Peer Effects and Consumption Behavior in Interconnected Networks

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Abstract

I consider peer effects of network externalities in a competitive telecommunication market. The magnitude of network externalities depends on whether the caller and the receiver are in the same peer group. Technically, interconnection between carriers can eliminate network externalities, but carriers may adopt termination-based price discrimination to reduce compatibility. When termination-based pricing is prohibited, carriers cannot exploit network externalities. Peer effects are equivalent to consumers with vertically heterogeneous tastes. There is a unique subscription equilibrium. On the other hand, when carriers offer termination-specific prices, peer effects have a substantial impact on the equilibrium. Because of the network externalities created by incompatibility, there might be multiple stable equilibria. Carriers may specialize to serve different consumer groups in a stable equilibrium even if they are identical ex ante. If carriers anticipate such an equilibrium, they would like to negotiate a high interconnection fee and offer intra-network discounts.

1 Introduction

Models of network externalities typically assume consumers are homogeneous, with identical externalities. Consequently, network effects only depend on the total number of users of a product. However, this assumption is not plausible in many situations. For example, consider the choice of e-mail service. My utility is positively correlated with the number of my friends using e-mail service but it decreases in the number of spammers on the Internet. This simple example illustrates the importance of *peer effects* in network externalities. Externalities are influenced by the composition of users, not just the number of users.

With telecommunication deregulation, many phone carriers often compete in the same market. This is especially true in cellular phone service markets. Competing carriers are

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generally interconnected. Interconnection allows customers of one carrier to make a phone call to customers of a different carrier. Technically, interconnection can eliminate network externalities between carriers. Nevertheless, carriers can use termination-based price discrimination to exploit network effects and compete with other carriers. When a receiver subscribes to the same carrier as the caller, the price of the phone call is often lower. Intra-network discounts reduce the compatibility of the services between different carriers. Despite interconnection, network externalities still exist among consumers subscribing to different carriers. In the extreme case, when the inter-network price is high enough, the two carriers essentially become incompatible.

For a phone call between two different networks, the caller's carrier needs to pay an *interconnection fee* to the receiver's carrier to compensate its cost of terminating the call. In the seminal works of Laffont, Rey, and Tirole (1998a,b), they analyze the policies on interconnection fees between competing networks.[1] Under their framework, carriers would like to negotiate an interconnection fee less than the actual cost of terminating a call in order to soften their competition whenever they can use two-part tariffs and termination-based price discrimination in the retail market (Gans and King, 2001). Such an agreement would result in a lower price for inter-network calls than for intra-network calls. However, it is rare, if at all, that a carrier offers discounts for inter-network calls in the real world. When I account for peer effects in this paper, carriers may prefer a higher interconnection fee in a stable equilibrium and offer discounts for intra-network calls. This model provides an explanation for the intra-network discounts that we often observe in the cellular service industry.

An important antitrust policy issue is to understand the impacts of terminational-based price discrimination. I propose a simple model to analyze competition among duopolistic carriers. Each carrier offers a two-part tariff, consisting of a subscription fee and a unit rate for a phone call. The same price schedule applies to all consumers. All consumers subscribe to one of the two carriers. There are two consumer groups. Consumers obtain a higher utility level when making a phone call which terminates within the same group. When termination-based price discrimination is not allowed, network externalities are entirely eliminated by interconnection. The peer effects simply affect the demand of a consumer. A consumer belonging to a large group has higher demand for the phone service. When both consumer groups are of the same size, carriers choose unit price equal to the perceived marginal cost and use the subscription fee to extract consumer surplus. On the other hand, when one group is larger than the other, price differs from the marginal cost. This distortion results because a single pricing scheme cannot extract all consumer surplus from different consumer groups. Regardless of the relative group sizes, profits are independent of the interconnection fee. In addition, the social optimum can be achieved by requiring the interconnection fee equal to the marginal cost of terminating a call.

When carriers charge different unit rates according to the termination of a call, network externalities are present. A consumer creates a positive externality for all other consumers subscribing to the same carrier when intra-network prices are lower. She would like to join a larger network, ceteris paribus. The subscription decision depends

[1] See Laffont and Tirole (2000) and Armstrong (2002) for surveys of interconnection in the telecommunication industry.

on the belief about the market shares. Under a rational expectation assumption, it is possible to have multiple stable equilibria for subscription choices. There are two types of stable equilibria, pooling and separating equilibria. While a pooling equilibrium always exist, a separating equilibrium is more likely to exist when (a) carriers are less differentiated, (b) intra-network discounts are larger, or (c) peer effects are stronger.

In a pooling equilibrium, the market share of a carrier is identical across different groups. Peer effects do not play any role when consumer groups are of the same size. Carriers choose intra- and inter-network prices equal to their respective marginal costs to maximize profits. The profits are maximized at an interconnection fee less than the cost of terminating a call. When carriers can negotiate the interconnection fee, they desire an interconnection deficit and offer a higher price for intra-network calls.

In a separating equilibrium, each carrier is dominant among one of the two consumer groups. Carriers specialize to serve different groups even though they are identical ex ante. Unlike the above cases, there is not marginal cost pricing rule even if the two consumer groups are of equal size. A numerical example shows that profits increase in the interconnection fee, and the intra-network price is less than inter-network price in a separating equilibrium. Therefore, carriers prefer a higher interconnection fee and offer intra-network discounts in anticipation of a separating equilibrium.

The rest of the paper is organized as follows. In the next section, I discuss the related literature. I then present a duopoly model with heterogeneous consumer groups. The social optimum is discussed in Section 4. I analyze the pricing decisions under duopolistic competition without termination-based price discrimination in Section 5. In Section 6, carriers are allowed to charge different prices based on the termination of a phone call. The conclusion is in the final section.

2 Related Literature

Although early researches of network externalities focus on a single network with homogeneous consumers, their main results hold in a more complicated environment. Because the utility of a product depends on the belief of the number of users, there are usually multiple equilibria. The outcome depends on consumers' beliefs. In the basic model of Rohlfs (1974), zero consumption in the market is obviously a stable equilibrium because, if everyone expects no one would purchase a network product, the product is useless. For positive quantities, the demand curve is a hump-shaped curve. Any point on the upward-sloping part of the demand curve is a unstable equilibrium, and it is the critical mass for the product to be introduced into the market. The downward-sloping part of the demand curve represents a stable equilibrium. The market would achieve a positive stable equilibrium only when consumers believe the equilibrium quantity would be greater than the critical mass.

Katz and Shapiro (1985) extend the discussion of network externalities into duopoly. Since there are more than one network in the market, compatibility between two networks is crucial. They assume that consumers are homogeneous so that their surplus only depends on the number of consumers buying a compatible product. There are multiple fulfilled equilibria, which depend on compatibility between different networks. In their model, products are either fully compatible or non-compatible. On the contrary, in my discussion of termination-based price discrimination, products are partially compatible. Carrier can change the degree of compatibility continuously by adjusting the discount on intra-network calls.

As I mentioned above, Laffont et al. (1998a,b) propose a framework to analyze competition between two interconnected networks.[2] The duopolistic networks are horizontally differentiated à la Hotelling. Consumers are otherwise identical. There exists a unique, stable, symmetric equilibrium when networks are poor substitutes. When termination-based price discrimination is not allowed, carriers choose their unit rate of a call equal to the perceived marginal cost under two part-tariffs, and their profits are independent of interconnection fee. Carriers are willing to accept an interconnection fee at the marginal cost of terminating a call, which is socially desirable.[3] When price discrimination is possible, they price intra- and inter-network calls to their respective marginal costs. However, Gans and King (2001) find that their profit decreases in the interconnection fee when the fee is nonnegative. As a result, carriers would like to negotiate an interconnection fee less than its marginal cost and offer discounts for inter-network calls. Social welfare is lower than the social optimum and consumers are worse off.

Peer effects in network industry are also analyzed in Jullien (2001, 2006). Contrary to my model, firms can use third-degree price discrimination based on the group identity of a consumer. An entrant can use a divide-and-conquer strategy to join the market. Specifically, it can subsidize some consumer groups and exploit the network externalities to recover the subsidy.

3 Model

There are two identical telephone carriers $k = 1, 2$. Carrier k offers a two-part tariff (t_k, p_{kk}, p_{kl}) to consumers, where t_k is the subscription fee; p_{kk} and p_{kl} are the unit rates for calls terminated in network k and network l, respectively. Receivers do not pay for incoming calls.[4] The phone tariff of q_I intra-network calls and q_O inter-network calls is $t_k + p_{kk}q_I + p_{kl}q_O$. There is a fixed cost $f \geq 0$ to serve a customer. Marginal cost c is the same for intra-network calls and inter-network calls. The marginal cost can be decomposed into two parts: $c = c_O + c_T$, where c_O is the originating-end cost and c_T is the terminating-end cost. For each phone call originated from network k and terminated in the other network l, carrier k needs to pay carrier l an interconnection fee a. This interconnection fee is reciprocal, determined exogenously. It can be either determined by the regulator or negotiated by the carriers in advance. A carrier pays as much for termination of a call on the rival's network as it receives for completing a call originated from the rival's network. The cost to operate a network is normalized to zero.

[2] See also Armstrong (1998) for this framework.

[3] Dessein (2003, 2004) and Hahn (2004) extend this framework to consider consumers with heterogeneous demand, the profit-neutrality holds for many generalized cases.

[4] This "calling party pays" principle applies to most telecommunication sectors in most countries. One notable exception is the cellular phone service in the United State, where receivers pay for incoming calls. See Jeon et al. (2004) for discussions on the "receivers-pay" principle.

Let $\mathcal{C}$ be the set of all consumers in the market. There are two group of consumers, $\mathcal{C}^1$ and $\mathcal{C}^2$, such that $\{\mathcal{C}^1, \mathcal{C}^2\}$ is a partition of $\mathcal{C}$. Normalize the population of consumers to be 1, $|\mathcal{C}| = 1$. Denote the size of group $\mathcal{C}^n$ by μ^n. The sizes $\{\mu^1, \mu^2\}$ are common knowledge, but the group identity of a consumer is unknown to carriers. Denote an individual i's group by $\mathcal{C}(i)$.

Consumers subscribe to the network which yields a higher utility level. Denote consumer i's selected carrier by $k(i)$. The utility function is quasi-linear[5] and the utility from making phone calls is additively separable across receivers. Consumer i's utility of subscribing to network k is

$$\int_{j\in\mathcal{C}} \bar{u}_{ij}(q_{ij})\, dj - T(\{q_{ij} : j \in \mathcal{C}\}) + \frac{1}{\sigma}\varepsilon_{ik}, \tag{1}$$

where q_{ij} is the quantity of calls from i to j, and $\bar{u}_{ij}(q_{ij})$ is i's utility from calling j. $T(\{q_{ij} : j \in \mathcal{C}\})$ is the tariff of these calls. The final term, ε_{ik}, is consumer i's idiosyncratic preference of subscribing to carrier k, which is not affected by the quantity choices $\{q_{ij} : j \in \mathcal{C}\}$. For example, ε_{ik} captures the effect of advertisements. The idiosyncratic preference ε_{ik} draws from the Type I extreme value distribution independently across i and k.[6] It is private information of consumer i. There is no utility from receiving phone calls.[7]

The coefficient $\sigma > 0$ captures the substitutability between the two carriers. Its inverse is the degree of differentiation between them. When the two carriers are more substitutable with each other, it is easier for the carrier that provides a higher utility level to attract more subscribers. As $\sigma \to \infty$, there is no differentiation among them, and carriers are competing with each other only in prices. On the contrary, when $\sigma \to 0$, each of them covers half of the market regardless their price schemes.

