

# **On the Neutrality of Profit Taxation in a Mixed Oligopoly**

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## **Abstract**

This paper investigates the neutrality of a profit tax in a mixed oligopoly. It is found that when the privatization level is exogenously given, the profit tax is no longer neutral. By contrast, if the optimal privatization level can be determined by the government, then the neutrality of the profit tax holds. These results are robust under a free-entry market structure.

***Keywords:*** *Mixed oligopoly, Privatization, Profit tax neutrality*

***JEL classification:*** *H21, L11, L22, L23*

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## 1. Introduction

Corporate profit taxation has become one of the most important sources of tax revenue of governments in many countries. For example, a report by PricewaterhouseCoopers (2013) shows that 95% of countries around the world implement corporate profit taxation.<sup>1</sup> Thus, the issue of profit taxation has attracted considerable attention in the public finance literature.

In their seminar work, Atkinson and Stiglitz (1980) propose that a profit tax does not affect output in either the short or the long run. Under an imperfectly competitive market, the neutrality of profit taxation has been the focus of much attention, particularly in relation to how tax evasion affects neutrality. For example, Wang and Conant (1988) and Yaniv (1996) find that the neutrality of profit taxation is not affected by tax evasion. By contrast, Kreutzer and Lee (1986), Lee (1998), and Goerke and Runkel (2006) show that the neutrality is not sustainable under tax evasion. In addition, studies also suggest that the neutrality of profit taxation will not hold under international oligopoly (Parai, 1999) or in a dynamic oligopoly model (Baldini and Lambertini, 2011).

The above papers all assume that firms belong to private sectors, which explains part of the reality. In industries such as transportation, telecommunications, power, etc., some firms are (partially) owned by the public sector. Nevertheless, whether profit taxation is neutral or not under a mixed oligopoly is unclear. Fujiwara (2007) investigates the optimal level of privatization within the context of a mixed oligopoly; however, the role of tax is introduced in the model. We extend his model to investigate the neutrality of profit taxation and how a profit tax affects the optimal privatization. This paper is also related to Mujumdar and Pal (1998) in which two forms of indirect taxation, a specific tax and an ad valorem tax, are examined. In a departure from them, we discuss direct taxation.<sup>2</sup> We show that if the privatization level is exogenously given, the neutrality of profit taxation does not hold. However, if the privatization level is endogenously determined, the tax neutrality is sustained. This result is robust under free entry. Moreover, the optimal privatization level increases with the profit tax.

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<sup>1</sup> Please refer to <https://www.pwc.com/gx/en/paying-taxes/pdf/pwc-corporate-income-tax-report.pdf>

<sup>2</sup> The difference between direct and indirect taxation is concerned with whether the firms can transfer the tax burden to buyers. The taxation takes the form of an indirect tax if a firm can transfer the tax to consumers.

## 2. The basic model

Assume that there are  $n$  private firms, each being firm  $i$ , and one public firm, firm 0, producing differentiated products and competing in Cournot fashion in the market. Firm 0 produces  $x_0$  and firm  $i$  produces  $x_i$  with a constant marginal cost,  $c$ , and a fixed cost,  $f$ . Following Dixit (1979), the utility function of the representative consumer is specified as follows:

$$u = \alpha \sum x_j - \frac{\beta - \gamma}{2} \sum x_j^2 - \frac{\gamma}{2} \left( \sum x_j \right)^2 + z, \quad j = 0, \dots, n. \quad (1)$$

where  $\alpha, \beta, \gamma, z > 0$ ,  $\alpha$  is the highest willingness to pay of the consumer,  $z$  is the consumption of numeraire goods, and  $\beta$  and  $\gamma$  respectively denote the own-price effect and the cross-price effect with the property  $\beta > \gamma$ . The products are more differentiated when the difference between  $\beta$  and  $\gamma$  becomes larger. Following standard procedures, we derive the inverse demand functions as follows:

$$p_0 = \alpha - (\beta - \gamma)x_0 - \gamma \sum x_j, \quad (2)$$

$$p_i = \alpha - (\beta - \gamma)x_i - \gamma \sum x_j, \quad i = 1 \dots n. \quad (3)$$

The consumer surplus (CS) can be expressed as follows:

$$CS = x_0 \left( \frac{\beta}{2} x_0 + \gamma \sum x_i \right) + \frac{\beta - \gamma}{2} \sum x_i^2 + \frac{\gamma}{2} \left( \sum x_i \right)^2,$$

The profit functions of the public firm and the private firms are specified as follows:

$$\pi_0 = (1-t) \left[ (\alpha - c - \beta x_0 - \gamma \sum x_i) x_0 - f \right], \quad (4)$$

$$\pi_i = (1-t) \left\{ \left[ \alpha - c - (\beta - \gamma)x_i - \gamma (x_0 + \sum x_i) \right] x_i - f \right\}, \quad (5)$$

where  $t$  is the profit tax.

