27$	(51): $q = 3$
19-1	73×4	$73 \times 1 + 18$	$73 \times 4 + 16$	$73 \times 1 + 34$	(52): $q = 2$
19-1	73×4	$73 \times 2 + 20$	$73 \times 4 - 8$	$73 \times 2 + 12$	(53): $q = 3$
19-2	73×4	$73 \times 2 + 32$	$73 \times 2 + 20$	$73 \times 0 + 52$	(54): $q = 3$

Fig. 2. An example for some cases in Section VI of [1]. Here $N_1 = 72$, $N_2 = 11$ and $Q = 5$. Bullets (●) denote sensors in $\mathbb{S}^{(Q)} \cap \mathbb{S}^{(Q-1)}$, black squares (■) represent sensors in $\mathbb{S}^{(Q-1)} \backslash \mathbb{S}^{(Q)}$, whereas black triangles (▲) show $\mathbb{S}^{(Q)} \backslash \mathbb{S}^{(Q-1)}$. The table on the bottom lists some cases and their numerical examples, as discussed in Section VI of [1].