Recent Progress in the Coastal Acoustic Tomography

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1. Principle of travel-time acoustic tomography

Reciprocal travel times obtained along the ray paths

\[ t_i^{\pm} = \int_{\Gamma_i^{\pm}} \frac{ds}{C_0(z) + \delta C(x, y, z) + \mathbf{v}(x, y, z) \cdot \mathbf{n}} \quad \ldots \ldots (1) \]

Travel times for the reference sound speed determined by the range-independent ray simulation

\[ t_{0,i} = \int_{\Gamma_{0,i}} \frac{ds}{C_0} \quad \ldots \ldots \ldots (2) \]

Travel time deviation from the reference sound speed

\[ \Delta \tau_i^{\pm} = \int_{\Gamma_i^{\pm}} \frac{ds}{C_0 \left( 1 + \frac{\delta C}{C_0} \pm \mathbf{v} \cdot \mathbf{n} \right)} - \int_{\Gamma_{0,i}} \frac{ds}{C_0} \quad \ldots \ldots (3) \]
Under the conditions

\[ C_0 >> \Delta C, \quad C_0 >> v \cdot n \]

the first term on the right-hand side of (3) is expanded into a Taylor function series and the terms higher than two orders are neglected to get

\[ \Delta \tau_i^\pm \approx \int_{\Gamma_{0,i}^\pm} \frac{ds}{C_0} \left( 1 - \frac{\delta C \pm v \cdot n}{C_0} \right) - \int_{\Gamma_{0,i}^\mp} \frac{ds}{C_0} \quad \ldots \ldots \quad (4) \]

The first term on the right-hand side is further expanded into a Taylor series around the reference ray path

\[ \Delta \tau_i^\pm = - \int_{\Gamma_{0,i}^\pm} \frac{(\delta C \pm v \cdot n) ds}{C_0^2} + \int_{\Gamma_{0,i}^\pm} \frac{1}{C_0} \left( 1 - \frac{\delta C \pm v \cdot n}{C_0} \right) \Delta(ds) \]

\[ \approx - \int_{\Gamma_{0,i}^\pm} \frac{(\delta C \pm v \cdot n)}{C_0^2} ds \quad \ldots \ldots \ldots \quad (5) \]
The subtraction and summation of the reciprocal travel times leads to

$$\delta \tau_i^y = \tau_i^+ - \tau_i^- \approx -2 \int_{\Gamma_{0,i}} \frac{\mathbf{v} \cdot \mathbf{n}}{C_0^2} ds \quad \cdots (6)$$

$$\delta \tau_i^C = \tau_i^+ + \tau_i^- \approx -2 \int_{\Gamma_{0,i}} \frac{\delta C}{C_0^2} ds \quad \cdots (7)$$

The ray paths with angle $\Phi$ from the horizontal are projected to the horizontal plane (X-axis)

$$\delta \tau_i^V \approx -2 \int_{\Gamma_{0,i}} \frac{V_s}{C_0^2} ds = -2 \int_0^L \frac{V \cos \phi_i}{C_0^2} \frac{dX}{\cos \phi_i} = -2 \int_0^L \frac{V}{C_0^2} dX \quad \cdots (6)'$$

$$\delta \tau_i^C \approx -2 \int_0^L \frac{\delta C}{C_0^2} \frac{dX}{\cos \phi_i} \quad \cdots (7)'$$

$(V_s \approx V, \ w \approx 0)$

V : Depth average current
\(\delta C\) : Depth average sound speed deviation
2. 2D mapping of depth-averaged current in the coastal seas around Japan

2.1 Inverse analysis for velocity fields

\[ \delta \tau_i^V = -2 \int_0^L \frac{V}{C_0^2} dX \]

\[ V = u \cos \theta + v \sin \theta \]

Integration along the x-axis

\[ \delta \tau_i^V = -2 \int_0^L \frac{u + v \tan \theta}{C_0^2} dx \]
Stream function for depth-averaged currents

\[ u = -\frac{\partial \Psi}{\partial y}, \quad v = \frac{\partial \Psi}{\partial x} \]