The utility of a phone call depends on the receiver's group. Specifically, let

$$\bar{u}_{ij}(q_{ij}) = \begin{cases} u^G(q_{ij}) & \text{for } j \in \mathcal{C}(i) \\ u^N(q_{ij}) & \text{for } j \notin \mathcal{C}(i), \end{cases} \tag{2}$$

with $u^G(q) \geq u^N(q)$ and $u^{G'}(q) \geq u^{N'}(q)$ for all $q \geq 0$. Therefore, a caller i gets a higher utility level and also higher marginal utility from calling a receiver in her peer group than from calling other people. Moreover, both u^G and u^N are strictly concave, and $u^{G'}(0) > c, u^{N'}(0) > c$.

Because a consumer's group identity is unknown to carriers, carriers have to offer a uniform price scheme for all consumers in the market. It is impossible to implement third degree price discrimination across peer groups. Therefore, the divide-and-conquer

[5] It is reasonable to assume quasi-linear because the expenditure on telecommunications is small relative to income.

[6] The logit-style preferences ensure market shares are always strictly between 0 and 1. I can analyze marginal effects of price in cases where one carrier is dominant in the market. In the framework of Laffont et al. (1998a,b), ε_{ik} is specified such that consumers are differentiated à la Hotelling. Market shares do not respond to an infinitesimal change in price at corner solutions.

[7] If consumers benefit from receiving incoming calls without paying for them, there are call externalities. See Hahn (2003) for discussions on call externalities under nonlinear pricing.

strategy discussed in Jullien (2001) does not work.

Denote the number of group-$\mathcal{C}^n$ consumers subscribing to network k by s_k^n, for $n = 1, 2$ and $k = 1, 2$. For each individual consumer i, let s_{ik}^n be her expected number of individuals subscribing to network k among group $\mathcal{C}^n$. Clearly, $s_{i1}^n + s_{i2}^n = \mu^n$. For most of the following analyses in this paper, I will focus on the results under rational expectation, $s_{ik}^n = s_k^n$ for all $i \in \mathcal{C}$.

For any given interconnection fee a, the interactions between consumers and carriers can be summarized in the following three stages.

1. Pricing stage: Carriers determine their price scheme (t_k, p_{kk}, p_{kl}) simultaneously. Each consumer i forms her belief about the market shares $\{(s_{ik}^n) : k = 1, 2,\ n = 1, 2\}$.

2. Subscription stage: Each consumer i selects one carrier $k(i)$ from $\{1, 2\}$ simultaneously.

3. Calling stage: Consumer i determines the quantities of phone calls, $\{q_{ij} : j \in \mathcal{C}\}$, and pay for the service.

Because utility from calling is additively separable across receivers, the quantity decision at Stage 3 is independent across receivers. For any consumer $i \in \mathcal{C}$, conditional on subscribing to carrier $k(i)$, the quantity of calls to a receiver j is determined by the utility maximization problem.

$$\max_{q_{ij}} \begin{cases} u^G(q_{ij}) - p_{k(i),k(j)} q_{ij}, & \text{if } j \in \mathcal{C}(i), \\ u^N(q_{ij}) - p_{k(i),k(j)} q_{ij}, & \text{if } j \notin \mathcal{C}(i), \end{cases} \tag{3}$$

where $p_{k(i),k(j)}$ is the unit rate of a call from i's network to j's network. It is obvious that the quantity depends on (a) peer effect: whether i and j are members of the same peer group, and (b) on-off effect: whether i and j subscribe to the same network. The strict concavity of utility functions guarantees a unique solution of (3). Hence, the quantity can be expressed as the following function.

$$q_{ij} = \begin{cases} q^G(p_{k(i),k(j)}), & \text{if } j \in \mathcal{C}(i), \\ q^N(p_{k(i),k(j)}), & \text{if } j \notin \mathcal{C}(i). \end{cases} \tag{4}$$

The on-off effect results from termination-based price discrimination. It diminishes when the intra-network price equals the inter-network price.

For notational convenience, let

$$\begin{aligned} v^G(p) &= u^G(q^G(p)) - p\, q^G(p), & \pi^G(p) &= (p - c)\, q^G(p), & \phi^G(p) &= (a - c_T)\, q^G(p), \\ v^N(p) &= u^N(q^N(p)) - p\, q^N(p), & \pi^N(p) &= (p - c)\, q^N(p), & \phi^N(p) &= (a - c_T)\, q^N(p). \end{aligned}$$

$v(p)$ is a consumer's surplus from calling a receiver at price p. The profit of a carrier made from its own customer calling someone else on its network is $\pi(p)$, and the profit from its customer calling someone outside its network is $\pi(p) - \phi(p)$. The profit of terminating calls from a rival's customer is $\phi(p)$. The superscripts in these expressions represent whether the caller and the receiver are in the same peer group (^{G}) or not (^{N}).

The subscription decision of a consumer at Stage 2 depends on consumer surpluses, $v^G(p)$ and $v^N(p)$, and her expected market shares among each peer group, $\{s^n_{ik} : k = 1, 2,\ n = 1, 2\}$. As I will show, it is possible to have multiple fulfilled equilibria when intra-network calls are cheaper than inter-network calls.

At Stage 1, each carrier chooses its price scheme (t_k, p_{kk}, p_{kl}) to maximize the profit,

$$\Pi_k(P) = \sum_{n=1,2} s^n_k \Big\{ s^n_k \pi^G(p_{kk}) + s^m_k \pi^N(p_{kk}) + s^n_l [\pi^G(p_{kl}) - \phi^G(p_{kl}) + \phi^G(p_{lk})] + s^m_l [\pi^N(p_{kl}) - \phi^N(p_{kl}) + \phi^N(p_{lk})] + t_k - f \Big\},$$

where $l = 3 - k$ and $m = 3 - n$. It depends on the belief of the realized equilibrium market shares at Stage 2.

4 Social Optimum

Since I assume zero fixed cost to operate a network, two active networks are better than one because consumers value the variety of subscription choices. For any given subscription pattern, there is no externality at the calling stage. Therefore, it is socially optimal that a consumer's marginal utility from making a call equals the marginal cost of a call. The socially optimal volume of intra-group calls, $\bar{q}^G$, is determined by $u^{G'}(\bar{q}^G) = c$. Similarly, the socially optimal volume of inter-group calls, $\bar{q}^N$, satisfies the condition $u^{N'}(\bar{q}^N) = c$. Because $u^{G'}(q) \geq u^{N'}(q)$ for any q, it is optimal to make more calls to intra-group receivers than to inter-group receivers, $\bar{q}^G \geq \bar{q}^N$. The total volume consumed by a consumer in a large group is higher than the total volume by a small-group consumer at the social optimum.

$$\mu^1 \bar{q}^G + \mu^2 \bar{q}^N \geq \mu^2 \bar{q}^G + \mu^1 \bar{q}^N$$

if and only if $\mu^1 \geq \mu^2$.

Because all consumers are required to subscribe to one of the two networks, there is no externality of the subscription choice as well. The optimal subscription pattern is determined by a consumer's idiosyncratic preferences $\{\varepsilon_{ik}, \varepsilon_{il}\}$. It is socially optimal to subscribe to network k if and only if $\varepsilon_{ik} \geq \varepsilon_{il}$. As a result, each network provides service to half of the consumers within each group, $s^n_k = 0.5\mu^n$, for $k = 1, 2$ and $n = 1, 2$.

Under nonlinear price schemes, the operator can recover the fixed cost f from the subscription fees $\{t_k\}$. These fees only change the distribution of welfare in the economy, but the overall welfare level is fixed. For Ramsey pricing, the industry breaks even at $t_1 = t_2 = f$.

Nonetheless, when the cost of serving a consumer is too high, it is optimal to shut down the phone service. Specifically, the social planner should provide the phone service if and only if

$$\sum_{n=1,2} \mu^n \log \left[2 \exp \left(\mu^n v^G(c) + \mu^l v^N(c) \right) \right] \geq f.$$

Equivalently,

$$\log 2 + \left[(\mu^1)^2 + (\mu^2)^2\right] v^G(c) + \left[2\mu^1\mu^2\right] v^N(c) \geq f.$$

5 Competition without Price Discrimination

The price of intra-network calls are the same as that of inter-network calls in this section. Denote the marginal price by $p_k = p_{kk} = p_{kl}$. For each group $\mathcal{C}^n$, denote the average utility level of subscribing to network k as

$$U_k^n(P) = \left[\mu^n v^G(p_k) + \mu^m v^N(p_k)\right] - t_k,$$

for $k = 1, 2$, $n = 1, 2$, and $m = 3 - n$. The utility level is independent of the belief about the market shares $\{s_{ik}^n\}$. Under the assumption on the distribution of the idiosyncratic preferences, ε_{ik}, the number of network-k subscribers among group $\mathcal{C}^n$ is

$$s_k^n(P) = \mu^n \frac{\exp\left(\sigma U_k^n(P)\right)}{\exp\left(\sigma U_k^n(P)\right) + \exp\left(\sigma U_l^n(P)\right)}$$

for $k = 1, 2$, $n = 1, 2$, and $l = 3 - k$.

5.1 Symmetric Consumer Groups

In this benchmark case, suppose peer groups are of the same size, $\mu^1 = \mu^2 = 1/2$. This implies $s_k^1(P) = s_k^2(P)$. Consequently, the profit of carrier k can be written as

$$\Pi_k(P) = 2s_k^1(P)\left[\frac{\pi^G(p_k)}{2} + \frac{\pi^N(p_k)}{2} + t_k - f\right] \\ + 2s_k^1(P)s_l^1(P)\left[\phi^G(p_l) - \phi^G(p_k) + \phi^N(p_l) - \phi^N(p_k)\right]$$

The first order conditions are

$$0 = \frac{\partial \Pi_k}{\partial t_k} = \frac{\partial s_k^1}{\partial t_k}\left[\pi^G(p_k) + \pi^N(p_k) + 2(t_k - f)\right] + 2s_k^1 \\ + 2\left[\frac{\partial s_k^1}{\partial t_k}s_l^1 + s_k^1\frac{\partial s_l^1}{\partial t_k}\right]\left[\phi^G(p_l) - \phi^G(p_k) + \phi^N(p_l) - \phi^N(p_k)\right] \quad (5)$$

and

$$0 = \frac{\partial \Pi_k}{\partial p_k} = \frac{\partial s_k^1}{\partial p_k}\left[\pi^G(p_k) + \pi^N(p_k) + 2(t_k - f)\right] + s_k^1\left[\pi^{G'}(p_k) + \pi^{N'}(p_k)\right] \\ + 2\left[\frac{\partial s_k^1}{\partial p_k}s_l^1 + s_k^1\frac{\partial s_l^1}{\partial p_k}\right]\left[\phi^G(p_l) - \phi^G(p_k) + \phi^N(p_l) - \phi^N(p_k)\right] - 2s_k^1 s_l^1\left[\phi^{G'}(p_k) + \phi^{N'}(p_k)\right]. \quad (6)$$

From equation (5), I obtain

$$-\left[\pi^G(p_k)+\pi^N(p_k)+2(t_k-f)\right]+\frac{1}{\sigma s_l^1}+2(s_k^1-s_l^1)\left[\phi^G(p_l)-\phi^G(p_k)+\phi^N(p_l)-\phi^N(p_k)\right]=0.$$

Substitute this into the equation (6). The first order conditions imply

$$p_k^* = c + (s_l^1 + s_l^2)(a - c_T). \tag{7}$$

The right hand size in equation (7) is the perceived marginal cost faced by a carrier when the interconnection fee is a and the ratio of calls terminating in the rival's network is $(s_l^1 + s_l^2)$. From equation (5), I obtain the optimal subscription fee,

$$t_k^* = f - \frac{1}{2}\left[\pi^G(p_k^*) + \pi^N(p_k^*)\right] + \frac{1}{2\sigma s_l^1} + (s_k^1 - s_l^1)\left[\phi^G(p_l^*) - \phi^G(p_k^*) + \phi^N(p_l^*) - \phi^N(p_k^*)\right]. \tag{8}$$

