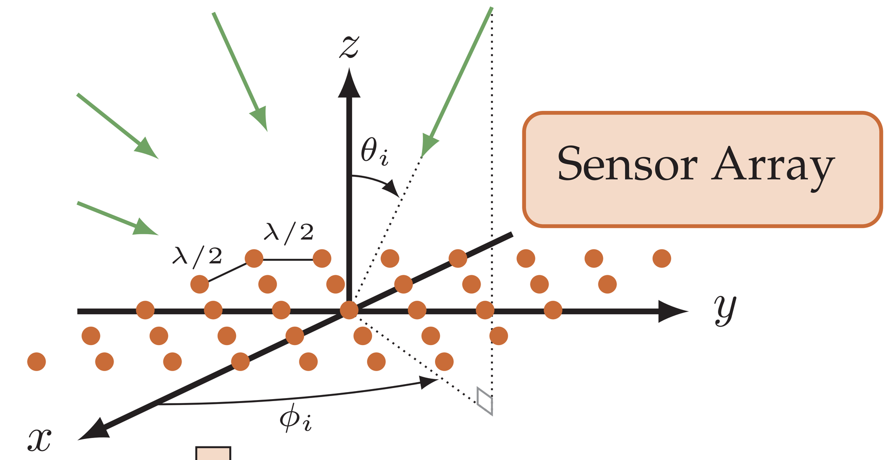
Two-Dimensional Sparse Arrays with Hole-Free Coarray and Reduced Mutual Coupling

Chun-Lin Liu (cl.liu@caltech.edu) and **P. P. Vaidyanathan** (ppvnath@systems.caltech.edu)
Electrical Engineering, California Institute of Technology, Pasadena, CA 91125, USA.

2D DOA Estimation

Monochromatic, Far-Field, Uncorrelated Sources

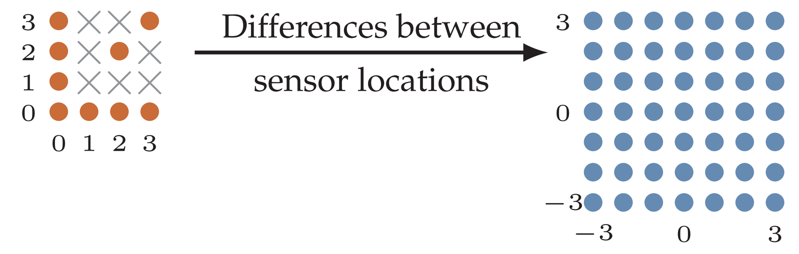


Mutual Coupling [1,2]
Interferences between closely-spaced sensors

DOA Estimators → Estimated DOA

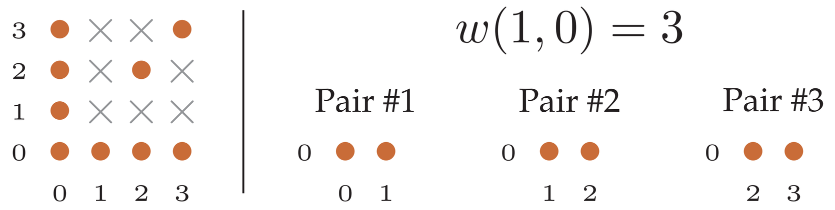
Array Design Criteria [6]

Hole-free Difference Coarrays



- Typically, $|\mathbb{D}| = \mathcal{O}(|\mathbb{S}|^2)$.
- *Hole-free* $\mathbb{D}$ *($\mathbb{D}$ is a URA)* admits efficient implementation of ESPRIT [3-7].
- *More sources than sensors* [4,6].

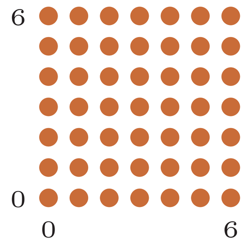
Small Weight Functions at Small Separations



- $w(m_x, m_y)$: The number of sensor pairs with separation (m_x, m_y).
- Qualitatively, small $w(1,0)$, $w(0,1)$, $w(1,1)$, and $w(1,-1)$ lead to *reduced mutual coupling* [6].