Fourier function expansion

\[ \Psi(x, y) = ax + by + \sum_{k=0}^{N_x} \sum_{l=0}^{N_y} \left\{ A_{k,l} \cos 2\pi \left( \frac{kx}{L_x} + \frac{ly}{L_y} \right) + B_{k,l} \sin 2\pi \left( \frac{kx}{L_x} + \frac{ly}{L_y} \right) \right\} \]

\[ = \sum_{j=1}^{(N_x+1)(N_y+1)} D_j Q_j(x, y) \]

\[ D = \{D_j\} = \begin{bmatrix} a, b, A_{00}, B_{00}, A_{01}, B_{01}, \cdots \cdots \cdots, A_{N_xN_y}, B_{N_xN_y} \end{bmatrix} \]

\[ Q(x, y) = \{Q_j\} = \begin{bmatrix} x, y, 1, 0, \cos \frac{2\pi y}{L_y}, \sin \frac{2\pi y}{L_y}, \cdots, \cos 2\pi \left( \frac{N_x x}{L_x} + \frac{N_y y}{L_y} \right), \sin 2\pi \left( \frac{N_x x}{L_x} + \frac{N_y y}{L_y} \right) \end{bmatrix} \]
Final equation for velocity inversion

\[
\delta \tau_i = \sum_{j=1}^{(N_x+1)(N_y+1)} 2D_j \int_0^{L_i} \left( \frac{\partial}{\partial y} Q_j - \tan \phi_i \frac{\partial}{\partial x} Q_j \right) dx \\
\]

Data

Unknown variables

Matrix expression with noises

\[
y = Ex + n
\]

- **y**: Data matrix for travel time difference
- **x**: Coefficient matrix for function expansion
- **E**: Transform matrix
- **n**: Travel time errors
Stochastic inverse (Gauss-Markov method)  
(Cornuelle et al., JGR, 1989)

Objective Function

\[ J = \left\langle (\tilde{X} - X)(\tilde{X} - X)^T \right\rangle \]
\[ \tilde{X} : \text{expected solution} \]
\[ \bar{X} : \text{mean solution} \]

Covariance of solution and travel-time errors

\[ R = \left\langle (X - \bar{X})(X - \bar{X})^T \right\rangle \]
\[ N = \left\langle nn^T \right\rangle \]

Expected solution is determined by minimizing \( J \)

\[ \tilde{X} = RE^T (ERE^T + N)^{-1} y \]

The stochastic inverse needs the a priori information on the variance of solution and error.
Damped (tapered) least squares method
(Park and Kaneko, GRL, 2000; JO, 2001)

Objective Function

\[ J = (y - Ex)^T (y - Ex) + \alpha^2 x^T x \]

Expected Solution

\[ \tilde{x} = (E^T E + \alpha^2 I)^{-1} E^T y \]

Expected error for travel time

\[ \tilde{e} = y - E\tilde{x} = \left\{ I - E(E^T E + \alpha^2 I)^{-1} E^T \right\} y \]
Correctness of inversion

Error covariance matrix $R$

$$R = \delta x \delta x^T = I - E^T (EE^T + \alpha^2 I)^{-1} E$$

$$(\delta x = \hat{x} - x)$$

Error covariance matrix $P$ in the physical space

$$P(x, y) = \sum_{i=1}^{M} \sum_{j=1}^{M} Q_i(x, y)Q_j(x, y)R_{ij}$$

$$= QRQ^T$$

where $Q$ denotes the matrix of expansion function. The error covariance matrix $P$ is a scalar function and called the uncertainty.
2.2 Mode function method with coastline constraint

The Fourier function expansion is not suitable for the tomography domain, bounded with a complicated coastline. The mode analysis method, adapted to the complicated coastline is preferable.

The mode analysis method has been developed to transform, interpolate and regularize the measured velocity field (HF radar et al.) with a coastline constraint. The boundary is composed of the closed (coastline) and open boundaries. All previous studies were done how well the flux across the open boundary is considered.
Three past important papers:

Lipphardt et al. (JGR, 2000)
The data are first projected onto closed-boundary modes, and then add a zero-order mode to simulate flow across the open boundary.

Chu et al. (JTech, 2003)
At the open boundary, the modes are constrained by a priori knowledge (data and simulation result) on the normal velocity.