Proposition 1. *Suppose termination-based price discrimination is not allowed and consumer groups are of the same size. There is a symmetric Nash equilibrium in the pricing stage if the interconnection markup* $(a-c_T)$ *is small in absolute value or the degree of substitutability* σ *is small. The unit price equals the perceived marginal cost* $p^* = c+(a-c_T)/2$ *and the subscription fee is* $t^* = f + 2/\sigma - [q^G(p^*) + q^N(p^*)](a - c_T)/4$ *for both carriers. The profit of each carrier is* $\Pi_k = 1/\sigma$*, which is independent of the cost parameters and the interconnection charge* a*. The market share is identical for both carriers among each group,* $s_k^n = 1/4$ *for* $k = 1, 2$ *and* $n = 1, 2$*. Furthermore, the equilibrium is unique if either (a) the interconnection markup* $(a - c_T)$ *is non-positive, or (b) the interconnection markup is positive but not too high.*

Proof. Existence and uniqueness of the symmetric Nash equilibrium is relegated to Appendix A. By symmetry, the market shares are $s_k^n = 1/4$ for $k = 1, 2$ and $n = 1, 2$. According to the first order conditions (7) and (8), I obtain $p^* = c + (a - c_T)/2$ and $t^* = f + 2/\sigma - [\pi^G(p_k^*) + \pi^N(p_k^*)]/2 = f + 2/\sigma - [q^G(p^*) + q^N(p^*)](a - c_T)/4$. The profit is $\Pi_k = (t^* - f)/2 + [\phi^G(p^*)/4 + \phi^N(p^*)/4]/2 = 1/\sigma$ for $k = 1, 2$. □

This result is fundamentally the same as Laffont et al. (1998a), in which carriers are differentiated à la Hotelling without consumer peer groups. In my model, they are differentiated in a logit model. When the group sizes are the same ($\mu^1 = \mu^2$), peer groups in the economy does not affect market equilibrium. The intuition of the result is straightforward. Since both groups are of the same size, there is no need to discriminate consumers against their group identity. The carriers can use a uniform two-part tariff to extract consumer surplus from both groups.

The social optimum is achieved in equilibrium when the interconnection fee equals the marginal cost of terminating a call. Because profit does not depend on this fee, the regulator may suggest carriers to set $a = c_T$ and carriers have no incentive to deviate to a different interconnection fee.

5.2 Asymmetric Consumer Groups

Suppose the group sizes are different, $\mu^1 \neq \mu^2$. Consider a symmetric equilibrium with $t_1 = t_2 = t^*$ and $p_1 = p_2 = p^*$. By continuity, the symmetric equilibrium is the unique one if the groups sizes are not too different, and either $|a - c_T|$ or σ is small. By symmetry, $s_1^n = s_2^n = \mu^n/2$ for each group $\mathcal{C}^n$. The first order condition $\partial\Pi_k(t^*, p^*, p^*; t^*, p^*, p^*)/\partial t_k = 0$ can be simplified as

$$t^* = f + \frac{2}{\sigma} - \left[(\mu^1)^2 + (\mu^2)^2\right] \pi^G(p^*) - 2\mu^1\mu^2\pi^N(p^*). \tag{9}$$

The other first order condition $\partial\Pi_k(t^*, p^*, p^*; t^*, p^*, p^*)/\partial p_k = 0$ is

$$\begin{aligned} 0 = &-\sigma[\mu^1 q^G(p^*) + \mu^2 q^N(p^*)]\mu^1[\mu^1\pi^G(p^*) + \mu^2\pi^N(p^*) + t^* - f] \\ &- \sigma[\mu^2 q^G(p^*) + \mu^1 q^N(p^*)]\mu^2[\mu^2\pi^G(p^*) + \mu^1\pi^N(p^*) + t^* - f] \\ &+ 2[(\mu^1)^2 + (\mu^2)^2]\pi^{G'}(p^*) + 4\mu^1\mu^2\pi^{N'}(p^*) - [(\mu^1)^2 + (\mu^2)^2]\phi^{G'}(p^*) - 2\mu^1\mu^2\phi^{N'}(p^*). \end{aligned}$$

Combining these two conditions, I obtain

$$\begin{aligned} p^* = c + \frac{a - c_T}{2} \times \\ \left(\frac{2[(\mu^1)^2 + (\mu^2)^2]q^{G'}(p^*) + 4\mu^1\mu^2 q^{N'}(p^*)}{2[(\mu^1)^2 + (\mu^2)^2]q^{G'}(p^*) + 4\mu^1\mu^2 q^{N'}(p^*) - \sigma\mu^1\mu^2(\mu^1 - \mu^2)^2[q^G(p^*) - q^N(p^*)]^2}\right). \end{aligned}$$

Because the term in the parenthesis in the above equation is between 0 and 1, the unit price p^* is (strictly) between c and $c + (a - c_T)/2$.

$$\begin{cases} c < p^* < c + \frac{1}{2}(a - c_T), & \text{if } a > c_T; \\ c = p^* = c + \frac{1}{2}(a - c_T), & \text{if } a = c_T; \\ c > p^* > c + \frac{1}{2}(a - c_T), & \text{if } a < c_T. \end{cases}$$

From the first order condition (9), I can obtain the subscription fee t^*. The profit for carrier k is $\Pi_k(t^*, p^*, p^*; t^*, p^*, p^*) = 1/\sigma$. The results are summarized in the following proposition.

Proposition 2. *Suppose termination-based price discrimination is not allowed. Each carrier uses a uniform two-part tariff to maximize its profit. Consumer groups have different sizes. Then, each carrier serves half of the consumers among each peer group in the symmetric Nash equilibrium. The unit price is between the actual marginal cost* c *and the perceived marginal cost* $c + (a - c_T)/2$*. Moreover, the profit is* $1/\sigma$*, independent of the interconnection charge.*

When the sizes of peer groups are different, the optimal unit price p_k is different from the perceived marginal cost $c + (a - c_T)/2$. This is in line with the result of implicit price discrimination discussed in Dessein (2004) in which consumers have vertically heterogenous demand. A consumer from a large peer group is logically identical to a heavy user in Dessein's model because she wants to make more phone calls for a given price. The intuition behind Proposition 2 is that the carriers cannot extract all consumer surplus from different groups by a single subscription fee. When the interconnection markup $(a - c_T)$ is positive, carriers would like to have a greater proportion of large-group users among their subscribers in order to receive more interconnection revenue from the rival for incoming phone calls. [8] By lowering the unit price p_k from the perceived marginal cost $c + (a - c_T)/2$, consumer surplus is higher, and the increase is relatively greater for large-group consumers. The carrier can extract the higher surplus by raising the subscription fee t_k. These changes in the price scheme would attract relatively more large-group users. As a result, the carrier would receive more interconnection revenue for any given price scheme of its rival. In equilibrium, however, both carriers cover half of the market and the net interconnection payment is zero.

Similar to the result with symmetric consumer groups, profits are independent of the interconnection charge. The social optimum can be reached by suggesting carriers to set $a = c_T$ in their interconnection agreement.

The discussions in this section can be easily extended if carriers can use more general nonlinear pricing instead of a two-part tariff. According to the revelation principle, I can assume, without loss of generality, that each carrier offers a menu of two optional plans. One rate plan for the large-group consumers, and the other one for the small-group consumers. The rate plans have to satisfy incentive constraints. The results are parallel to Section 4 of Dessein (2004).

6 Competition with Price Discrimination

Now, I allow carriers to charge different unit prices based on the terminating network of a phone call. The following lemma shows their incentive to adopt termination-based price discrimination.

Lemma 1. *Suppose the two consumer groups are of the same size. At the symmetric non-discriminatory equilibrium, both carriers have an incentive to decrease the price for intra-network calls and raise the price for inter-network calls if and only if the interconnection markup $(a - c_T)$ is positive.*

Proof. Let $p^* = c + (a - c_T)/2$ and $t^* = f + 2 - [\pi^G(p^*) + \pi^N(p^*)]/2$. Then the partial derivatives of carrier k's profit with respect to p_{kk} and p_{kl} are

$$\frac{\partial \Pi_k(t^*, p^*, p^*; t^*, p^*, p^*)}{\partial p_{kk}} = \frac{1}{16}(a - c_T)[q^{G'}(p^*) + q^{N'}(p^*)] \quad \text{and}$$
$$\frac{\partial \Pi_k(t^*, p^*, p^*; t^*, p^*, p^*)}{\partial p_{kl}} = -\frac{1}{16}(a - c_T)[q^{G'}(p^*) + q^{N'}(p^*)],$$

respectively. Consequently, $\partial\Pi_k/\partial p_{kk} < 0$ and $\partial\Pi_k/\partial p_{kl} > 0$ at $P = (t^*, p^*, p^*; t^*, p^*, p^*)$ if and only if $a - c_T > 0$. □

[8] Carriers do not worry about the interconnection payments due to outgoing calls because they can use the subscription fee t_k to recover this cost from their own subscribers.

6.1 Conditions for Rational Expectation Equilibria

After carriers announce a pricing scheme $P = (t_1, p_{11}, p_{12}; t_2, p_{22}, p_{21})$, consumers make their subscription decisions $k(i)$. When carriers discriminate phone calls based on the carrier of the receiver, the price difference between the intra-network and inter-network calls creates network externalities among consumers of different carriers. Termination-based price discrimination essentially makes the networks less compatible. When the intra-network price is lower than the inter-network price, a consumer would like to subscribe to the larger network, ceteris paribus. As a result, the subscription choice depends on the belief about the group-specific market shares $\{s_k^n, k = 1, 2,\ n = 1, 2\}$. I assume consumers have rational expectation about the market shares for the remaining of this paper.

Assumption 1 (Rational Expectation)**.** *Consider the subscription stage. For any given price scheme* $P = (t_1, p_{11}, p_{12}; t_2, p_{22}, p_{21})$*, the expected market distribution equals the realized market distribution in equilibrium,* $s_{ik}^n = s_k^n(P)$*, for all* $i \in \mathcal{C}$*,* $k = 1, 2$*, and* $n = 1, 2$*.*

For each group $\mathcal{C}^n$, the average utility of subscribing to network $k \in \{1, 2\}$ for a given price scheme P is

$$U_k^n(P; s_k^n, s_k^m, s_l^n, s_l^m) \equiv s_k^n v^G(p_{kk}) + s_k^m v^N(p_{kk}) + s_l^n v^G(p_{kl}) + s_l^m v^N(p_{kl}) - t_k, \tag{10}$$

where $m = 3 - n$ and $l = 3 - k$. Under the logit specification, the number of carrier-k subscribers among group $\mathcal{C}^n$ satisfies

$$s_k^n = \mu^n \times \frac{\exp[\sigma U_k^n(P; s_k^n, s_k^m, s_l^n, s_l^m)]}{\exp[\sigma U_k^n(P; s_k^n, s_k^m, s_l^n, s_l^m)] + \exp[\sigma U_l^n(P; s_k^n, s_k^m, s_l^n, s_l^m)]}. \tag{11}$$

for each $k = 1, 2$, $n = 1, 2$.

The following proposition shows the existence of a rational expectation equilibrium.

