The Location Choice under Cournot Competition with *n* Firms in a Linear City Model

Current Version: July 12, 2006

Introduction

- It is well recognized since Hamilton *et al.* (1989), and Anderson and Neven (1991) that central agglomeration is a *unique* location equilibrium when firms play Cournot competition.
- In the real world, we can frequently observe that there are more than two firms competing in an industry.
- The interactions among firms are more complicated in the *n*-firm case.

- Liang *et al*. (2006) show that quantity competition creates a competition effect, which is definitely a centrifugal force in the two-firm case. But in the n-firm case, the competition effect between firms i -1 and i forces firm i to move toward right, while this effect between firms i and i +1 forces firm i to move toward left. The competition effect is no longer a definite centrifugal force.

- Relevant literature includes: AN, HTW, Gupta *et al*. (1997), Mayer (2000) and Matsushima and Shimizu (2005) with a linear city model, as well as Pal (1998), Chamorro-Rivas (2000), Matsushima (2001), Yu and Lai (2003), and Gupta *et al*. (2004) with a circular city model.

- AN, and Matsushima and Shimizu (2005) show that for linear demand and linear transport costs, the central agglomeration is a unique equilibrium to the *n*-firm location-quantity game in a linear city model.

- Liang *et al*. (2005) have indicated that AN and HTW take a slightly stringent standard by requiring the output of each firm shipped to its remote endpoint be positive even if *the two firms are located at two opposite ends of the line segment*.

- The purpose of this paper is to explore where firms select to locate in the *n*-firm case by using an appropriate definition of the market-serving condition and taking into consideration the stability conditions, when firms play Cournot competition in the commodity market.

The Basic Model

- There are n firms, located at xi with $0 \leq x_1 \leq x_2 \leq \ldots \leq x_n \leq 1$, along a line segment with the length $L = 1$. The firms sell a homogeneous product. Assume that the demand function at each point $x \in [0, 1]$ is $P = 1 - Q$, and that the transport cost function is $T(x - x_i) = t\left|x - x_i\right|$

- Firm *i*'s profit at point *x* is:

$$\pi_i(x) = [1 - Q(x) - t|x - x_i|]q_i(x), \quad i = 1, \ldots, n, \qquad (1)$$

where $Q(x) = \sum_{i=1}^{n} q_i(x)$ and $q_i(x)$ is firm *i*'s sales at *x*.

- The game consists of two stages -- firms choose their locations in the first stage followed by Cournot quantity competition in each market in the second stage.

- In stage 2, we yield:

$$q_i(x) = [1/(n+1)][1 + t\sum_{j=1}^{n}\left|x - x_j\right| - (n+1)t\left|x - x_i\right|], i = 1, \ldots, n, \qquad (2.1)$$

$$\pi_i(x) = \{[1/(n+1)][1 + t\sum_{j=1}^{n}\left|x - x_j\right| - (n+1)t\left|x - x_i\right|]\}^2, i =, 1, \ldots, n. \qquad (2.2)$$

- The total profit function of firm *i* is:

$$\pi_i = \int_0^1 \pi_i(x)dx, \quad i = 1, \ldots, n. \qquad (3)$$

- In stage 1, totally differentiating (3) with respect to *x*, we have:

$$
\begin{aligned}
&[(n+1)^2/2nt](\partial\pi_i/\partial x_i) \\
&= -\int_0^{x_1}\left[1+t\sum_{j=1}^{n}\left|x-x_j\right|-(n+1)t(x_i-x)\right]dx-\ldots \\
&-\int_{x_{i-1}}^{x_i}\left[1+t\sum_{j=1}^{n}\left|x-x_j\right|-(n+1)t(x_i-x)\right]dx \\
&+\int_{x_i}^{x_{i+1}}\left[1+t\sum_{j=1}^{n}\left|x-x_j\right|-(n+1)t(x-x_i)\right]dx+\ldots \\
&+\int_{x_n}^{1}\left[1+t\sum_{j=1}^{n}\left|x-x_j\right|-(n+1)t(x-x_i)\right]dx=0, i=1,\ldots,n. \qquad (4)
\end{aligned}
$$

The Equilibrium Location for an Even Number of Firms

- The case of n = 2, substituting n = 2 into (4), we can re-express it as:

$$(9/4t)(\partial\pi_1/\partial x_1) = (2-t)[(1/2)-x_1] - t[1-(x_2-x_1)](x_2-x_1) = 0, \quad (5.1)$$

$$(9/4t)(\partial\pi_2/\partial x_2) = (2-t)[(1/2)-x_2] + t[1-(x_2-x_1)](x_2-x_1) = 0. \quad (5.2)$$

- The first term is named the cost-saving effect: firm i desires to move toward the center in order to save on the transportation cost. Moreover, a higher transport rate, which leads to lower sales, weakens the cost-saving effect.

