

Homework 3

1. If $A_{ij} = B_{mn} e_{im} e_{jn}$, write down terms A_{13} and A_{32} .

2. (a) Write down all the components of tensor notations $\frac{\partial}{\partial x_j} \left(\frac{\partial u_i}{\partial x_i} \right)$ and $u_i \frac{\partial u_j}{\partial x_i}$.

(b) What are their vector forms?

3. Show that if B_{ij} is a symmetrical tensor and C_{ij} is an anti-symmetrical tensor, then

(a) $B_{ij}C_{ji} = 0$,

(b) if $A_{ij} = B_{ij} + C_{ij}$ then $A_{ij} + A_{ji} = 2B_{ij}$.

4. If \vec{v} is a vector function, show by expansion in Cartesian coordinates that the following hold:

(a) $\nabla \cdot (\nabla \times \vec{v}) = 0$,

(b) $(\vec{v} \cdot \nabla)\vec{v} = (\nabla \times \vec{v}) \times \vec{v} + \nabla \left(\frac{1}{2} \vec{v} \cdot \vec{v} \right)$.