1. True or False: Indicate whether the statement is true or false clearly with a brief reason.

(a) If there are increasing returns to scale, then average costs are a decreasing function of output.

(b) If a competitive firm uses two inputs and has the production function 
\[ f(x_1, x_2) = x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}} , \] 
then its marginal cost is higher than its average cost.

(c) If a firm is operating with constant marginal cost, it will also have constant average cost; and if marginal cost is increasing, average cost will be increasing also.

(d) If a firm is operating at minimum short-run average cost, it is also operating at a point on its long-run average cost curve.

(e) Long-run average cost can never exceed short-run average cost; and long run marginal cost can never exceed short-run marginal cost.

2. Hildegard, an intelligent and charming Holstein cow, grazes in a very large, mostly barren pasture with a few patches of lush grass. When she finds a new grassy area, the amount of grass she gets from it is equal to the square root of the number of hours, \( h \), that she spends grazing there. Finding a new patch of grass on which to graze takes her 1 hour. Since Hildegard does not have pockets, the currency in which her costs are measured is time.

(a) What is the total cost to Hildegard of finding a new plot of grass and getting \( y \) units of grass from it? Draw her marginal cost and average cost of doing so.

(b) If Hildegard spends \( H > 2 \) hours finding and grazing on two plots of grass, what is the maximal amount of grass she can get?
(c) What is the total cost to Hildegard of finding two new plots of grass and getting $y$ units of grass from it? Draw her marginal cost and average cost of doing so.

(d) What is the total cost to Hildegard of finding three new plots of grass and getting $y$ units of grass from it? Draw her marginal cost and average cost of doing so.

(e) Inductively, what is the total cost to Hildegard of finding $n$ (where $n$ is a positive integer) new plots of grass and getting $y$ units of grass from it? Find her marginal cost and average cost of doing so.

(f) Combining the results above, draw Hildegard’s marginal cost and average cost of getting $y$ units of grass. (Hint: Remember that the long run average cost curve is the lower envelope of the short run cost curve.)

3. The desert town of Dry Gulch buys its water solely from LowTech Inc. In other words, LowTech is the monopolist supplying water to Dry Gulch. LowTech hires residents to walk to the nearest oasis and carry back buckets of water. Thus, the inputs to the production of water are workers and buckets. The walk to the oasis and back takes one full day. Every worker can carry either 1 or 2 buckets of water but no more. Assume that both workers and buckets are measured in discrete amount. Buckets rent for $1 a day and workers earn $2 per day. In the following, define the marginal cost of the first unit to be the total cost of producing 1 unit minus the total cost of producing 0 unit, the marginal cost of the 2nd unit to be the total cost of producing 2 units minus the total cost of producing 1 unit and so on.

(a) Draw some of the isoquants of LowTech.

(b) Does the production of LowTech satisfy constant returns to scale? If so, briefly prove it; otherwise, name a counterexample.
(c) In the short run, suppose LowTech is constrained to rent 4 buckets no matter what. Draw LowTech’s total cost curve, marginal cost curve and average cost curve when the bucket number is fixed at 4.

(d) In the long run, suppose LowTech can choose to rent 0, 2, 4 or 6 buckets. The number of buckets it rents cannot be other than 0, 2, 4 or 6 due to regulations. Draw LowTech’s long-run total cost curve, long-run marginal cost curve and long-run average cost curve.

(e) At what level of output will 4 buckets be rented in the long run? Show by graph that the long-run cost curve is the lower envelope of the short-run cost curves.

4. Lady P makes silk purses out of sows’ ears. She is the only person in the world who knows how to do so. It takes one sow’s ear and one hour of her labor to make a silk purse. She can buy as many sows’ ears as she likes for $1 each. Lady P has no other source of income than her labor. Her utility function is \[ U(c, r) = c^{\frac{1}{3}} r^{\frac{2}{3}}, \] where \( c \) is the amount of money per day she has to spend on consumption goods and \( r \) is the amount of leisure that she has. Lady P has 24 hours a day that she can devote either to leisure or to working.

(a) Lady P can either make silk purses or she can earn $5 an hour as a seamstress in a sweatshop. If she worked in the sweatshop, how many hours would she work?

(b) If the price of silk purse is \( p \), how much money will Lady P earn per purse after she pays for the sows’ ears that she uses?

(c) If she can earn $5 an hour as a seamstress, what is the lowest price at which she will make any silk purses?

(d) What is the supply function for silk purses? (Hint: The price of silk purses determines the "wage rate" that Lady P can earn by making silk purses. This determines the number of hours she will choose to work and hence the supply of silk purses.)
5. Suppose there is a fall in the demand for shoes, which are provided by a competitive industry in which every firm has the same technology and the input prices are given.

(a) Does the price of shoes change by more in the short run or in the long run?

(b) Does the industry-wide quantity change by more in the short run or in the long run?

(c) Does the quantity provided by each individual shoemaker (which still stays in the industry after the demand falls) change by more in the short run or in the long run?

(d) Does the profit of each shoemaker (which still stays in the industry after the demand falls) change by more in the short run or in the long run?
Suggested Solution Key

1(a) True. Since if we double inputs, outputs are more than doubled. Hence to produce exactly two times as before, we do not need to double inputs.

1(b) True. Note that \( f(tx_1, tx_2) = (tx_1)^{\frac{1}{t}} + (tx_2)^{\frac{1}{t}} = t^\frac{1}{t}(x_1^{\frac{1}{t}} + x_2^{\frac{1}{t}}) < t(x_1^{\frac{1}{t}} + x_2^{\frac{1}{t}}) \) if \( t > 1 \). Hence the production function exhibits decreasing returns to scale. Thus average cost is increasing as output increases. Since AC is increasing, MC must be larger than AC.