- The second term is called the competition effect. It indicates that as the two firms are more distant from each other, their transport costs at site *x* become dissimilar and therefore the competition lessens. Moreover, the cost deviation will be larger if the transport rate is higher.

- The second-order and the stability conditions:

$$\partial^2 \pi_1 / \partial x_1^2 = (-8t/9)[(1-t) + t(x_2 - x_1)] < 0, \qquad (6.1)$$

$$J = (\partial^2 \pi_1 / \partial x_1^2)(\partial^2 \pi_2 / \partial x_2^2) - (\partial^2 \pi_1 / \partial x_1 \partial x_2)(\partial^2 \pi_2 / \partial x_2 \partial x_1)$$
$$= (4t/9)^2 (2-t)[(1/4)(2-3t) + t(x_2 - x_1)] > 0. \qquad (6.2)$$

- The market-serving condition:

$$q_1(1; x_1, x_2) = (1/3)[1 - 2t|x_1 - 1| + t|x_2 - 1|] > 0. \qquad (7)$$

- Solving (4), we obtain the following location equilibria:

$$x_1 = 1/2, \text{ and } x_2 = 1/2, \tag{8.1}$$

$$x_1 = (2-t)/4t, \text{ and } x_2 = (5t-2)/4t. \tag{8.2}$$

- Substituting (8.1) and (8.2) into the stability condition yields:

$$J(x_1 = x_2 = 1/2)$$

$$= (2t/9)^2(2-t)(2-3t) > 0, \text{ if } t < 2/3 \text{ or } t > 2, \tag{9.1}$$

$$J(x_1 = (2-t)/4t, x_2 = (5t-2)/4t)$$

$$= (2t/9)^2(2-t)(3t-2) > 0, \text{ if } 2/3 < t < 2. \tag{9.2}$$

- Substituting (8.1) and (8.2) into the market-serving condition obtains:

$$q_1(1; x_1 = x_2 = 1/2) = (1/3)[1-(t/2)] > 0, \text{ if } t < 2, \quad (10.1)$$

$$q_1(1; x_1 = (2-t)/4t, x_2 = (5t-2)/4t)$$
$$= (1/6)(10-11t) > 0, \text{ if } t < 10/11. \quad (10.2)$$

- The second-order conditions require:

$$\partial^2 \pi_1 / \partial x_1^2 (x_1 = x_2 = 1/2) = (-8t/9)(1-t) < 0, \text{ if } t < 1, \quad (11.1)$$

$$\partial^2 \pi_1 / \partial x_1^2 (x_1 = (2-t)/4t, x_2 = (5t-2)/4t) = -4t^2/9 < 0. \quad (11.2)$$

- After taking into account all three conditions, we have the following location configuration: The two firms should agglomerate at the market center if $t \leq 2/3$; but they choose to locate separately if $2/3 < t < 10/11$.

- Intuitively, the cost-saving effect moves firms toward the market center; but the competition effect pulls the two firms away from the center. When the transport rate is higher than 2/3, there exists a disperse equilibrium. On the contrary, the two firms are located at the market center.

► The case of n = 4. Substituting n = 4 into (4), we have:

$$(25/8t)(\partial\pi_1/\partial x_1) = (2-t)[(1/2)-x_1] - t[1-(x_2-x_1)](x_2-x_1) \\ - t[1-(x_3-x_1)](x_3-x_1) - t[1-(x_4-x_1)](x_4-x_1) = 0, \qquad (12.1)$$

$$(25/8t)(\partial\pi_2/\partial x_2) = (2-t)[(1/2)-x_2] + t[1-(x_2-x_1)](x_2-x_1) \\ - t[1-(x_3-x_2)](x_3-x_2) - t[1-(x_4-x_2)](x_4-x_2) = 0, \qquad (12.2)$$

$$(25/8t)(\partial\pi_3/\partial x_3) = (2-t)[(1/2)-x_3] + t[1-(x_3-x_1)](x_3-x_1) \\ + t[1-(x_3-x_2)](x_3-x_2) - t[1-(x_4-x_3)](x_4-x_3) = 0, \qquad (12.3)$$

$$(25/8t)(\partial\pi_4/\partial x_4) = (2-t)[(1/2)-x_4] + t[1-(x_4-x_1)](x_4-x_1) \\ + t[1-(x_4-x_2)](x_4-x_2) + t[1-(x_4-x_3)](x_4-x_3) = 0. \qquad (12.4)$$