Lekien et al. (JGR, 2004)
The open-boundary modes is given as a set of time and data independent eigenfunctions.
The 2D current (vector) field is expressed by the summation of two scalar functions \( \Phi \) and \( \Psi \) as

\[
\mathbf{u}(\mathbf{r}) = u_\phi(\mathbf{r}) + u_\psi(\mathbf{r}) = \nabla \Phi(\mathbf{r}) + \nabla \times \Psi(\mathbf{r}) \mathbf{k}
\]

The irrotational component and solenoidal component are:

- \( u_\phi(\mathbf{r}) \) and \( u_\psi(\mathbf{r}) \) as \( \nabla \Phi(\mathbf{r}) \) and \( \nabla \times \Psi(\mathbf{r}) \mathbf{k} \)

Taking the scalar (\( \nabla \cdot \cdot \)) and vector (\( \nabla \times \)) products of the above equation, we get the following two equations:

\[
\Delta \Phi(\mathbf{r}) = \nabla \mathbf{u}(\mathbf{r})
\]

Divergence of \( \mathbf{u} \)

\[
\Delta \Psi(\mathbf{r}) = -k \cdot (\nabla \times \mathbf{u}(\mathbf{r}))
\]

Rotation of \( \mathbf{u} \)
Two scalar functions are expanded into a set of mode functions fitting to the complicated coastline.

\[ \Phi = \sum_{j=1}^{M} a_j \phi_j , \quad \Psi = \sum_{k=1}^{S} a_{M+k} \psi_k \]

The mode functions are determined by solving the following two approximated Helmholtz equations:

**Neumann condition**

\[ \nabla^2 \phi_m + \eta_m \phi_m = 0 , \quad (\mathbf{n} \cdot \nabla \phi_m)_{\text{boundary}} = 0 \]

**Dirichlet condition**

\[ \nabla^2 \psi_m + \mu_m \psi_m = 0 , \quad \psi_m \big|_{\text{boundary}} = 0 \]

Non-normal current
When the velocity data are obtained inside the observational domain as in HF radar, the mode expansion coefficients are determined like

\[ u(r_i) = \sum_{j=1}^{M} a_j \nabla \phi_j(r_i) + \sum_{k=1}^{S} a_{M+k} \nabla \times \psi_k(r_i) k \]

Data unknown unknown

As for travel-time acoustic tomography, the mode expansion coefficients are determined from the travel time difference data

\[ \Delta \tau_i = -\frac{2}{C_0^2} [\Phi(r_2) - \Phi(r_1)] - \frac{2}{C_0^2} \int_{\Gamma_{0i}} [\nabla \times \Psi(x, y) k] \cdot n ds \]

Data

\[ = -\sum_{j=1}^{M} a_j 2 \left( \phi_j(r_2) - \phi_j(r_1) \right) \]

unknown

\[ - \sum_{k=1}^{S} a_{M+k} 2 \int_{L_i} \left( -\frac{\partial \psi_k}{\partial y} + \frac{\partial \psi_k}{\partial x} \tan \theta_i \right) dx \]
2.3 Field application of CAT

① Kanmon Strait Experiment
(Yamaguchi et al., JO, 2005; Lin et al., GRL, 2005)

- May 17-20, 2003
- 8 CAT systems
- $f = 5.5$ kHz
- Pseudo random signal: $10^{th}$ order Gold code
- range = 1-2 km
Location map

CAT system on the wharf
Hourly map of tidal current structures
Contour plots of error covariance matrix (uncertainty matrix) (unit in m/s)
Animation
(Kanmon Strait data)

- 9:00 18 March ~
  21:00 19 March 2003
  (12 hours)
Hiroshima Bay Experiment
(Yamaguchi, PhD Thesis, 2006)

The mode analysis method is applied to Hiroshima Bay with a complicated coastline. All boundaries, including the open boundaries are closed. To relax this unnatural constraint, the model domain is taken much wider than the tomography domain.