Following the standard setup in the mixed oligopoly literature,<sup>3</sup> the objective function of the partial public firm is a weighted average of its own profit,  $\pi_0$ , and

social welfare,  $W = CS + \sum_{j=0}^n \pi_j + TR$ , where TR denotes the total revenue in the form

of the profit tax. The objective function of firm 0 can be expressed as follows:

$$\theta \pi_0 + (1-\theta)W, \quad (6)$$

where  $\theta$  denotes the privatization level. By differentiating (5) and (6) with respect

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<sup>3</sup> See, for example, Matsumura (1998), Mujumdar and Pal (1998), and Matsumura and Sunada (2013), among others.

to  $x_i$  and  $x_0$ , respectively, the first-order conditions of the private and public firms are derived as follows:

$$\gamma x_0 + [2\beta + \gamma(n-1)]x_i = (\alpha - c), \quad i = 1, \dots, n, \quad (7)$$

$$\beta(1 + \hat{\theta})x_0 + \gamma \sum x_i = (\alpha - c), \quad (8)$$

where  $\hat{\theta} \equiv \theta(1-t)/(1-\theta t)$ . To simplify our analysis, we further assume that all private firms are symmetric, producing the same amount of output, say,  $x_1$ . We can re-write (7) and (8) as follows:

$$\gamma x_0 + [2\beta + \gamma(n-1)]x_1 = (\alpha - c), \quad (9)$$

$$\beta(1 + \hat{\theta})x_0 + \gamma n x_1 = (\alpha - c), \quad (10)$$

By solving (9) and (10) simultaneously, the equilibrium outputs for the private and public firms are derived as follows:

$$x_1^s(\theta, t) = \frac{(\alpha - c)[\beta(1 + \hat{\theta}) - \gamma]}{\Omega}, \quad \text{and} \quad x_0^s(\theta, t) = \frac{(\alpha - c)(2\beta - \gamma)}{\Omega}, \quad (11)$$

where  $\Omega = [2\beta + \gamma(n-1)]\beta(1 + \hat{\theta}) - n\gamma^2 > 0$ .

By differentiating (11) with respect to  $\theta$  and  $t$ , we can derive that  $\partial x_0^s / \partial \theta < 0$ ,  $\partial x_1^s / \partial \theta > 0$ ,  $\partial x_0^s / \partial t > 0$ ,  $\partial x_1^s / \partial t < 0$ . This implies that an increase in the privatization level reduces the public firm's output but increases the private firms' output. This is because privatization makes the public firm become less aggressive in production, causing its strategic-substitute rivals to produce more. By contrast, an increase in the profit tax raises the output of the public firm but reduces the combined outputs of the private firms, owing to that a higher profit tax reduces the weight on the public firm's profit in its objective function. This finding suggests that the neutrality of profit taxation is not sustainable. We establish the following proposition:

***Proposition 1. For any given privatization level, a profit tax policy increases the output of the public firm and the total output, but reduces the output of the private firms. That is to say, the neutrality of profit taxation does not hold.***

### **3. The neutrality of a profit tax under an optimal privatization policy**

We then investigate the neutrality of profit taxation under optimal privatization policies. The game in question consists of two stages. In the first stage, the government determines its privatization level. In the second stage, the public firm and the private

firms compete in quantity terms in the market. We solve the sub-game perfect Nash equilibrium via backward induction. The equilibrium in the second stage game is the same as that in Section 2. We proceed to the first-stage game. In the first stage, the objective function of the government is expressed as follows.

$$\max_{\theta} W = CS + \sum_{i=0}^n \pi_i + TR.$$

By means of routine calculations, we derive the optimal privatization level as follows:

$$\theta^S(t) = \frac{n\gamma(\beta - \gamma)}{(1-t)(2\beta - \gamma)^2 + n\gamma(\beta - \gamma)}. \quad (12)$$

We then derive that  $d\theta^S/dt > 0$ , implying the optimal privatization level increases with the profit tax. By Proposition 1, a higher profit tax incurs larger outputs of the public firm, resulting a higher level of optimal privatization level. Thus, we can establish the proposition as follows.

***Proposition 2. The optimal privatization level increases with the profit tax.***

By setting  $t = 0$ , the optimal privatization level,  $\theta^S(0)$ , restores to the result in Fujiwara (2007) who does not consider profit taxation. In addition, by substituting (12) into (11), we find that  $x_0^S(\theta^S(t), t) = x_0^S(\theta^S(0), 0)$  and  $x_1^S(\theta^S(t), t) = x_1^S(\theta^S(0), 0)$ , implying that profit taxation has no effects on the market equilibrium. As there is a one-to-one relationship between  $\theta$  and  $t$ , both of which affect the weights of the public firm's objective function, the government can choose the privatization level to cure the distortion caused by the profit tax, and the neutrality of profit taxation holds. We establish the following proposition.