- The impact of the cost-saving effect is the same as that in the case of $n = 2$, while the impact of the competition effect is more complicated. For example, we find from (12.2) that the competition effect between firms (1, 2), which behaves as a centripetal force, moves firm 2 toward right, but those between firms (3, 2) and (4, 2), which behave as centrifugal forces, move firm 2 toward left. This occurs because firm 2 locates at the right side of firm 1, but at the left side of firms 3 and 4. Similarly, these impacts of competition effects apply to (12.1), (12.3), and (12.4).

- Accordingly, this shows that the competition effect between any two firms moves them away from each other. This effect is not definite a centrifugal force any more depending upon the relative locations of firms.

- Solving (12), we yield three location equilibria:

$$x_1 = x_2 = x_3 = x_4 = 1/2, \text{ for } t \le 2/5, \qquad (13.1)$$

$$x_1 = x_2 = (2-t)/8t,$$
$$\text{and } x_3 = x_4 = (9t-2)/8t, \text{ for } 2/5 < t < 18/29, \qquad (13.2)$$

$$x_1 = (1-t)/3t, \; x_2 = x_3 = 1/2,$$
$$\text{and } x_4 = (4t-1)/3t. \qquad (13.3)$$

- Intuitively, when the transport rate is higher than 2/5, there exists a disperse equilibrium due to higher competition effect. The competition effect between firms (1, 2) and that between firms (3, 4) are vanished. Thus, firms (1, 2) and (3, 4) locate at the same point, respectively. On the contrary, when the transport rate is no greater than 2/5, central agglomeration is the location equilibrium. This arises because the cost-saving effect outweighs the net competition effects.

- For the case of n = 6:

$$x_1 = \ldots = x_6 = 1/2, \text{ for } t \leq 2/7, \tag{14.1}$$

$$x_1 = x_2 = x_3 = (2-t)/12t,$$

$$\text{and } x_4 = x_5 = x_6 = (13t-2)/12t., \text{ for } 2/7 < t < 26/55 \tag{14.2}$$

- For the case of $n = 8$:

$$x_1 = \ldots = x_8 = 1/2, \text{ for } t \le 2/9, \tag{15.1}$$

$$x_1 = x_2 = x_3 = x_4 = (2-t)/16t,$$
$$\text{and } x_5 = x_6 = x_7 = x_8 = (17t-2)/16t, \text{ for } 2/9 < t < 34/89. \tag{15.2}$$

- We can thus reduce a general pattern of the location equilibrium as follows by using a "weak definition" of the stability conditions:

$$x_1 = ... = x_n = 1/2, \text{ for } t \le 2/(n+1), \tag{16.1}$$

$$x_1 = ... = x_{n/2} = (2-t)/2nt,$$
$$\text{and } x_{(n/2)+1} = ... = x_n = 1 - x_1, \text{ for } 2/(n+1) < t < t_c^e, \tag{16.2}$$

where $t^e{}_c$ denotes the critical transport rate that all firms can serve the entire market.

Proposition 1.

(1)*Taking the weak definition of the stability conditions, the central agglomeration is a location equilibrium when the transport rate is low, say no greater than* 2/(n+1), *while the disperse equilibrium emerges in which two equal groups of firms locate at the opposite side of the line symmetrically, when the transport rate is high. Furthermore, the first half of the firms locates at* [(2-t)/2nt], *while the last half locates at* [1- (2-t)/2nt] *at the disperse equilibrium.*

(2)*Taking the strict definition of the stability conditions, the disperse equilibria for the cases of* $n \geq 4$ *are unstable.*