Known Planar Arrays

Uniform Rectangular Arrays ★

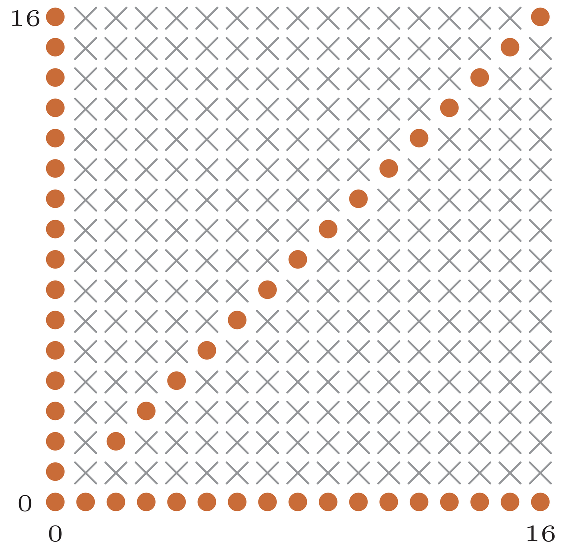


49 sensors

Hole-free $\mathbb{D}$ ★

$w(1,0) =$	42	✗
$w(0,1) =$	42	✗
$w(1,1) =$	36	✗
$w(1,-1) =$	36	✗

Billboard Arrays [3] ★★

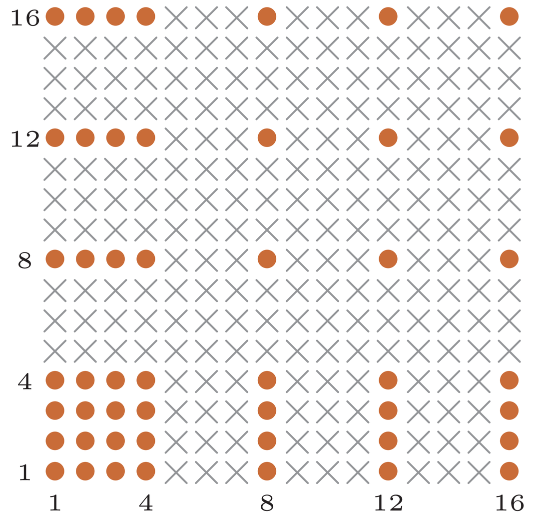


48 sensors

Hole-free $\mathbb{D}$ ★

$w(1,0) =$	16	✗
$w(0,1) =$	16	✗
$w(1,1) =$	14	✗
$w(1,-1) =$	1	★

2D Nested Arrays [4] ★★★

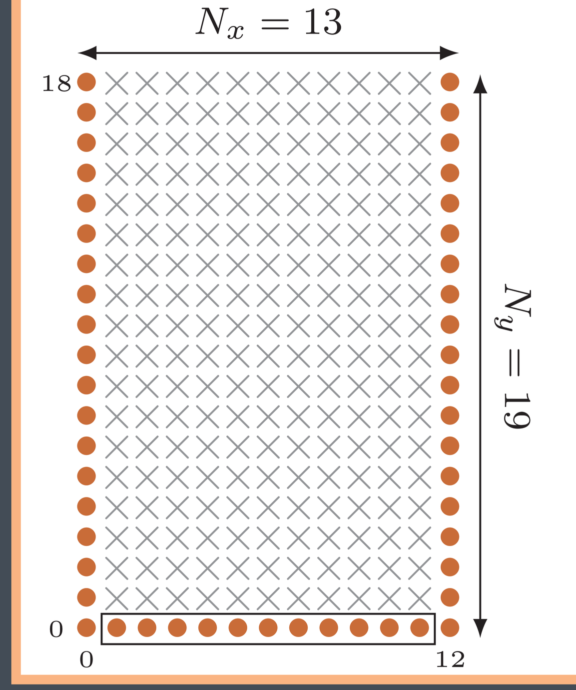


49 sensors

Hole-free $\mathbb{D}$ ★

$w(1,0) =$	21	✗
$w(0,1) =$	21	✗
$w(1,1) =$	9	★
$w(1,-1) =$	9	★

Open Box Arrays [5] ★★★



49 sensors

Hole-free $\mathbb{D}$ ★

$w(1,0) =$	12	✗
$w(0,1) =$	36	✗
$w(1,1) =$	1	★
$w(1,-1) =$	1	★

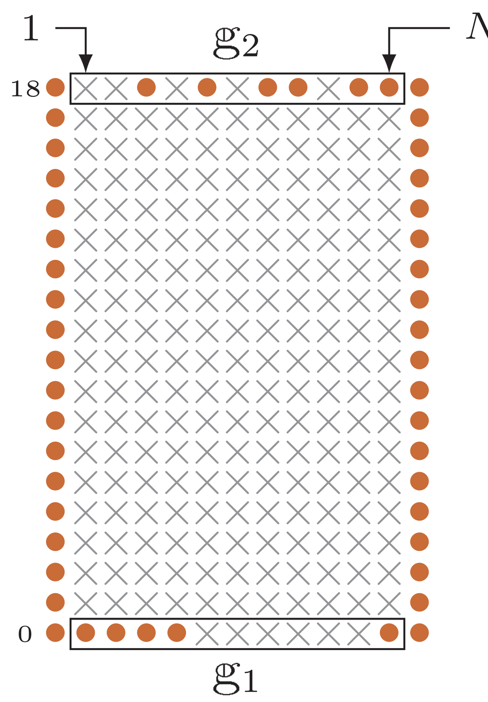
New Contributions

Are there any five-star arrays?

Construct novel arrays with