- September 18-27, 2003
- 7 CAT systems
- \( f = 5.5 \text{ kHz} \)
- Pseudo random signal : 10\(^{th}\) order Gold code
- range = 2-7 km
Wharf type system

Aquaculture raft type system
irrotational component
\[ \phi_i(x, y) \quad i = 1, 2 \]

Solenoidal component
\[ \psi_i(x, y) \quad i = 1, \ldots, 6 \]
Vector plots for the M2 tidal current

(a) 9:00 to 14:00

(b) 15:00 to 20:00
Tidal current simulation of Hiroshima Bay (Mutsuda)
3. System design and signal processing

**Reception**

- Pre-Amp
- BPF
- Amp
- Complex Demodulator
- LPF
- GPS receiver
- SH2 micro computer
- A/D
- Cross Corr.
- SD Memory Card (2GB)

**Transmission**

- Transducer
- Power Amp
- Transmission signal (M sequence)
- External PC
- RS232C serial cross cable
- Atomic clock
- WLAN Antenna
- GPS Antenna
- WLAN Router
- GPS receiver
- Cross Corr.
- SH2 micro computer
- M sequence
One digit of M sequence is its minimum unit, and equivalent to the time resolution \( t_r \) for multi-path arrival. Most of errors come from the time resolution.

\[ t_r = 0.55 \text{ ms for Kanmon Strait Experiment (f=5.5kHz)} \]
\[ = 0.75 \text{ ms for Aki-nada and Kurushima Strait Exp (f=4kHz).} \]
4. Application to Kuroshio study in the Seto Inland Sea

The Seto Inland Sea faces the Kuroshio at the southwestern and southeastern exits (Bungo Channel and Kii Channel, respectively).

Part of Kuroshio water intrudes into the Seto Inland Sea through the BC (Kyucho) and KC (Bifurcation Current).

How is the central part of the Seto Inland Sea (Aki-nada and Kurushima Strait) influenced by the Kuroshio?
Effect of the Kuroshio on the Seto Inland Sea
Tides in Aki-Nada

Flood tide $\rightarrow$ Eastward current $\downarrow$ positive along-line current

Ebb tide $\rightarrow$ Westward current $\downarrow$ negative along-line current
Range-averaged current for the depth-average layer

The reciprocal travel times are given

\[ t_1 = \frac{L}{C_m + V_m} \quad \ldots \ldots \quad (4.1) \quad t_2 = \frac{L}{C_m - V_m} \quad \ldots \ldots \quad (4.2) \]

By solving the coupled equations (8) and (9), \( C_m \) and \( V_m \) are

\[ V_m = \frac{C_0^2}{2L} \Delta t \quad \ldots \ldots \quad (4.3) \]

\[ C_m = \frac{L}{t_m} \quad \ldots \ldots \quad (4.4) \]

\[ \Delta t = t_2 - t_1 \]

\[ t_m = \frac{(t_1 + t_2)}{2} \]
By taking $t_m$-derivative of (3.4) under fixed L, the sound speed error is related to the travel-time error like

$$\delta C_m = -\frac{L}{t_m^2} \delta t_m$$  \hspace{1cm} (4.5)

By using the standard sound speed formula (Mackenzie, 1981), the range-averaged sound speed ($C_m$) is transformed into the range-averaged temperature ($T_m$) under the fixed salinity ($S$) and depth ($D$).

$$C = 1448.96 + 4.591T - 5.304 \times 10^{-2} T^2 + 2.374 \times 10^{-4} T^3$$

$$+ 1.340(S - 35) + 1.630 \times 10^{-2} D + 1.675 \times 10^{-7} D^2$$

$$- 1.025 \times 10^{-2} T(S - 35) - 7.139 \times 10^{-13} TD^3$$
For the coastal seas like Aki-nada and Kurushima Strait, arrival peaks in the correlation waveforms are not resolvable and construct a broad arrival peak. Thus the resolution of velocity and sound speed (temperature) is determined by the time resolution ($t_r$) for multi-path arrival. For $f=4\text{kHz}$ and $Q=3$ (cycles per digit), $t_r = 0.75\text{ms}$. By substituting this $t_r$ value into (4.3) and (4.4), we get