***Proposition 3. In a mixed oligopoly, the neutrality of profit tax sustains if the optimal privatization level is endogenously determined by the government.***

#### 4. Free entry

We then extend our basic model to the free entry case. Taking  $\theta$  as given, we can derive the equilibrium number of private firms. By substituting (11) into (5), we derive the zero-profit condition as follows.

$$\pi_1 \Big|_{x_0=x_0^S, x_1=x_1^S} = \beta \left\{ \frac{(\alpha - c) \left[ (1 - \theta t)(\beta - \gamma) + \beta \theta (1 - t) \right]}{\Omega} \right\}^2 - f = 0. \quad (13)$$

Solving (13) yields the equilibrium number of private firms as follows:

$$n^L(\theta, t) = \frac{(\alpha - c)(\beta(1 - \hat{\theta}) - \gamma) - \beta(2\beta - \gamma)(1 + \hat{\theta})}{\gamma[\beta(1 + \hat{\theta}) - \gamma]} \sqrt{\frac{\beta}{f}}. \quad (14)$$

By substituting (14) into (11), we further derive that:

$$x_0^L(\theta, t) = \frac{(2\beta - \gamma)}{\beta(1 + \hat{\theta}) - \gamma} \sqrt{\frac{f}{\beta}}, \quad \text{and} \quad x_1^L(\theta, t) = \sqrt{\frac{f}{\beta}}. \quad (15)$$

By differentiating (14) and (15) with respect to  $t$ , we derive the comparative static effects as follows:  $\partial n^L / \partial t < 0$ ,  $\partial x_0^L / \partial t > 0$ ,  $\partial x_1^L / \partial t = 0$ . It follows that an increase in the profit tax increases the output of the public firm, reduces the number of private firms, and has no effect on the private firms' outputs. In addition, given the zero-profit condition, the private firm's optimal output decision is not affected by the profit tax. This result is different from that without free entry. This leads us to the following proposition:

**Proposition 4.** *Under free entry, an increase in the profit tax increases the output of the public firm, reduces the number of private firms, and has no impact on the output of the private firms if the privatization level is given.*

We then discuss the case where the government can determine the optimal privatization level prior to the output stage. The setups are similar to those in Section 3. Under free entry, the profits of the private firms are zero. The social welfare function can be rewritten as follows.

$$\max_{\theta} SW(x_0^L(\theta), n^L(\theta), x_1^L) = \left( \alpha - c - \frac{\beta}{2} x_0^L \right) x_0^L + \frac{\beta + \gamma(n^L - 1)}{2} n^L (x_1^L)^2 - f.$$

By routine calculations, the optimal privatization level is derived as follows.

$$\theta^L(t) = \frac{(\beta - \gamma)^2}{\beta(1 - t)(3\beta - \gamma) + (\beta - \gamma)^2 t}. \quad (16)$$

It can be derived that  $\partial \theta^L / \partial t = \frac{(\beta - \gamma)^2 (2\beta^2 - \gamma^2 + \gamma\beta)}{(t(\beta - \gamma)^2 + (1 - t)\beta(3\beta - \gamma))^2} > 0$ . This

suggests that an increase in the profit tax increases the optimal privatization level. This result and the underlying intuition are the same as those in Section 3. In addition, By substituting (16) into (14) and (15), it is found that  $n^L(\theta^L(t), t) = n^L(\theta^L(0), 0)$ ,  $x_0^L(\theta^L(t), t) = x_0^L(\theta^L(0), 0)$  and  $x_1^L(\theta^L(t), t) = x_1^L(\theta^L(0), 0)$ . This implies that the neutrality of profit taxation is sustained if the government can choose the optimal

privatization level. This is because the government can use the privatization policy to remedy the welfare distortion caused by the profit tax. This result is the same as that in Section 3. We construct the following proposition:

***Proposition 5. In a mixed oligopoly, the neutrality of profit taxation holds in the long run as long as the optimal privatization level is determined by the government.***

## **5. Concluding remarks**

Profit taxation plays an important role in many economies, and the issue of the neutrality of profit taxation has been extensively discussed in the public finance literature. This paper investigates the neutrality of profit taxation in a mixed oligopoly. Our results suggest that if the privatization level is exogenously given, a profit tax policy increases the output of the public firm and the total output, but decreases the output of the private firms. Namely, the neutrality of profit taxation does not hold. However, if the government can determine the optimal privatization level, profit taxation is neutral. In addition, the optimal privatization level increases with the profit tax. These results hold if the market is characterized by free entry.

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