1(c) False. For instance \( C(y) = 1 + y \). Then \( MC = \frac{1}{y} + 1 \). So MC is constant, but AC is not. For the second part, suppose \( C(y) = 1 + y^2 \). Then \( MC = 2y \). \( AC = \frac{1}{y} + y \). So even though MC is increasing, AC is U-shaped.

1(d) False. Look at Figure 21.7. The min SAC point is not on the LAC.

2(a) \( C_1(y) = 1 + y^2 \). \( MC_1(y) = 2y \). \( AC_1(y) = \frac{1}{y} + y \).

2(b) Hildegard should solve \( \max_s,t \sqrt{s} + \sqrt{t} \) s.t. \( s + t + 2 = H \). FOC gives \( s = t = \frac{H-2}{2} \). SOC is OK. Hence the grass she can get is \( 2\sqrt{\frac{H-2}{2}} \).

2(c) Hildegard should solve \( \min_s,t \) \( s + t + 2 \) s.t. \( \sqrt{s} + \sqrt{t} = y \). In fact, 2(c) is the dual problem to 2(b). At optimum, \( s = t = \frac{y^2}{4} \). Hence \( C_2(y) = 2 + \frac{y^2}{2} \). \( MC_2(y) = y \). \( AC_2(y) = \frac{y}{2} + \frac{y}{2} \).

2(d) Hildegard should solve \( \min_s,t,u \) \( s + t + u + 3 \) s.t. \( \sqrt{s} + \sqrt{t} + \sqrt{u} = y \). At optimum, \( s = t = u = \frac{y^2}{4} \). Hence \( C_3(y) = 3 + \frac{y^2}{2} \). \( MC_3(y) = 2y \). \( AC_3(y) = \frac{3}{2} + \frac{y}{2} \).

2(e) \( C_n(y) = n + \frac{y^2}{n} \). \( MC_3(y) = \frac{2y}{n} \). \( AC_3(y) = \frac{n}{y} + \frac{y}{n} \).
2(f) When will Hildegard start grazing on two plots of grass? It must be at the point where \( C_1(y) = C_2(y) \) or \( y = \sqrt{2} \). Similarly, when will she start grazing on \( n + 1 \) plots of grass? It must be at the point where \( C_n(y) = C_{n+1}(y) \) or \( y = \sqrt{n(n+1)} \). So \( C(y) = C_n(y) \) when \( y \in [\sqrt{(n-1)n}, \sqrt{n(n+1)}] \).

3(a) The production function is essentially \( y = \min\{2w, b\} \) except \( w \) and \( b \) only take value at non-negative integers.

3(b) The production function indeed satisfies CRS.

\[
\begin{array}{cccccc}
  y & FC & VC & TC & MC & AC \\
  0 & 4 & 0 & 4 \\
  1 & 4 & 2 & 6 & 2 & 6 \\
  2 & 4 & 2 & 6 & 0 & 3 \\
  3 & 4 & 4 & 8 & 2 & 8/3 \\
  4 & 4 & 4 & 8 & 0 & 2 \\
\end{array}
\]

\[
\begin{array}{cccccc}
  y & b & w & TC & MC & AC \\
  0 & 0 & 0 & 0 \\
  1 & 2 & 1 & 4 & 4 & 4 \\
  2 & 2 & 1 & 4 & 0 & 2 \\
  3 & 4 & 2 & 8 & 4 & 8/3 \\
  4 & 4 & 2 & 8 & 0 & 2 \\
  5 & 6 & 3 & 12 & 4 & 12/5 \\
  6 & 6 & 3 & 12 & 0 & 2 \\
\end{array}
\]

3(c) When outputs are 3 or 4, 4 buckets will be rented.

4(a) Lady P solves \( \max_{c,r} c \frac{x}{x^2+r^2} \) s.t. \( c = (24 - r) * 5 \). FOC gives \( r = 16 \), \( c = 40 \). Hence she works 8 hours.

4(b) \( p - 1 \).

4(c) \( p - 1 = 5 \) or \( p = 6 \).
4(d) As long as $\rho \geq 6$, Lady P will solve $\max_c\ c^\frac{1}{3}r^\frac{2}{3}$ s.t. $c = (24-r)*(p-1)$. FOC gives $r = 16$. Hence she always works 8 hours. Thus, when $p < 6$, the quantity supplied is 0. When $p \geq 6$, the quantity supplied is fixed at 8.

5(a) In the short run when the number of firms is fixed, since typically supply (SR) is upward sloping, price will go down when demand falls. In the LR, supply is fixed at min AC, so price won’t change. Hence price changes more in the SR.

5(b) In the short run the industry-wide quantity decreases because given an upward sloping supply (SR), when demand falls, quantity falls. However, since price in the SR falls as well, a typical firm will make a loss. Hence some has to exit in the LR. When this occurs, price gradually goes up. Eventually in the LR, LR supply is still fixed at min AC. Hence with a smaller demand, the quantity supplied is even lower. Hence the industry-wide supply changes more in the LR.

5(c) In the LR, the staying firms produce exactly as before the demand falls. In the SR, they produce less since price is lower. Hence the quantity of each individual firm changes more in the SR.

5(d) Similarly to 5(c). In the LR, the staying firms have the same profit as that before the demand falls. In the SR, their profit becomes lower since price is lower. Hence the profit of each individual firm changes more in the SR.