Proposition 3. *For any given price scheme* P*, there exists a rational expectation equilibrium of the subscription decisions,* $\{s_k^n,\ k = 1, 2,\ n = 1, 2\}$*.*

Proof. Let $\Delta \equiv \{(s_k^n)_{k=1,2,\,n=1,2} : s_1^n + s_2^n = \mu^n; s_k^n \geq 0, \forall k, n\}$. Δ is the set of all possible market distributions. Since $\{(s_k^n)_{k=1,2} : s_1^n + s_2^n = \mu^n; s_k^n \geq 0, \forall k\}$ is a simplex in $\mathbf{R}^2$ for each n, it is a nonempty, convex, and compact space. Because Δ is the product of two such spaces, it is nonempty and convex. Moreover, by Tychonoff theorem, Δ is compact.

Fix the price scheme P. Equations (10) and (11) define a continuous mapping from Δ onto itself. Therefore, by Brouwer's fixed point theorem, the mapping has a fixed point $s \in \Delta$. This fixed point is a rational expectation equilibrium. □

Because $s_2^1 = \mu^1 - s_1^1$ and $s_2^2 = \mu^2 - s_1^2$, and the sizes of consumer groups (μ^1, μ^2) are fixed, the market shares in equilibrium can be expressed by two variables $(s_1^1, s_1^2) \in [0, \mu^1] \times [0, \mu^2]$. Consequently, with some abuse of notation, the average utility level (10) can be rewritten as

$$U_k^n(P; s_1^1, s_1^2) = \left[v^G(p_{k1}) - v^G(p_{k2})\right] s_1^n + v^G(p_{k2})\mu^n + \left[v^N(p_{k1}) - v^N(p_{k2})\right] s_1^m + v^N(p_{k2})\mu^m - t_k$$

for $k=1,2$, $n=1,2$, and $m=3-n$. For a given price scheme P, the rational expectation equilibria of carrier 1's market shares are fixed points of the mapping $\mathcal{T}:[0,\mu^1]\times[0,\mu^2]\to[0,\mu^1]\times[0,\mu^2]$

$$\mathcal{T}\begin{bmatrix} s_1^1 \\ s_1^2 \end{bmatrix} = \begin{bmatrix} \mu^1 \times \frac{\exp[\sigma U_1^1(P;s_1^1,s_1^2)]}{\exp[\sigma U_1^1(P;s_1^1,s_1^2)]+\exp[\sigma U_2^1(P;s_1^1,s_1^2)]} \\ \mu^2 \times \frac{\exp[\sigma U_1^2(P;s_1^1,s_1^2)]}{\exp[\sigma U_1^2(P;s_1^1,s_1^2)]+\exp[\sigma U_2^2(P;s_1^1,s_1^2)]} \end{bmatrix}.$$

This operator maps the belief of the distribution to the actual distribution.

Define an equilibrium (s_1^1, s_1^2) to be stable if it is a stable fixed point of the mapping $\mathcal{T}$. In other words, if the belief is slightly deviated from an equilibrium, applying the operator $\mathcal{T}$ recursively would converge to the original equilibrium.

Lemma 2 (Stability). *The subscription distribution (s_1^1, s_1^2) is stable if and only if the following conditions hold.*

$$\left(1-\frac{\sigma s_1^1 s_2^1}{\mu^1}\triangle v^G(P)\right)\left(1-\frac{\sigma s_1^2 s_s^2}{\mu^2}\triangle v^G(P)\right) > \frac{\sigma s_1^1 s_2^1}{\mu^1}\frac{\sigma s_1^2 s_2^2}{\mu^2}(\triangle v^N(P))^2,$$
$$\left(1+\frac{\sigma s_1^1 s_2^1}{\mu^1}\triangle v^G(P)\right)\left(1+\frac{\sigma s_1^2 s_s^2}{\mu^2}\triangle v^G(P)\right) > \frac{\sigma s_1^1 s_2^1}{\mu^1}\frac{\sigma s_1^2 s_2^2}{\mu^2}(\triangle v^N(P))^2, \textit{ and}$$
$$-1 < \frac{1}{2}\left(\frac{\sigma s_1^1 s_2^1}{\mu^1}+\frac{\sigma s_1^2 s_2^2}{\mu^2}\right)\triangle v^G(P) < 1,$$

where $\triangle v^G(P) \equiv v^G(p_{11})+v^G(p_{22})-v^G(p_{12})-v^G(p_{21})$ and $\triangle v^N(P) \equiv v^N(p_{11})+v^N(p_{22})-v^N(p_{12})-v^N(p_{21})$.

Proof. A necessary and sufficient condition for stability is that the radii of the eigenvalues of the Jacobian of $\mathcal{T}$ are both less than 1. The characteristic function of the Jacobian is

$$\begin{aligned}\Psi(\lambda) &= \left[\frac{\sigma s_1^1 s_2^1}{\mu^1}\triangle v^G(P)-\lambda\right]\left[\frac{\sigma s_1^2 s_2^2}{\mu^2}\triangle v^G(P)-\lambda\right] - \frac{\sigma s_1^1 s_2^1}{\mu^1}\triangle v^N(P)\frac{\sigma s_1^2 s_2^2}{\mu^2}\triangle v^N(P) \\ &= \lambda^2 - \left[\frac{\sigma s_1^1 s_2^1}{\mu^1}+\frac{\sigma s_1^2 s_2^2}{\mu^2}\right]\triangle v^G(P)\lambda + \frac{\sigma s_1^1 s_2^1}{\mu^1}\frac{\sigma s_1^2 s_2^2}{\mu^2}\left[(\triangle v^G(P))^2-(\triangle v^N(P))^2\right].\end{aligned}$$

Since

$$\left[\frac{\sigma s_1^1 s_2^1}{\mu^1}+\frac{\sigma s_1^2 s_2^2}{\mu^2}\right]^2(\triangle v^G(P))^2 - 4\frac{\sigma s_1^1 s_2^1}{\mu^1}\frac{\sigma s_1^2 s_2^2}{\mu^2}\left[(\triangle v^G(P))^2-(\triangle v^N(P))^2\right] > 0,$$

both roots of $\Psi(\lambda)=0$ are real numbers. Therefore,

$$\Psi(1) > 0,$$
$$\Psi(-1) > 0, \text{ and}$$
$$-1 < \frac{1}{2}\left(\frac{\sigma s_1^1 s_2^1}{\mu^1}+\frac{\sigma s_1^2 s_2^2}{\mu^2}\right)\triangle v^G(P) < 1.$$

are the necessary and sufficient conditions for the absolute value of the roots to be less than one. □

To simplify the analysis, suppose the ratio of the surplus generated from calling a receiver within the same peer group and the utility from calling other receiver satisfies the following relation.[9]

Assumption 2. *$\gamma \equiv v^N(p)/v^G(p)$ is a constant for all price p.*

Obviously, $0 \leq \gamma \leq 1$. The inverse of the ratio γ represents the strength of the peer effects. When $\gamma = 0$, consumers receive no utility from people outside their own group. On the contrary, when $\gamma = 1$, peer effects vanish.

Under rational expectation, the market shares among group $\mathcal{C}^1$ has to satisfy

$$s_1^1 = \mu^1 \times \frac{\exp[\sigma U_1^1(P; s_1^1, s_1^2)]}{\exp[\sigma U_1^1(P; s_1^1, s_1^2)] + \exp[\sigma U_2^1(P; s_1^1, s_1^2)]}. \tag{12}$$

When $\triangle v^N(P) \neq 0$, this equation can be equivalently expressed as

$$s_1^2 = \frac{-\log\left(\frac{\mu^1}{s_1^1} - 1\right)}{\sigma \triangle v^N(P)} - \frac{1}{\gamma} s_1^1 + \frac{[v^G(p_{22}) - v^G(p_{12})]\mu^1 + [v^N(p_{22}) - v^N(p_{12})]\mu^2 + (t_1 - t_2)}{\triangle v^N(P)}. \tag{13}$$

For any given brief of s_1^2, this equations solve for the number of consumers among group $\mathcal{C}^1$ who subscribe to carrier 1 in a rational expectation equilibrium. Because of network externalities among the group members, there might be multiple solutions.

The right hand side of equation (13) is a function of s_1^1.[10] The numerator of the first term, $-\log(\mu^1/s_1^1 - 1)$, is strictly increasing. It goes to $-\infty$ as $s_1^1 \to 0$ and to $+\infty$ as $s_1^1 \to \mu^1$. The second term is a downward-sloping linear function of s_1^1, and the final term is a fixed value for any given price scheme. In Figure 1, I demonstrate four possible shapes of the graph of this function. When $\triangle v^N(P) < 0$, it is strictly decreasing in s_1^1. When $\triangle v^N(P) = 0$, s_1^1 does not depend on s_1^2. Finally, when $\triangle v^N(P) > 0$, the right hand side of (13) may either strictly increase in s_1^1 or contain a downward-sloping region.[11] It depends on the magnitude of $\sigma \triangle v^N(P)$ relative to γ.

Likewise, for any subscription distribution among group $\mathcal{C}^1$, the market shares in group $\mathcal{C}^2$ satisfies the following condition under rational expectation.

$$s_1^2 = \mu^2 \times \frac{\exp[\sigma U_1^2(P; s_1^1, s_1^2)]}{\exp[\sigma U_1^2(P; s_1^1, s_1^2)] + \exp[\sigma U_2^2(P; s_1^1, s_1^2)]}. \tag{14}$$

Similar to (13), I can express s_1^1 as a function of s_1^2. The rational expectation Nash equilibria in the subscription stage are the market shares (s_1^1, s_1^2) which satisfy conditions (12) and (14) together. Figure 2 demonstrates several possible scenarios of rational expectation equilibria. All intersections on the graphs are equilibria at the subscription stage. When $\triangle v^N(P) \geq 0$, multiple equilibria may exist.

[9]Most results remains true even then γ is not a constant. Only the results in Lemma 4 and Proposition 5 rely on this sufficient condition.

[10]To be more rigorous, it is a function of s_1^1 when $\triangle v^N(P) \neq 0$.

[11]When it exists, the region is a connected subset of the open interval $(0, \mu^1)$.