- *the same number of sensors* as OBA.
- *the same difference coarray* as OBA (no holes in $\mathbb{D}$).
- *Small weight functions* $w(1,0)$, $w(0,1)$, $w(1,1)$, and $w(1,-1)$.

Partially Open Box Arrays

Redistribute the sensors at $\{(n_x, 0) \mid n_x = 1, 2, \ldots, N_x - 2\}$ in OBA.



$\mathbb{g}_1 = \{1, 2, 3, 4, 11\}$
$\mathbb{g}_2 = \{3, 5, 7, 8, 10, 11\}$

$\mathbb{D}$ has holes ✗

$w(1,0) =$	8	★
$w(0,1) =$	36	✗
$w(1,1) =$	1	★
$w(1,-1) =$	2	★

Design $\mathbb{g}_1$ and $\mathbb{g}_2$ so that $\mathbb{D}$ is hole-free.
Exponential Search Space

Theorem 1: Hole-free $\mathbb{D}_{\text{POBA}}$

$\mathbb{D}_{\text{POBA}} = \mathbb{D}_{\text{OBA}}$

⇕ if and only if

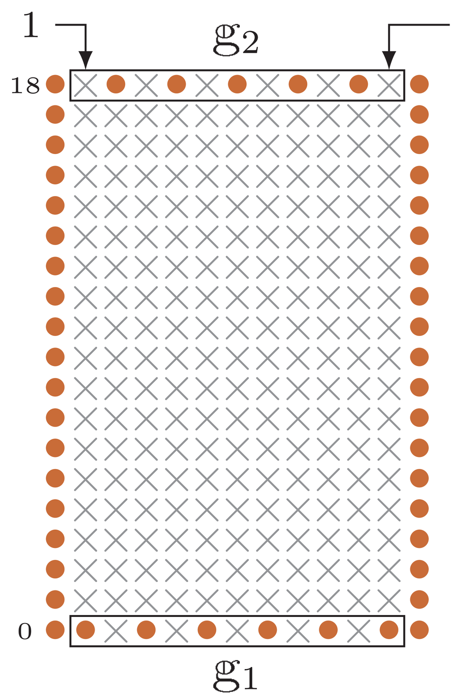
$\{\mathbb{g}_1, N_x - 1 - \mathbb{g}_2\}$ is a partition of $\{1, 2, \ldots, N_x - 2\}$.