The Equilibrium Location for an Odd Number of Firms

- For the case of n = 3: Substituting n = 3 into (4):

$$(16/6t)(\partial\pi_1/\partial x_1) = (2-t)[(1/2)-x_1] - t[1-(x_2-x_1)](x_2-x_1) - t[1-(x_3-x_1)](x_3-x_1) = 0, \quad (17.1)$$

$$(16/6t)(\partial\pi_2/\partial x_2) = (2-t)[(1/2)-x_2] + t[1-(x_2-x_1)](x_2-x_1) - t[1-(x_3-x_2)](x_3-x_2) = 0, \quad (17.2)$$

$$(16/6t)(\partial\pi_3/\partial x_3) = (2-t)[(1/2)-x_3] + t[1-(x_3-x_1)](x_3-x_1) + t[1-(x_3-x_2)](x_3-x_2) = 0. \quad (17.3)$$

- Solving (17), we have:

$$x_1 = x_2 = x_3 = 1/2, \text{ for } t \le 1/2, \qquad (18.1)$$

$$x_1 = (4 - 3t)/10t, \ x_2 = 1/2,$$

$$\text{and } x_3 = (13t - 4)/10t, \text{ for } 1/2 < t < 26/37. \qquad (18.2)$$

- Intuitively, when the transport rate is no greater than 1/2, central agglomeration is the location equilibrium. This occurs because the cost-saving effect outweighs the net competition effects. When the transport rate is higher than 1/2, there exists a disperse equilibrium. The competition effect between firms (1, 2) and that between firms (2, 3) are balanced. The net competition effect vanishes. Thus, firm 2 locates at the center of the market.

- For the case of $n = 5$:

$$x_1 = \ldots = x_5 = 1/2, \text{ for } t \le 1/3, \tag{19.1}$$

$$x_1 = x_2 = (4-3t)/18t,\ x_3 = 1/2,$$
$$\text{and } x_4 = x_5 = (21t-4)/18t, \text{ for } 1/3 < t < 14/27. \tag{19.2}$$

- For the case of n = 7:

$$x_1 = \ldots = x_7 = 1/2, \text{ for } t \leq 1/4, \qquad (20.1)$$

$$x_1 = x_2 = x_3 = (4-3t)/26t, \; x_4 = 1/2,$$
$$\text{and } x_5 = x_6 = x_7 = (29t-4)/26t, \text{ for } 1/4 < t < 58/141. \qquad (20.2)$$

- For the case of n = 9:

$$x_1 = \ldots = x_9 = 1/2, \text{ for } t \le 1/5, \tag{21.1}$$

$$x_1 = \ldots = x_4 = (4-3t)/34t,\ x_5 = 1/2,$$
$$\text{and } x_6 = \ldots = x_9 = (37t-4)/34t, \text{ for } 1/5 < t < 74/217. \tag{21.2}$$

- We can thus reduce a general pattern of the location equilibrium by setting the number of firms, *n* equal to 2*m*+1, as follows:

$$x_1 = \ldots = x_n = 1/2, \text{ for } t \le 2/(n+1), \tag{22.1}$$

$$\begin{aligned} & x_1 = \ldots = x_m = (4-3t)/(4n-2)t, \ x_{m+1} = 1/2, \\ & \text{and } x_{m+2} = \ldots = x_n = 1-(4-3t)/(4n-2)t, \text{ for } 2/(n+1) < t < t_c^o, \end{aligned} \tag{22.2}$$

where $t^o{}_c$ denotes the critical transport rate that all firms can serve the entire market.

Proposition 2.

The central agglomeration is a location equilibrium when the transport rate is low, say no greater than 2/(n+1)*, while the disperse equilibrium prevails in which the medium firm locates at the center of the market and two equal groups of firms locate at the opposite side of the line symmetrically, when the transport rate is high. Furthermore, the first group of the firms locates at* [(4-3t)/(4n-2)t]*, while the second group locates at* [1- (4-3t)/(4n-2)t] *at the disperse equilibrium.*

Concluding Remarks

- For the case of an even number of firms, the central agglomeration is a location equilibrium when the transport rate is no greater than $2/(n+1)$, while the disperse equilibrium emerges in which two equal groups of firms locate at the opposite side of the line symmetrically, when the transport rate is high. Note that this equilibrium may not satisfy the strict definition of the stability conditions for $n \geq 4$.

- For the case of an odd number of firms, the central agglomeration is a location equilibrium when the transport rate is no greater than $2/(n+1)$, while the disperse equilibrium prevails in which the medium firm locates at the center of the market and two equal groups of firms locate at the opposite side of the medium firm symmetrically, when the transport rate is high.

- Taking into consideration the stability conditions, we show that the multiple location equilibria do not occur. There is only one location equilibrium applying to its corresponding transport rate.