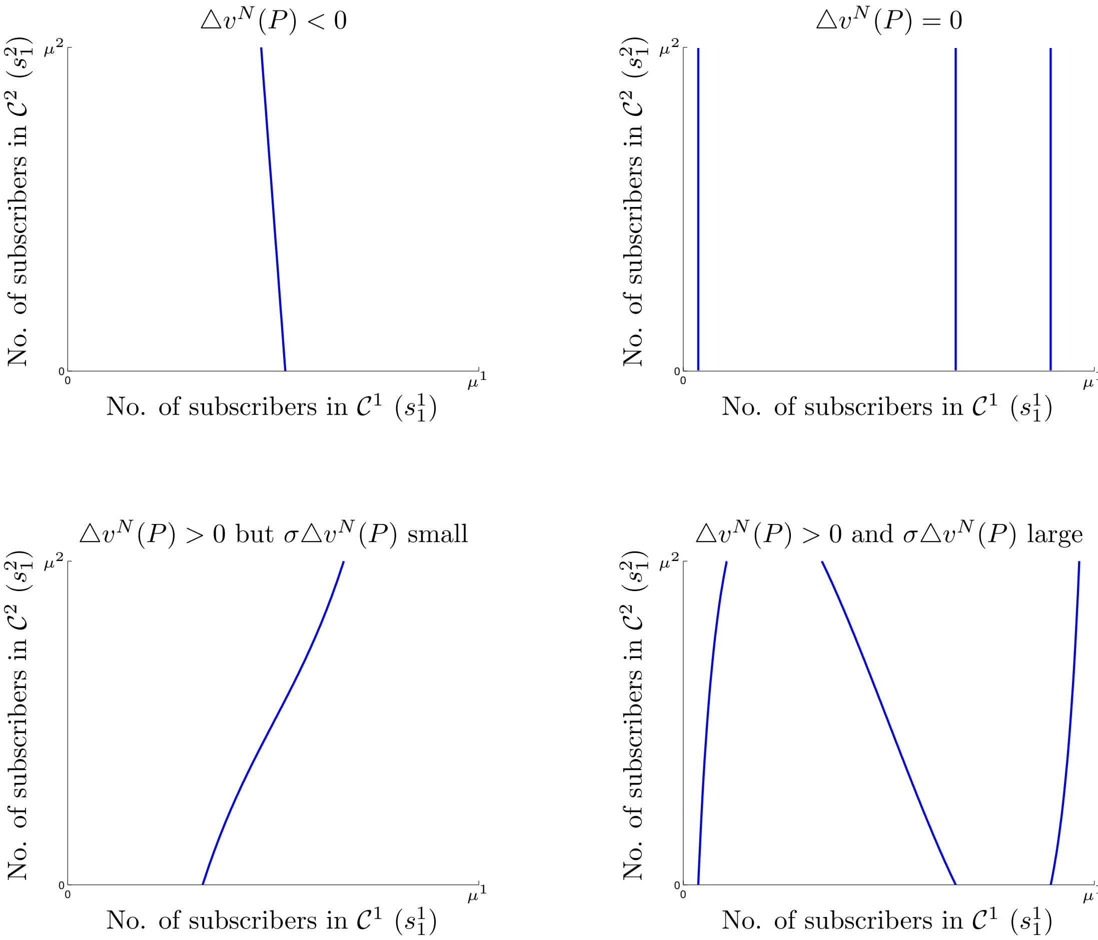

Figure 1: The market share among group 1 for a given share among group 2

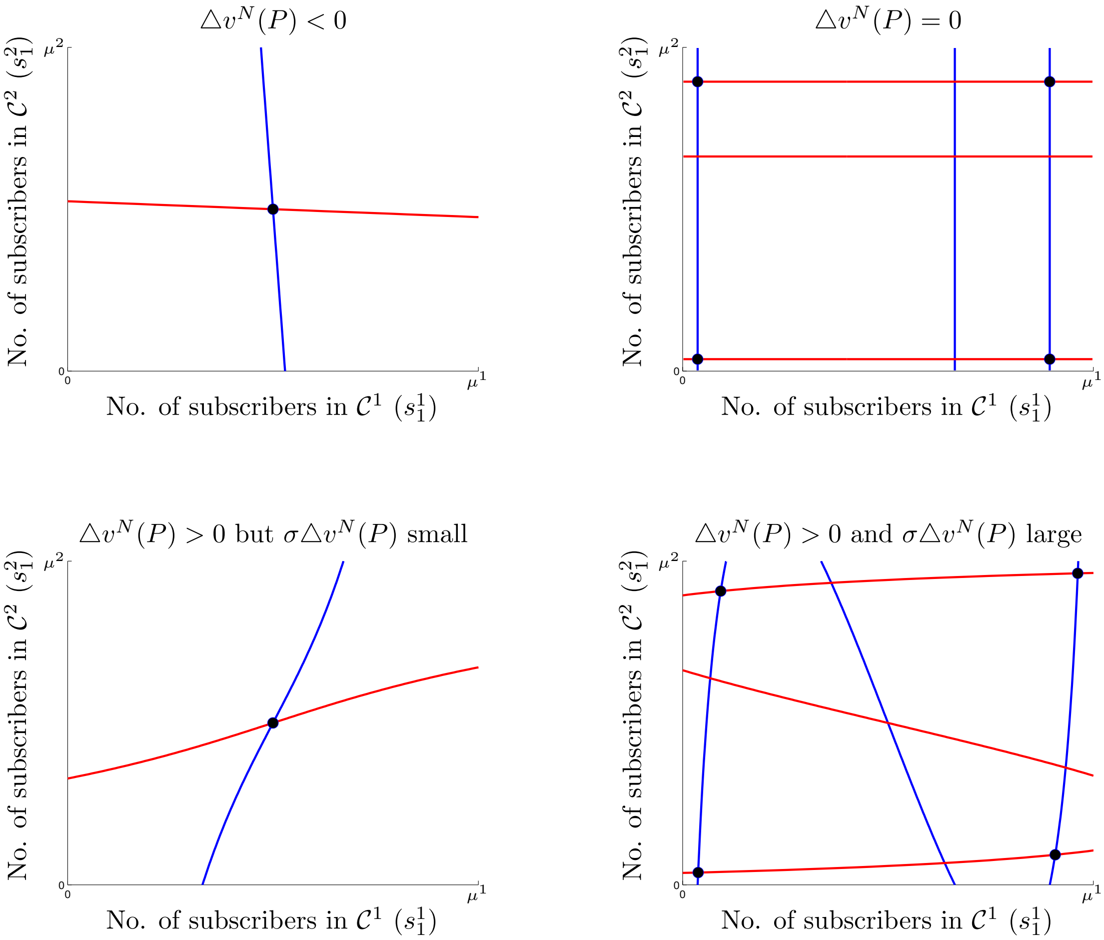

Figure 2: Rational expectation equilibria for subscription

The following lemma provides a necessary condition for a stable equilibrium.

Lemma 3. *When* $\triangle v^G(P) > 0$*, the slope of the graph for equation* (12) *on the* (s_1^1, s_1^2) *space is positive for a stable equilibrium. The corresponding result holds for the graph for equation* (14) *on the* (s_1^2, s_1^1) *space.*

Proof. Without loss of generality, I only prove the result for the slope of the graph for the equation (12) on the (s_1^1, s_1^2) space. The other one holds by symmetry. According to the first condition stated in Lemma 2,

$$\left(\frac{\mu^1}{\sigma s_1^1 s_2^1 \triangle v^N(P)} - \frac{1}{\gamma}\right)\left(\frac{\mu^2}{\sigma s_1^2 s_2^2 \triangle v^N(P)} - \frac{1}{\gamma}\right) > 1.$$

Combine this condition with the third condition in Lemma 2. It is obvious that the slope of equation (13), $\mu^1/[\sigma \triangle v^N(P) s_1^1(\mu^1 - s_1^1)] - 1/\gamma$, is positive. □

The black dots on the graphs in Figure 2 are stable equilibria. It is possible to have up to four stable equilibria as illustrated by the two right graphs. The stable equilibria can be categorized into two types:[12] *pooling* and *separating equilibria.* For a pooling equilibrium, market shares are the same across different consumer groups, $s_k^1 = s_k^2$. For a separating equilibrium, one group of consumers flocks to one carrier while the other group flocks to the other carrier, $s_k^1 \neq s_k^2$. When $\triangle v^N(P) < 0$, however, there is only one equilibrium, which is a stable pooling equilibrium.

Multiple stable equilibria exist when $\sigma \triangle v^N(P)$ is large enough relative to γ. Because $\triangle v^N(P)$ increases in intra-network discounts, and γ decreases in the peer effects of network externalities, I have the following result.

Proposition 4. *When the surplus of intra-network prices is smaller than that of inter-network prices* ($\triangle v^N(P) < 0$)*, there is a unique stable pooling equilibrium at the subscription stage.*

On the other hand, when the surplus of intra-network prices is greater than that of inter-network prices ($\triangle v^N(P) > 0$)*, there are up to four stable equilibria. It is more likely to have multiple stable equilibria at the subscription stage when (a) the substitutability* σ *between carrier is larger, (b) the intra-network discounts are larger, or (c) the peer effects are stronger.*

When there are multiple equilibria, it is inappropriate to restrict our attention to the symmetric equilibrium $s_k^n = \mu^n/2$ for all $k = 1, 2$ and $n = 1, 2$. For example, consider the case shown in the lower right graph of Figure 2. Both carriers offer identical price scheme in this example. Although the symmetric solution $s_k^n = \mu^n/2$ is an equilibrium for the subscription decision, it is unstable.

Lemma 4. *Under Assumption 2, when* $\triangle v^N(P) > 0$*, the graph of the rational expectation equilibrium condition for* s_1^1*, equation* (12)*, moves upward in the space* $(s_1^1, s_1^2) \in [0, \mu^1] \times [0, \mu^2]$ *as any of carrier* 1*'s price components* (t_1, p_{11}, p_{12}) *increases. In particular, this*

[12] This definition is for the case $\mu^1 = \mu^2 = 0.5$. When consumer groups are of different sizes, a pooling equilibrium still refers to an equilibria where a carrier is dominant among both groups, but the market shares would differ across groups.

movement is a parallel shift for change of t_1. When $\triangle v^N(P) < 0$, the graph moves downward when any of the price components increases.

Proof. See Appendix B □

As a result of Lemma 4, I can perform comparative statics at a stable equilibrium for small changes in the price schemes.

Proposition 5. *Under Assumption 2, the number of consumers subscribing to carrier k in either group declines locally at a stable equilibrium of subscription if the carrier marginally increases the subscription fee t_k, the intra-network price p_{kk}, or the inter-network price p_{kl}.*

Proof. This result follows from Lemma 3 and Lemma 4 immediately. □

The properties mentioned in Proposition 5 do not hold for an unstable equilibrium. A carrier may marginally increase its market shares by raising its price at an unstable equilibrium. The analyses in this paper rule out these equilibria.

6.2 Pooling Equilibria

Now, I consider the pricing decisions under the belief of a pooling equilibrium. To eliminate the effects due to different group sizes, suppose the two consumer groups are of the same size, $\mu^1 = \mu^2 = 1/2$. A pooling equilibrium has $s_k^1 = s_k^2$ for $k = 1, 2$. Since $\triangle v^N(P) = \gamma \triangle v^G(P)$, the stability condition in Lemma 2 implies

$$2(s_1^1 s_2^1)(1+\gamma)\left|\triangle v^G(P)\right| < \frac{1}{\sigma}. \tag{15}$$

The first order partial derivatives of carrier k's profit can be written as

$$\begin{aligned}\frac{\partial \Pi_k}{\partial t_k} = 2\frac{\partial s_k^1}{\partial t_k}\Big\{&2s_k^1\pi^G(p_{kk}) + 2s_k^1\pi^N(p_{kk}) + (s_l^1 - s_k^1)[\pi^G(p_{kl}) - \phi^G(p_{kl}) + \phi^G(p_{lk})]\\ &+ (s_l^1 - s_k^1)[\pi^N(p_{kl}) - \phi^N(p_{kl}) + \phi^N(p_{lk})] + t_k - f\Big\} + 2s_k^1,\end{aligned}$$

$$\begin{aligned}\frac{\partial \Pi_k}{\partial p_{kk}} = 2\frac{\partial s_k^1}{\partial p_{kk}}\Big\{&2s_k^1\pi^G(p_{kk}) + 2s_k^1\pi^N(p_{kk}) + (s_l^1 - s_k^1)[\pi^G(p_{kl}) - \phi^G(p_{kl}) + \phi^G(p_{lk})]\\ &+ (s_l^1 - s_k^1)[\pi^N(p_{kl}) - \phi^N(p_{kl}) + \phi^N(p_{lk})] + t_k - f\Big\}\\ &+ 2(s_k^1)^2[q^G(p_{kk}) + (p_{kk} - c)q^{G'}(p_{kk})] + 2(s_k^1)^2[q^N(p_{kk}) + (p_{kk} - c)q^{N'}(p_{kk})],\end{aligned}$$

and

$$\frac{\partial \Pi_k}{\partial p_{kl}} = 2\frac{\partial s_k^1}{\partial p_{kl}}\Big\{2s_k^1\pi^G(p_{kk}) + 2s_k^1\pi^N(p_{kk}) + (s_l^1 - s_k^1)[\pi^G(p_{kl}) - \phi^G(p_{kl}) + \phi^G(p_{lk})]$$
$$+ (s_l^1 - s_k^1)[\pi^N(p_{kl}) - \phi^N(p_{kl}) + \phi^N(p_{lk})] + t_k - f\Big\}$$
$$+ 2s_k^1 s_l^1[q^G(p_{kl}) + (p_{kl} - c_O - a)q^{G'}(p_{kl})] + 2s_k^1 s_l^1[q^N(p_{kl}) + (p_{kl} - c_O - a)q^{N'}(p_{kl})].$$