- $N_x - 1 - \mathbb{g}_2 \triangleq \{N_x - 1 - g \mid g \in \mathbb{g}_2\}$.
- $\{\mathbb{A}, \mathbb{B}\}$ is a partition of $\mathbb{C}$ if and only if $\mathbb{A} \cap \mathbb{B} =$ the empty set and $\mathbb{A} \cup \mathbb{B} = \mathbb{C}$.

Half Open Box Arrays

Main idea

Select $\mathbb{g}_1$ and $\mathbb{g}_2$ to be
$$\mathbb{g}_1 = \{1 + 2\ell \mid \ell = 0, 1, \ldots, \lfloor \tfrac{N_x-3}{2} \rfloor\},$$
$$\mathbb{g}_2 = \{N_x - 1 - 2\ell \mid \ell = 1, 2, \ldots, \lfloor \tfrac{N_x-2}{2} \rfloor\}.$$



$\mathbb{g}_1 = \{1, 3, 5, 7, 9, 11\}$
$\mathbb{g}_2 = \{2, 4, 6, 8, 10\}$

Hole-free $\mathbb{D}$ ★

$w(1,0) =$	2	★
$w(0,1) =$	36	✗
$w(1,1) =$	1	★
$w(1,-1) =$	1	★

Theoretical Guarantees of HOBA

Number of sensors

$$N_x + 2N_y - 2$$

Theorem 2: Difference coarrays

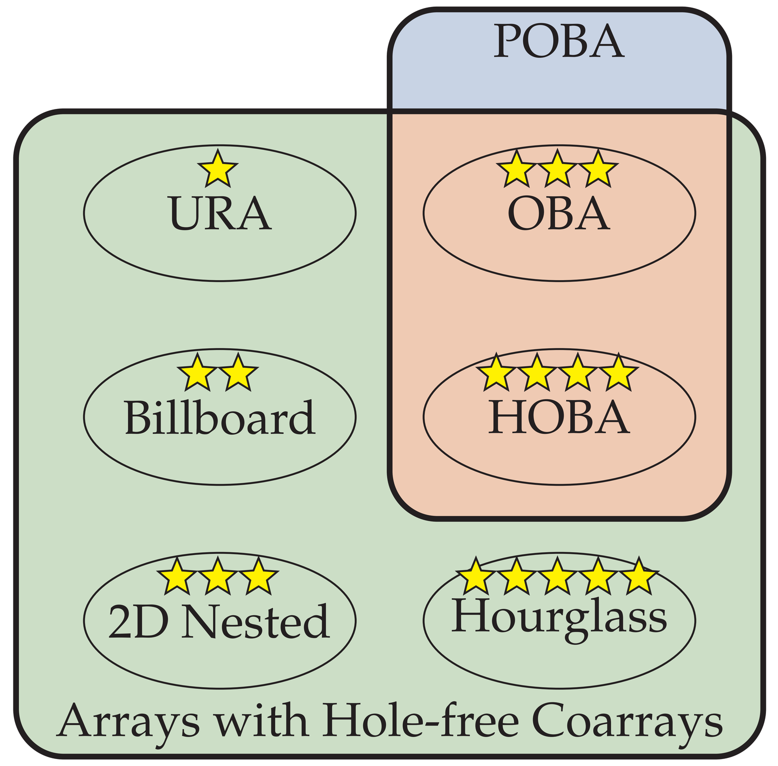
$$\mathbb{D}_{\text{HOBA}} = \mathbb{D}_{\text{OBA}} = \{0, \pm 1, \ldots, \pm(N_x - 1)\} \times \{0, \pm 1, \ldots, \pm(N_y - 1)\}$$

