Micro HW #1 Due: 2009/3/9

1. The Cobb-Douglas production function $f(x_1, x_2) = x_1^2 x_2^2$. Does this exhibit constant, increasing, or decreasing returns to scale?
2. In a production process is it possible to have decreasing marginal product in an input and yet increasing returns to scale?
3. If $pMP_1 > w_1$, then should the firm increase or decrease the amount of factor 1 in order to increase profits?
4. The Ontario Brassworks products brazen effronteries. As you know brass is an alloy of copper and zinc, used in fixed proportions. The production function is given by: $f(x_1, x_2) = \min\{x_1, 2x_2\}$, where x_1 is the amount of copper it used and x_2 is the amount of zinc that it uses in production.
   a. Illustrate a typical isoquant for this production function in a graph.
   b. Does this production function exhibit increasing, decreasing, or constant returns to scale?
   c. If the firm wanted to produce 10 effronteries, how much copper would it need? Hoe much zinc would it need?
   d. If the firm faces factor prices (1, 1), what is the cheapest way for it to produce 10 effronteries? How much will this cost?
   e. If the firm faces factor prices (w_1, w_2), what is the cheapest way for it to produce 10 effronteries?
   f. If the firm faces factor prices (w_1, w_2), what will be the minimal cost of producing y effronteries?
5. A firm uses labor and machines to produce output according to the production function $f(L, M) = 4L^{\frac{1}{2}}M^{\frac{1}{2}}$, where L is the number of units of labor used and M is the number of machines. The cost of labor is $40 per unit and the cost of using a machine is $10.
   a. Draw an isocost line for this firm, showing combinations of machines and labor that cost $400 and another isocost line showing combinations that cost $200. What is the slope of these isocost lines?
   b. Suppose that the firm wants to produce its output in the cheapest possible way. Find the number of machines it would use per worker. (Hint: The firm will produce at a point where the slope of the production isoquant equals the slope of the isocost line.)

c. On the graph, sketch the production isoquant corresponding to an output of 40. Calculate the amount of Labor and the number of machines that are used to produce 40 units of output in the cheapest possible way, given the above factor prices. Calculate the cost of producing 40 units at these factor prices: c(40,10.40)
d. How many units of Labor and hoe many machines would the firm use to produce y units in the cheapest possible way? How much would this cost? (Hint: Notice that there are constant returns to scale.)

6. Mary Magnolia wants to open a flower shop, the Petal Pusher, in a new mall. She has her choice of there different floor sizes, 200 square feet, 500 square feet, or 1000 square feet. The monthly rent will be $1 a square feet. Mary estimates that is she has F square feet of floor space and sells y bouquets a month, her variable costs will be $c_v(y) = y^2/F$ per month.
   a. If she has 200 square feet of floor space, write down her marginal cost function and her average cost function. At what amount of output is average cost minimized? At this level of output, how much is average cost?
   b. If she has 500 square feet, write down her marginal cost function and her average function. At what amount of output is average cost minimized? At this level of output, how much is average cost?
   c. If she has 1000 square feet, write down her marginal cost function and her average function. At what amount of output is average cost minimized? At this level of output, how much is average cost?
   d. Use red ink to show Mary's average cost curve and her marginal cost curves if she has 200 square feet. Use blue ink to show her average cost curve and her marginal cost curve if she has 500 square feet. Use black ink to show her average cost curve and her marginal cost curve if she has 1000 square feet. Label the average cost curves AC and the marginal cost curves MC.
   e. Show Mary's long-run average cost curve and her long-run marginal cost curve in your graph. Label them LRAC and LRMC.

7. Consider the cost function $c(y) = 4y^2 + 16$
   a. The average cost function is?
   b. The marginal cost function is?
   c. The level of output that yields the minimum average cost of production is?
   d. The average variable cost function is?
   e. At what level of output does average variable cost equal marginal cost?