The partial derivatives of s_k^1 can be computed from (12) and (14) by using the Implicit Function Theorem.

$$\frac{\partial s_k^1}{\partial t_k} = -\frac{\sigma}{\frac{1}{s_k^1} + \frac{1}{s_l^1} - \sigma(1+\gamma)\triangle v^G(P)},$$
$$\frac{\partial s_k^1}{\partial p_{kk}} = -\frac{\sigma[q^G(p_{kk}) + q^N(p_{kk})]s_k^1}{\frac{1}{s_k^1} + \frac{1}{s_l^1} - \sigma(1+\gamma)\triangle v^G(P)},$$
$$\frac{\partial s_k^1}{\partial p_{kl}} = -\frac{\sigma[q^G(p_{kl}) + q^N(p_{kl})]s_l^1}{\frac{1}{s_k^1} + \frac{1}{s_l^1} - \sigma(1+\gamma)\triangle v^G(P)}.$$

According to (15), the denominator $\frac{1}{s_k^1} + \frac{1}{s_l^1} - \sigma(1+\gamma)\triangle v^G(p)$ is positive for a stable equilibrium. Plugging these partial derivatives into the first order conditions, I obtain the marginal-cost pricing strategy for both carriers.

$$\begin{cases} p_{kk} = c \\ p_{kl} = c_O + a, \end{cases} \tag{16}$$

for $k = 1, 2$, $l = 3 - k$, and the subscription fee t_k satisfies

$$-\sigma\frac{(s_l^n - s_k^n)[\phi^G(c_O + a) + \phi^N(c_O + a)] + t_k - f}{\frac{1}{s_1^1} + \frac{1}{s_2^1} - \sigma\Delta^*} + s_k^n = 0, \tag{17}$$

where $\Delta^* \equiv 2[v^G(c) + v^N(c) - v^G(c_O + a) - v^N(c_O + a)]$ is a constant.

The conditions in (16) show that the unit prices are equal to the termination-specific marginal costs. In addition, carrier k chooses the subscription fee t_k that satisfies equation (17). As a result,

$$\begin{cases} (s_2^1 - s_1^1)[\phi^G(c_O + a) + \phi^N(c_O + a)] + t_1 - f = \frac{1}{\sigma}\left(1 + \frac{s_1^1}{s_2^1}\right) - s_1^1\Delta^* \\ (s_1^1 - s_2^1)[\phi^G(c_O + a) + \phi^N(c_O + a)] + t_2 - f = \frac{1}{\sigma}\left(1 + \frac{s_2^1}{s_1^1}\right) - s_2^1\Delta^* \end{cases} \tag{18}$$

are necessary conditions for a Nash equilibrium of (t_1, t_2) at the pricing stage.

Lemma 5. *Consider Nash equilibria in the pricing stage. The symmetric pricing decision $t_1 = t_2$ is the unique solution to equations* (18) *when*

$$\sigma\left\{3\Delta^* - 4(1+\gamma)\phi^G(c_O + a)\right\} \leq 24. \tag{19}$$

On the other hand, there are multiple solutions to the system of equations (18) *if the inequality* (19) *does not hold.*

Proof. See Appendix C □

When the inequality (19) does not hold, there exists subscription fees $t_1 \neq t_2$ which satisfy both carriers' first order conditions and constitute a Nash equilibrium at the pricing stage. The market shares are asymmetric, $s_1^n \neq s_2^n$. Nevertheless, these pricing decisions are profit-maximizing behaviors only when the profit functions are locally concave for both carriers. The following lemma describes its condition.

Lemma 6. *An equilibrium satisfies the second order condition of the carrier k's profit function if and only if*

$$\sigma\left[\Delta^* - (1+\gamma)\phi^G(c_O + a)\right] < \frac{1}{2s_1^1} + \frac{1}{2s_2^1} + \min\left\{\frac{1}{(2s_1^1)^2}, \frac{1}{(2s_2^1)^2}\right\}. \tag{20}$$

Proof. See Appendix D □

Proposition 6. *Suppose consumer groups are of the same size and the market shares are believed to be the same across groups at the subscription stage in the sense that $s_k^1 = s_k^2$ for each carrier k. When carriers can charge different prices based on the termination of a call, it is optimal to set the termination-specific usage price equals to the effective marginal cost of a call, $p_{kk} = c$ and $p_{kl} = c_O + a$.*

When the interconnection markup is nonnegative ($a - c_T \geq 0$), the symmetric pricing strategy, $t_k = f + 2/\sigma - \Delta^*/4$ for $k = 1, 2$, is the unique equilibrium. Each carrier serves half of the market, $s_k^n = 1/4$ for $k = 1, 2$, $n = 1, 2$. The profit is $1/\sigma + [(1+\gamma)\phi^G(c_O + a) - \Delta^*]/8$ for both carriers.

On the other hand, when the interconnection markup is negative ($a - c_T < 0$), it is possible to have asymmetric pricing strategies, $t_1 \neq t_2$, in addition to the symmetric one, $t_1 = t_2$. The market shares differ across carriers, $s_1^n \neq s_2^n$, under asymmetric pricing strategies.

Proof. I have shown in equation (16) that both carriers choose marginal-cost pricing for intra-network and inter-network calls, respectively. It remains to show the decision on the subscription fee t_k for $k = 1, 2$.

Consider the case where the interconnection markup is zero or positive ($a - c_T \geq 0$). Without loss of generality, suppose $t_1 > t_2$. Then, (s_1^1, s_2^1) is an asymmetric equilibrium with $s_1^1 < s_2^1$. Let $x = 1/4 - s_1^1$, which lies in $(0, 1/4)$. From the proof of Lemma 5, I have

$$\begin{aligned}\frac{1}{3}\left\{\frac{1}{x}\log\left(\frac{\frac{1}{4}+x}{\frac{1}{4}-x}\right) + \frac{1}{\frac{1}{16}-x^2}\right\} &= \sigma\left\{\Delta^* - \frac{4}{3}(1+\gamma)\phi^G(c_O+a)\right\} \\ &< \sigma\left\{\Delta^* - (1+\gamma)\phi^G(c_O+a)\right\} \\ &< \frac{1}{2(\frac{1}{4}-x)} + \frac{1}{2(\frac{1}{4}-x)} + \frac{1}{[2(\frac{1}{4}+x)]^2},\end{aligned}$$

The last inequality follows from Lemma 6. However, the last term is always smaller than the first term for any $x \in (0, 1/4)$. This is a contradiction. Therefore, there exists no

Table 1: Example of multiple pooling equilibria

	Equilibrium 1	Equilibrium 2	Equilibrium 3
(t_1, t_2)	(20.348, 17.022)	(18.602, 18.602)	(17.022, 20.348)
(s_1^n, s_2^n)	(0.200, 0.300)	(0.250, 0.250)	(0.300, 0.200)
(Π_1, Π_2)	(1.339, 3.013)	(2.051, 2.051)	(3.013, 1.339)

Notes: $\sigma(1+\gamma)\phi^G(c_O+a) = -50$, $\sigma\Delta^* = -58.408$, and $f = 2$.

asymmetric equilibrium. It is impossible for these two carriers to set different fees $t_1 \neq t_2$ in equilibrium.

For the symmetric equilibrium with $t_1 = t_2$, it is clear that the market share is $s_k^n = 1/4$ for $k = 1, 2$ and $n = 1, 2$. The subscription fee is $t_k = f + 2/\sigma - \Delta^*/4$. Therefore, the profit is $1/\sigma + [(1+\gamma)\phi^G(c_O+a) - \Delta^*]/8$ for each carrier.

On the other hand, when the interconnection markup is negative ($a - c_T < 0$), the conditions in Lemma 5 and Lemma 6 can be satisfied simultaneously. Carriers can choose different subscription fees, $t_1 \neq t_2$, in a Nash equilibrium. □

The following example demonstrates the existence of an asymmetric Nash equilibrium.

Example 1. *Suppose* $\sigma(1+\gamma)\phi^G(c_O+a) = -50$ *and* $\sigma\Delta^* = -58.408$. *The fixed cost is* $f = 2$. *Then there are three possible equilibria for the choice of subscription fees. Their subscription fee, market share, and profit are summarized in Table 1.*

It is easy to find a demand function which satisfies the conditions, $\sigma(1+\gamma)\phi^G(c_O+a) = -50$ *and* $\sigma\Delta^* = -58.408$. *For instance,* $c = 1.430$, $c_O + a = 0.430$, $q^G(p) = 29.204\,p^{-0.8}$, $\gamma = 0$, *and* $\sigma = 1$.

The results in Proposition 6 do not rely on the peer effects. In fact, when $\gamma = 1$, all consumers are homogeneous. In the Hotelling model of Laffont et al. (1998b), the usage prices chosen by firms are also termination-specific marginal costs. The two firms essentially compete in one dimension, namely the subscription fees. Laffont et al. only focus on the symmetric choice of the subscription fees $t_1 = t_2$. Here, I also consider conditions for multiple Nash equilibria in the pricing game. As Example 1 shows, the subscription fees may differ across firms even though their cost structure is assumed to be identical. This would result in asymmetric stable market shares in the subscription stage. Hence, restricting the attention to symmetric solutions is inappropriate.

Proposition 7. *Suppose consumer groups are of the same size. Consider a pooling equilibrium. The profit is decreasing in the interconnection fee whenever the interconnection markup is nonnegative* ($a - c_T \geq 0$)*.*

Proof. From Proposition 6, the profit can be expressed as

$$\Pi_k = \frac{1}{\sigma} + \frac{1+\gamma}{8}\left[(a - c_T)q^G(a + c_O) + 2v^G(a + c_O) - 2v^G(c)\right].$$

Its derivative with respect to a is

$$\frac{\partial \Pi_k}{\partial a} = \frac{1+\gamma}{8}\left[(a-c_T)q^{G'}(a+c_O) - q^G(a+c_O)\right],$$

which is always negative whenever $a \geq c_T$. □

As a result of this proposition, when carriers can negotiate the interconnection charge a, it is always optimal to choose $a < c_T$. By doing so, they will offer discounts for inter-network calls in the retail market. Therefore, competition for subscribers is softened. This conclusion is identical to Gans and King (2001).

6.3 Separating Equilibria

The market shares of a carrier differ across groups in a separating equilibrium, $s_k^1 \neq s_k^2$. A separating equilibrium can exist only when there are multiple equilibria in the subscription stage. Consequently, I only need to consider $a - c_T \geq 0$. Suppose the peer groups are of the same size $\mu^1 = \mu^2 = 1/2$ and consider a separating equilibrium, $s_1^1 = s_2^2$ and $s_1^2 = s_2^1$. Without loss generality, assume $s_1^1 > s_2^1$.