$\mathbb{D}_{\text{HOBA}}$ is hole-free. ★

Theorem 3: Weight functions

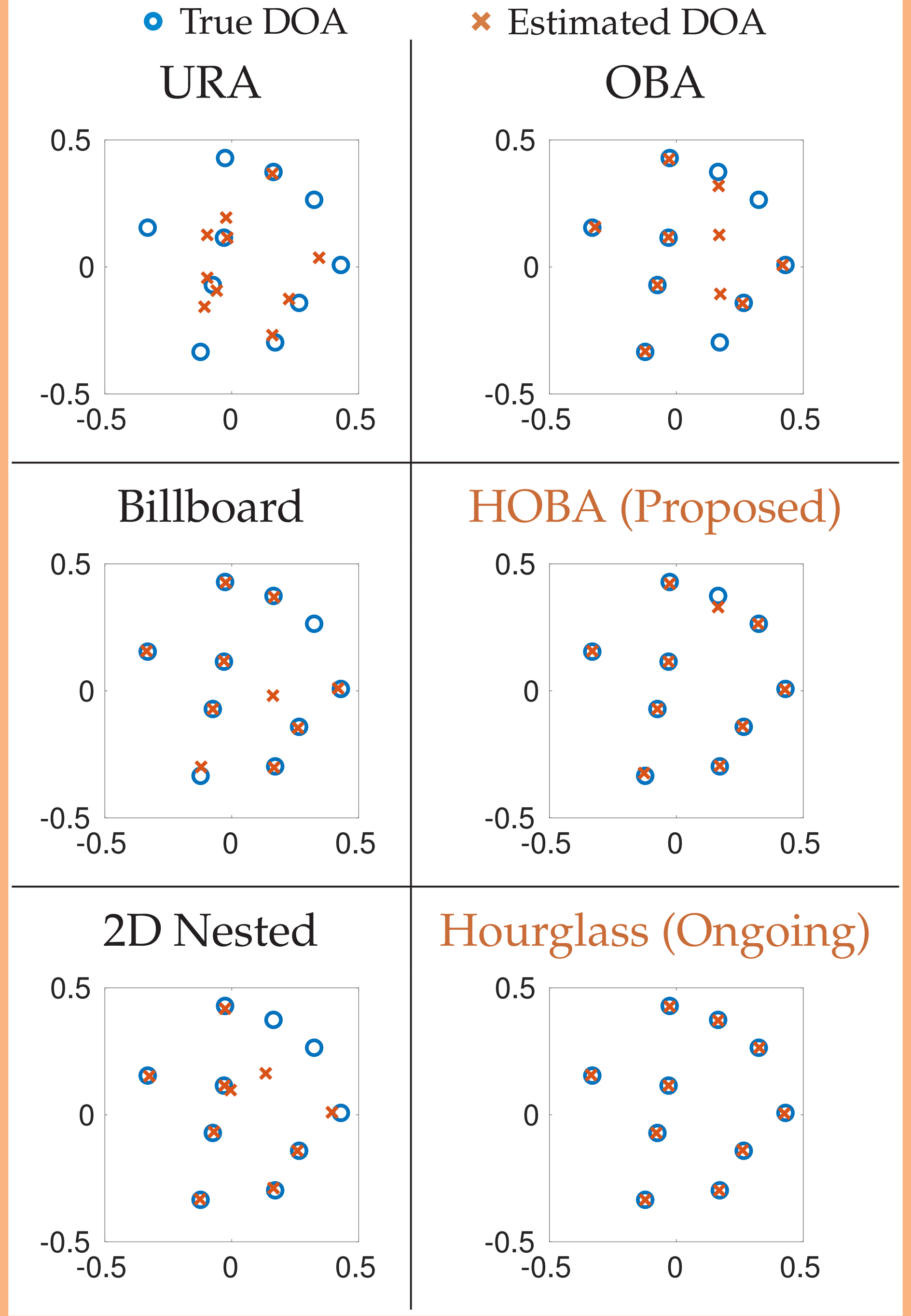
$w(1,0) = 2.$	★
$w(0,1) = 2(N_y - 1).$	✗
$w(1,1) = 1.$	★
$w(1,-1) = 1.$	★
Overall rating:	★★★★