**Aki-nada:**
- $L=30\text{km}$, $C_0=1500\text{m/s}$, $\Delta t=0.75\text{ms}$
- $V_m = 2.8 \text{ cm/s}$, $C_m = 0.056\text{m/s}$, $T_m = 0.018\degree\text{C}$
- 6 repeat and hourly mean (5 ensemble average) $\sqrt{30}$
- $V_m = 0.5 \text{ cm/s}$, $C_m = 0.010\text{m/s}$, $T_m = 0.003\degree\text{C}$

**Kurushima Strait:**
- $L=5\text{km}$, $C_0=1530\text{m/s}$, $\Delta t=0.75\text{ms}$
- $V_m = 16.9 \text{ cm/s}$, $C_m = 0.338\text{m/s}$, $T_m = 0.137 \degree\text{C}$
- Hourly mean (16 ensemble average) $\sqrt{16}$
- $V_m = 4.2 \text{ cm/s}$, $C_m = 0.084\text{m/s}$, $T_m = 0.034\degree\text{C}$
Aki-nada
Kurushima Strait

Depth-average
layer

S1 S2

Aki-nada

Depth-average
layer

KR4 KR5

Section-average

Range-independent ray
simulation (Snell’s law
of refraction)

Kurushima Strait
Aki-nada Experiment
(Yudi et al., JO, 2011)

- March 2 ~ May 12, 2010
- 2 CAT systems
- range = 30 km
- $f = 4$ kHz
- transmission interval = 15 min
- Pseudo random signal:
  - $12^{th}$ order M sequence
  - $\log_{20}\sqrt{2^{12} - 1} = 36$ dB  \((\text{M sequence gain})\)
  - six repeat transmission
  - $\log_{20}\sqrt{6} = 8$ dB  \((\text{Ensemble average gain})\)
Data lack may mainly be caused by the stratification, destructed in the spring tide and reconstructed in the neap tide.

Time plot of the hourly mean current

$V_m (\text{cm/s})$

3.8 ± 1.8 cm/s

4.4 ± 1.7 cm/s
As well known, temperature is decreased from west to east in the Seto Inland Sea. Warm water appears in the observational site with the positive current ($S_1 \rightarrow S_2$) during the flood tide. The eastward current is measured as the positive current, projected to the sound transmission line.
Hourly temperature

Temperature band-pass filtered for one-hour to 30-day
Power spectral density diagram for temperature data
② Kurushima Strait Experiment
(July 2 ~ August 4, 2010)

- 3 CAT systems
- range = 4-6 km
- f = 4 kHz
- transmission interval= 4 min
- Pseudo random signal :
  - 10th order M sequence
  \[
  20 \log \sqrt{2^{10} - 1} = 30 \text{ dB} \quad \text{(M sequence gain)}
  \]
- Hourly mean procedure (15 ensembles)
  \[
  20 \log \sqrt{16} = 6 \text{ dB} \quad \text{(Ensemble average gain)}
  \]
$V_n$ (mean) = 7.6 cm/s

$V_e$ (mean) = 6.6 cm/s
Hourly mean

Q (mean) = 0.727 \times 10^4 \text{ m}^3/\text{s}
2-week High-Pass Filtered Temperature

$0.15 \, ^\circ C$

5 days

Date from July 1, 2009
5 days
Coherence=0.9

Phase=-0.065 rad
Summary

Aki-nada

1) Eastward mean current, implied by the positive current along the sound transmission line

\[(3.8 \sim 4.4) \pm (1.7 \sim 1.8) \text{ cm/s}\]

2) Temperature variation in the sub-tidal range
   Amplitude: 0.2 °C
   Periods: 5 days, 7 days, 21 days

Kurushima Strait

1) The volume transport due to tidal current
   7,270 m³/s

2) Temperature variation in the sub-tidal range
   Amplitude: 0.15°C, Periods: 5 days

3) Eastward phase velocity of 5-day wave 0.45 m/s
Further progress

1) The 3D mapping of the coastal seas has never been attempted. It may be realized in the shallow seas with depths greater than 50 m.

2) The CAT systems are constructed commercially only by the Aqua Environmental Monitoring Limited Liability Partnership (AEMLLP), recognized by Hiroshima University. It may be mainly caused by the difficulty in data analysis.