Contrary to a pooling equilibrium, the first order conditions do not hold for marginal cost pricing ($p_{kk} = c$ and $p_{kl} = c_O + a$). Consider an equilibrium with $p_{11} = p_{22}$ and $p_{12} = p_{21}$. The first order condition of carrier k's profit with respect to the subscription fee t_k can be simplified as

$$-\frac{\pi^G(p_{kk}) + \pi^N(p_{kk}) + 2(t_k - f)}{\frac{1}{2\sigma s_1^1 s_2^1} - \Delta(P)} + \frac{1}{2} = 0.$$

The partial derivatives with respect to the unit prices are

$$\frac{\partial \Pi_k}{\partial p_{kk}} = \frac{(s_k^1 - s_l^1)^2\left[q^G(p_{kk}) - q^N(p_{kk})\right]\{[q^G(p_{kl}) - q^N(p_{kl})](p_{kl}-c) - \left[q^G(p_{kk}) - q^N(p_{kk})\right](p_{kk}-c)\}}{\frac{1}{2\sigma s_1^1 s_2^1} - \triangle v^G + \triangle v^N}$$
$$+\left[(s_k^1)^2 + (s_k^2)^2 - \frac{1}{8}\right][q^G(p_{kk}) - q^N(p_{kk})] + (p_{kk}-c)\{[(s_k^1)^2 + (s_k^2)^2]g^{G'}(p_{kk}) + 2s_k^1 s_k^2 q^{N'}(p_{kk})\}$$

and

$$\frac{\partial \Pi_k}{\partial p_{kl}} = \frac{(s_k^1 - s_l^1)^2[q^G(p_{kl}) - q^N(p_{kl})]\{[q^G(p_{kk}) - q^N(p_{kk})](p_{kk}-c) - [q^G(p_{kl}) - q^N(p_{kl})](p_{kl}-c)\}}{\frac{1}{2\sigma s_1^1 s_2^1} - \triangle v^G + \triangle v^N}$$
$$-\left[(s_k^1)^2 + (s_l^1)^2 - \frac{1}{8}\right][q^G(p_{kl}) - q^N(p_{kl})] + (p_{kl} - c_O - a)\{2s_k^1 s_k^2 q^{G'}(p_{kl}) + [(s_k^1)^2 + (s_k^2)^2]q^{N'}(p_{kl})\}.$$

Since $q^G(p_{kk}) > q^N(p_{kk})$, $q^G(p_{kl}) > q^N(p_{kl}) > 0$, and $(s_k^1)^2 + (s_l^1)^2 > 1/8$, the first order condition $\partial \Pi_k / \partial p_{kk} = 0$ implies $p_{kk} \leq c$ and $p_{kl} < c$ cannot jointly hold. When the two first order conditions are combined, I obtain the following necessary condition.

$$\frac{(p_{kk}-c)\{[(s_k^1)^2+(s_k^2)^2]q^{G'}(p_{kk})+2s_k^1s_k^2q^{N'}(p_{kk})\}}{q^G(p_{kk})-q^N(p_{kk})} + \frac{(p_{kl}-c_O-a)\{2s_k^1s_k^2q^{G'}(p_{kl})+[(s_k^1)^2+(s_k^2)^2]q^{N'}(p_{kl})\}}{q^G(p_{kl})-q^N(p_{kl})}=0. \quad (21)$$

It is impossible to have

$$\begin{cases} p_{kk}<c \\ p_{lk}<c_O+a \end{cases} \text{or} \quad \begin{cases} p_{kk}>c \\ p_{lk}>c_O+a \end{cases}.$$

Moreover, equation (21) implies that $p_{kk}=c$ if and only if $p_{kl}=c_O+a$. The first order conditions do not hold for this pricing strategy. Consequently, the usage prices must satisfy

$$\begin{cases} p_{kk}>c \\ p_{lk}<c_O+a \end{cases}.$$

Proposition 8. *Suppose the consumer groups are of the same size and the market shares are believed to be a separating equilibrium with $s_1^1=s_2^2$ and $s_2^1=s_1^2$. The optimal usage prices are different from the marginal costs of a call. While $p_{kk}>c$, $p_{kl}<c_O+a$.*

The following example illustrates the existence of a separating equilibrium.

Example 2. *Suppose $v^G(p)=100\exp(-0.8p)$, $v^N(p)=0$, $c=3$, $a-c_O=8$, $\sigma=1$, and $f=0$. When carriers expected consumers to coordinate to a separating stable equilibrium in the subscription stage, both of them choose unit prices $p_{kk}=3.4553$, $p_{kl}=7.7692$, and the subscription fee $t=3.8966$. The market shares are $s_1^1=s_2^2=0.4669$ and $s_2^1=s_1^2=0.0331$. This pricing strategy satisfy the first order conditions, $\partial\Pi_k/\partial t_k=\partial\Pi_k/\partial p_{kk}=\partial\Pi_k/\partial p_{kl}=0$ and its Hessian is*

$$\begin{bmatrix} -0.3325 & -1.5530 & 0.0255 \\ -0.1530 & -4.6908 & 0.0226 \\ 0.0255 & 0.0226 & -0.0032 \end{bmatrix},$$

which is negative definite. Their profits are $\Pi_1=\Pi_2=2.4749$. Furthermore, the profit increases in the exogenously determined access charge a. $\partial\Pi_k/\partial a=0.270$ for $k=1,2$.

Consequently, when peer effects are incorporated into the network effect model. The rule of marginal-cost pricing ceases to hold in a separating equilibrium. Even when carriers are identical in their cost structure and offer the same price scheme, they may be dominant among different consumer groups.

Contrary to a pooling equilibrium, the profit of carriers may increase in the interconnection charge. Therefore, when this charge is negotiated by carriers, they have an incentive to choose a higher interconnection fee. As Example 2 demonstrates, the intra-network price could be lower than inter-network price at a stable equilibrium.

7 Conclusion

I analyze network externalities with peer effects. When termination-based price discrimination is not allowed, the peer effects would simply affect the taste for the service. A consumer in a larger peer group has stronger taste because there are more friends in her group to make phone calls. If all groups are of the same size, consumers are essentially homogeneous. The results in my model is the same as Laffont et al. (1998a). If the group size varies across consumers, it is equivalent to consumers with vertically heterogeneous taste as in Dessein (2004).

When carriers can discriminate phone calls based on the terminating network, network externalities across carriers have substantial effects on the market. There might be multiple stable rational expectation equilibria if the interconnection charge is greater than the marginal cost of terminating a call. The realized one would depend on consumers' beliefs. For a pooling equilibrium, the market shares are identical across peer groups. Carriers use marginal cost pricing and compete in the subscription fee. They may choose different subscription fees in a stable equilibrium. On the other hand, for a separating equilibrium, carriers specialize to serve different consumer groups even though they are identical ex ante. The profit-maximizing pricing strategy is different from marginal cost pricing. The intra-network price is higher than its marginal cost, but the inter-network price is lower than the perceived marginal cost. While carriers prefer a lower interconnection fee in anticipation of a pooling equilibrium, they could favor a higher interconnection fee in anticipation of a separating equilibrium. There is no clear relation between the profit and this fee. My model provides a reasonable explanation for intra-network discounts when carriers can negotiate the interconnection fee and compete in two-part tariffs to maximizing their profits.

Carriers are assumed to be identical ex ante in this paper. An extension of the model is to consider an entrant competing with an incumbent. The incumbent has some locked-in customers who have a long-term contract with the carrier and cannot switch to the entrant. Intra-network discounts give the incumbent an advantage through the installation base.

A Proof of Proposition 1

Uniqueness and Existence of a Symmetric Equilibrium. I need to show that there is no Nash equilibrium other than the symmetric one, $(s_1^n, s_2^n) = (1/4, 1/4)$ for $n = 1, 2$. Denote $s_k \equiv s_k^1 + s_k^2$, $v(p) \equiv v^G(p)/2 + v^N(p)/2$, $q(p) \equiv q^G(p)/2 + q^N(p)/2$, $\pi(p) \equiv \pi^G(p)/2 + \pi^N(p)/2$, $\phi(p) \equiv \phi^G(p)/2 + \phi^N(p)/2$ and $U_k \equiv U_k^1 = U_k^2$.

When $a \leq c_T$, it is impossible to have multiple equilibria regardless the magnitude of $(a - c_T)$

and σ. The difference of equation (8) evaluated at $k = 1$ and at $k = 2$ is

$$\begin{aligned}
t_1 - t_2 &= \frac{1}{\sigma s_2} - \frac{1}{\sigma s_1} + [\pi(p_2) - \pi(p_1)] \\
&= \frac{1}{\sigma s_2} - \frac{1}{\sigma s_1} + \left\{ \int_{p_1}^{p_2} [q(p) + (p-c)q'(p)]\, dp \right\} \\
&= \frac{1}{\sigma s_2} - \frac{1}{\sigma s_1} + \int_{p_1}^{p_2} (p-c)q'(p)\, dp + [v(p_1) - v(p_2)]\,.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\log(s_1) - \log(s_2) &= \sigma U_1 - \sigma U_2 \\
&= \sigma\left[v(p_1) - t_1\right] - \sigma\left[v(p_2) - t_2\right] \\
&= \frac{1}{s_1} - \frac{1}{s_2} - \sigma \int_{p_1}^{p_2} \left[(p-c)q'(p)\right]\, dp.
\end{aligned}$$

Consider the case $\delta \equiv a - c_T \leq 0$. Without loss of generality, assume $s_1 < 1/2 < s_2$. The condition (7) implies $p_2 \geq p_1$. As a result, the right hand size of the above equation is strictly positive. Nonetheless, the left hand side of the equation is always negative for $s_1 < 1/2 < s_2$. This is a contradiction.

When $a > c_T$, the uniqueness holds only when $(a - c_T)$ or σ is small. The choice of the subscription fee t_k can be equivalently viewed as the choice of the average utility level U_k^n. Then $t_k = v(p_k) - {}_k^n$. Given the rival's pricing decision, the profit can be express as a function of U_k^n and p_k.

$$\Pi_k(p_k, U_k) = s_k(U_k, U_l)\left[\pi(p_k) + v(p_k) - U_k - f + s_l(U_k, U_l)\left[\phi(p_l) - \phi(p_k)\right]\right].$$

The first order condition with respect to p_k implies marginal cost pricing, $p_k = c + s_l(a - c_T)$. Moreover, the first order condition with respect to U_k is

$$\frac{\partial \Pi_k}{\partial U_k} = \sigma s_k s_l \left[\pi(p_k) + v(p_k) - U_k - f + (s_l - s_k)[\phi(p_l) - \phi(p_k)]\right] - s_k = 0.$$

Since both carriers would choose marginal cost pricing, the above condition implicitly defines U_k as a function of U_l. The slope of of this reaction function $U_k(U_l)$ is

$$\frac{dU_k}{dU_l} = \frac{2\sigma s_k s_l[\phi(p_l) - \phi(p_k)] + \frac{s_k}{s_l} + (a - c_T)\sigma\left[(p_k - c)q'(p_k) - 2(s_l - s_k)(a - c_T)q(p_l)\right]}{1 + 2\sigma s_k s_l[\phi(p_l) - \phi(p_k)] + \frac{s_k}{s_l} + (a - c_T)\sigma\left[(p_k - c)q'(p_k) - 2(s_l - s_k)(a - c_T)q(p_l)\right]}$$

When the absolute values of the slopes of $U_1(U_2)$ and $U_2(U_1)$ are both less than 1, the intersection of the two reaction functions is a stable equilibrium. Furthermore, if the absolute slopes are both globally less than one, there is only one intersection. Therefore, a sufficient condition for the uniqueness is

$$2\sigma s_k s_l[\phi(p_l) - \phi(p_k)] + \frac{s_k}{s_l} + (a - c_T)\sigma\left[(p_k - c)q'(p_k) - 2(s_l - s_k)(a - c_T)q(p_l)\right] > -\frac{1}{2}.$$

for any equilibrium distribution (s_1, s_2). Equivalently,

$$\frac{s_k}{s_l} + 2\sigma(a - c_T)[q(p_l) - q(p_k)] + \sigma(a - c_T)^2 s_l q'(p_k) - 2\sigma(a - c_T)^2(s_l - s_k)q(p_l) > -\frac{1}{2}.$$

Consequently, the condition hold when $(a - c_T)$ or σ is small.