Summary



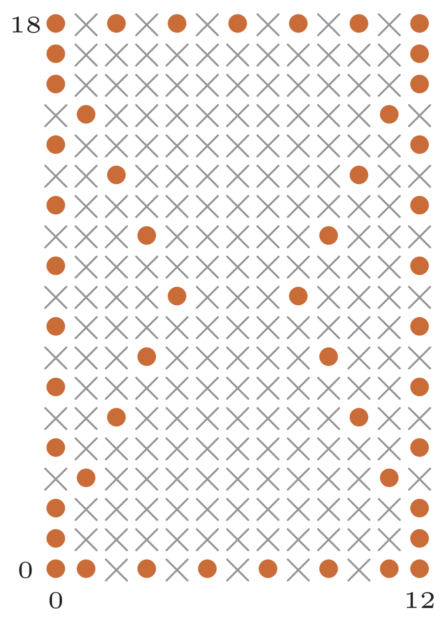
Numerical Examples

10 sources, 0dB SNR, 100 snapshots, *in the presence of mutual coupling*, 2D unitary ESPRIT on $\mathbb{D}$ [7] (*no decoupling or calibration*).



Ongoing Work

Hourglass arrays ★★★★★



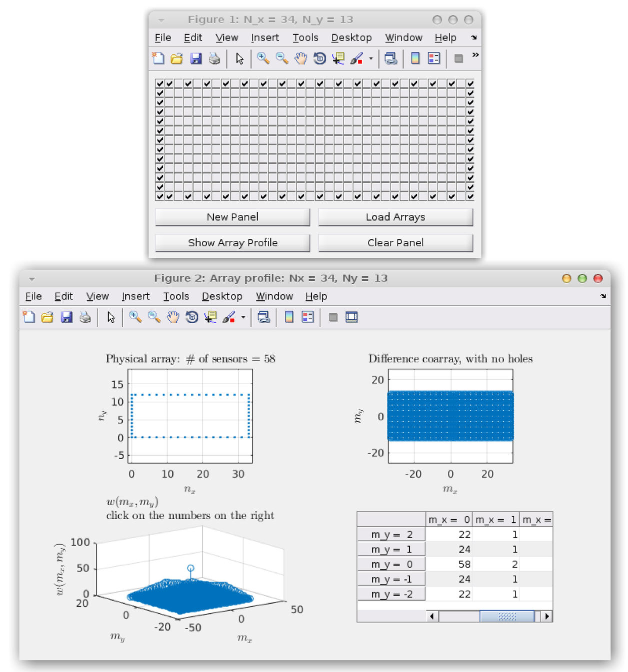
49 sensors

Hole-free $\mathbb{D}$ ★

$w(1,0) =$	2	★
$w(0,1) =$	8	★
$w(1,1) =$	5	★
$w(1,-1) =$	5	★

Concluding Remarks

- OBA → POBA → HOBA.
- *Decoupling or calibration algorithms* can be built on top of the proposed arrays.
- Applications: Radio astronomy, DOA estimation, beamforming, incoherent imaging.
- *Interactive interface* [8]:



References

[1] Friedlander and Weiss, *IEEE Trans. Antennas Propag.*, 1991.
[2] BouDaher, Ahmad, Amin, and Hoorfar, *Digit. Signal Process.*, 2016.
[3] Greene and Wood, *J. Acoust. Soc. Am.*, 1978.
[4] Pal and Vaidyanathan, *IEEE Trans. Signal Process.*, 2012.
[5] Haubrich, *Bull. Seismol. Soc. Am.*, 1968.
[6] Liu and Vaidyanathan, *IEEE Trans. Signal Process.*, 2016.
[7] Zoltowski, Haardt, and Mathews, *IEEE Trans. Signal Process.*, 1996.
[8] http://systems.caltech.edu/dsp/students/clliu/