Homework 3

1. A scalar function is given as $f(x_1, x_2, x_3) = 2x_1x_3^2 - x_2^3$ find $\frac{\partial f}{\partial x_i}$ and $\frac{\partial^2 f}{\partial x_i \partial x_i}$.

2. Show that $\nabla \times (\nabla \phi) = 0$ and $\nabla \cdot (\nabla \times \boldsymbol{\psi}) = 0$, where ϕ is a scalar function and $\boldsymbol{\psi}$ is a vector function.

3. Write down the tensor forms of $\nabla \phi$, $\nabla \cdot \boldsymbol{v}$ and $\nabla \times \boldsymbol{v}$.

4. If $A_{ij} = B_{mn}\, e_{im}\, e_{jn}$, write down terms A_{13} and A_{32}.

5. (a) Write down all the components of tensor notations $\frac{\partial}{x_j}\left(\frac{\partial u_i}{\partial x_i}\right)$ and $u_i \frac{\partial u_j}{\partial x_i}$.

   (b) What are their vector forms?

6. Show that if B_{ij} is a symmetrical tensor and C_{ij} is an anti-symmetrical tensor, then

   (a) $B_{ij}C_{ji} = 0$,

   (b) if $A_{ij} = B_{ij} + C_{ij}$ then $A_{ij} + A_{ji} = 2B_{ij}$.