For the symmetric equilibrium, I have $s_1 = s_2$ and $p_1 = p_2$. Therefore, the symmetric equilibrium is stable whenever

$$1 + \frac{1}{2}\sigma(a - c_T)^2 q'\left(c + \frac{a - c_T}{2}\right) > -\frac{1}{2}.$$

I now prove the existence of a symmetric equilibrium by showing that the profit function is concave. Given the rival's price scheme, the profit can be expressed as a function of U_k^n.

$$\hat{\Pi}_k = s_k(U_k, U_l)\left[v(c + s_l(U_k, U_l)\delta) - U_k - f + s_l\delta q(p_l)\right],$$

where $\delta \equiv a - c_T$. Therefore,

$$\frac{\partial \hat{\Pi}_k}{\partial U_k} = \sigma s_k s_l[v(c + s_l\delta) - U_k - f + s_l\delta q(p_l)] + s_k[\sigma s_k s_l \delta q(c + s_l\delta) - 1 - \sigma s_k s_l \delta q(p_l)]$$

and

$$\begin{aligned}\frac{\partial^2 \hat{\Pi}_k}{\partial U_k^2} = \sigma s_k s_l(s_l - s_k)&[v(c + s_l\delta) - U_k - f + s_l\delta q(p_l)] \\ &+ 2\sigma s_k s_l[q(c + s_l\delta)\,\sigma\,\delta\, s_l s_k - 1 - \sigma\, s_k s_l\,(a - c_T)\,q(p_l)] \\ &+ s_k[s_k s_l(s_l - s_k)\delta q(c + s_l\delta) - (\sigma s_k s_l)^2\delta^2 q'(c + s_l\delta) - \sigma^2 s_k s_l(s_l - s_k)\delta q(p_l)],\end{aligned}$$

When $s_k = s_l = 1/2$, $p_1 = p_2$.

$$\frac{\partial^2 \hat{\Pi}_k}{\partial U_k^2} = -\frac{\sigma}{2} - \frac{\sigma^2}{32}(a - c_T)^2 q'\left(c + \frac{a - c_T}{2}\right).$$

Therefore, when $|a - c_T|$ or σ is small enough, there exist a symmetric equilibrium $s_1 = s_2 = 1/2$. Moreover, this is the unique equilibrium. □

B Proof of Lemma 4

Proof. Without loss of generality, consider only the price change of carrier 1.

The result for t_1 can be easily established from (13) and this holds even without Assumption 2. For any given s_1^1, the first two terms on the right-hand side are independent of t_1, and the third term is an affine function of t_1. The coefficient on t_1 is positive if and only if $\triangle v^N(P) > 0$. Hence, increasing t_1 shifts the graph upward if and only if $\triangle v^N(P) > 0$.

Under Assumption 2, the right-hand side of equation (13) can be expressed as the following two parts.

$$s_1^2 = \left[\frac{-\log\left(\frac{\mu^1}{s_1^1}-1\right)}{\triangle v^N(P)}\right] + \left[-\frac{s_1^1}{\gamma} + \frac{[v^G(p_{22})-v^G(p_{12})]\mu^1 + [v^N(p_{22})-v^N(p_{12})]\mu^2 + t_1 - t_2}{\triangle v^N(P)}\right]. \tag{22}$$

The first part equals 0 at $s_1^1 = \mu^1/2$ and goes exponentially to $-\infty$ and $+\infty$ as μ_1^1 approaches 0 and s^1, respectively. The second part is an affine function of s_1^1 with a negative slope.

As p_{11} increases, the denominators of the first and the third terms in equation (22) decrease. When $\triangle v^N(P) > 0$, the first term magnifies proportionally away from the s_1^1-axis. The intercept of the affine function in the second part also expands by the same ratio, but the slope is unchanged. The change on equation (22) is the sum of the two parts. Both changes are positive for $s_1^1 > \mu^1/2$. As for $s_1^1 < \mu^1/2$, whenever equation (22) takes values on $[0, \mu^2]$, the positive change is greater than the negative change in the absolute values. Therefore, the graph of equation (22) moves up inside the box $[0, \mu^1] \times [0, \mu^2]$. On the other hand, when $\triangle v^N(P) < 0$, the first part and the intercept shrink by the same ratio. It can be similarly shown that the total change is negative whenever equation (22) takes values on $s_1^2 > 0$.

Next, claim that the change in the right-hand side of (22) is a decreasing function of s_1^1 for a given increase of p_{12}. Because the denominator of the first term in equation (22) increases, its graph shrinks (expands) proportionally with respect to the s_1^1-axis when $\triangle v^N(P) > (<)\ 0$. In either case, the change decreases in s_1^1. The change in p_{12} causes a parallel shift of the affine function. Therefore, the total change of equation (22) is a decreasing function of s_1^1 regardless the sign of $\triangle v^N(P)$.

Claim that the graph always moves upward inside the box $[0, \mu^1] \times [0, \mu^2]$ as p_{12} increases if and only if $\triangle v^N(P) > 0$. Let me calculate the intersection of the graph with the line $s_1^2 = \mu^2$. When $s_1^2 = \mu^2$, equation (12) can be written as

$$\log\left(\frac{\mu^1}{s_1^1} - 1\right) = -\triangle v^G(P) s_1^1 + [v^G(p_{22}) - v^G(p_{12})]\mu^1 + [v^N(p_{21}) - v^N(p_{11})]\mu^2 + t_1 - t_2. \tag{23}$$

Denote the affine function of s_1^1 on the right-hand side by $F(s_1^1)$. Observe that $F(\mu^1) = [v^G(p_{21}) - v^G(p_{11})]\mu^1 + [v^N(p_{21}) - v^N(p_{11})]\mu^2 + t_1 - t_2$, which is independent of p_{12}. Increasing p_{12} rotates the graph of F clockwise along the point $(\mu^1, F(\mu^1))$ (the diamond point on Figure 3). As the graph illustrates, F moves from the red solid line to the green dashed line. Hence, the first and the last intersections (solid circles on the graph) move to the left (open circles on the graph) while the second intersection moves to the right (from the solid square to the open square). (When there is only one intersection, the same argument shows the only intersection must move left.) Therefore, the first and the last intersections of the equation (22) with the line $s_1^2 = \mu^2$ move left on Figure 1. This implies that the graph of equation (22) (the curve on Figure 1) must move upward (downward) at these intersection points. This is sufficient to prove the entire graph moves upward (downward) inside the region $[0, \mu^1] \times [0, \mu^2]$ for $\triangle v^N(P) > (<)\ 0$ because I have shown the total movement is a decreasing function of s_1^1. □

C Proof of Lemma 5

Proof. When $t_1 = t_2 = f + \frac{2}{\sigma} + \frac{\Delta^*}{4}$, I obtain a symmetric equilibrium $s_1^1 = s_1^2 = 1/4$. Obviously, this solution satisfies condition (18). To find out any other equilibrium, let $s_1^1 = s_1^2 = 1/4 - x$ and $s_2^1 = s_2^2 = 1/4 + x$. Without loss of generality, consider $0 < x < 1/4$. Because of the logit

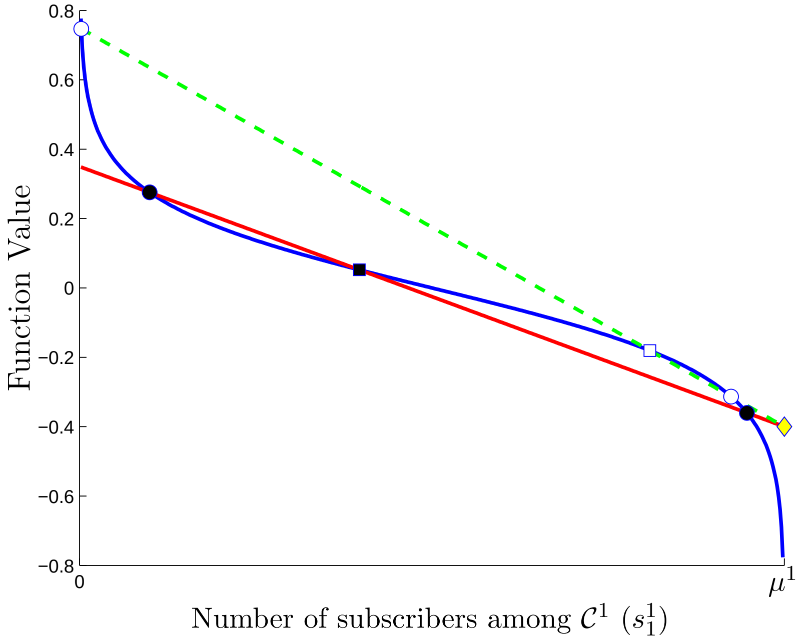

Figure 3: Solutions for equation (23)

assumption on the idiosyncratic preferences ε_{ik}, I have

$$\log\left(\frac{\frac{1}{4}+x}{\frac{1}{4}-x}\right) = x\sigma\Delta^* + \sigma(t_1 - t_2).$$

Together with the conditions (18), x has to satisfy

$$\log\left(\frac{\frac{1}{4}+x}{\frac{1}{4}-x}\right) + \frac{x}{\frac{1}{16}-x^2} - x\sigma\left\{3\Delta^* - 4(a-c_T)[q^G(c_O+a) + q^N(c_O+a)]\right\} = 0.$$

Since $x > 0$,

$$\frac{1}{x}\log\left(\frac{\frac{1}{4}+x}{\frac{1}{4}-x}\right) + \frac{1}{\frac{1}{16}-x^2} = \sigma\left\{3\Delta^* - 4(a-c_T)\left[q^G(c_O+a) + q^N(c_O+a)\right]\right\}.$$

The left-hand side equals 24 at $x = 0$ and increases to infinity as $x \to +\infty$. The right-hand side does not depend on x. There is a solution $x > 0$ if and only if the right hand side is greater than 24. □

D Proof of Lemma 6

Proof. Under the termination-specific marginal cost pricing, $p_{kk} = c$ and $p_{kl} = c_O + a$ for both carriers. The two carriers essentially compete with each other in the subscription fees t_k. The profit of carrier k can be written as

$$\hat{\pi}_k(t_k, t_l) = 2s_k^1(t_k, t_l)s_l^1(t_k, t_l)[\phi^G(c_O + a) + \phi^N(c_O + a)] + 2s_k^1(t_k, t_l)[t_k - f].$$

Consequently,

$$\frac{\partial \hat{\pi}_k}{\partial t_k} = -\frac{2\sigma}{\frac{1}{s_k^1} + \frac{1}{s_l^1} - \sigma\Delta^*}\left[(s_l^1 - s_k^1)(1+\gamma)\phi^G(c_O + a) + t_k - f\right] + 2s_k^1.$$

The second derivative is

$$\frac{\partial^2 \hat{\pi}_k}{\partial t_k^2} = -\frac{4\sigma}{\left(\frac{1}{s_k^1} + \frac{1}{s_l^1} - \sigma\Delta^*\right)^2}\left\{\sigma(1+\gamma)\phi^G(c_O + a) - \sigma\Delta^* + \frac{1}{2s_k^1} + \frac{1}{2s_l^1} + \frac{1}{(2s_l^1)^2}\right\}.$$

If (s_k^1, s_l^1) is the market shares in equilibrium, the second order condition requires $\partial^2\hat{\pi}_k/\partial t_k^2 < 0$ and $\partial^2\hat{\pi}_l/\partial t_l^2 < 0$. This condition needs to hold for both carriers $k = 1, 2$. Therefore,

$$\sigma\left[\Delta^* - (1+\gamma)\phi^G(c_O + a)\right] < \frac{1}{2s_1^1} + \frac{1}{2s_2^1} + \min\left\{\frac{1}{(2s_1^1)^2}, \frac{1}{(2s_2^1)^2}\right\}.$$

□

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