TAYLOR–COUETTE FLOW: THE EARLY DAYS

Fluid caught between rotating cylinders has been intriguing physicists for over 300 years with its remarkably varied patterns and its chaotic and turbulent behavior.

Russell J. Donnelly

The striking flow shown in figure 1 is produced in a simple apparatus: A fluid is confined between two concentric cylinders, with the inner and perhaps the outer cylinder able to rotate. The cellular motion that develops with rotation was discovered and described mathematically by Geoffrey I. Taylor in 1923. A similar apparatus, with the inner cylinder suspended from a torsion fiber and the outer cylinder rotating, was used even earlier as a viscometer. Maurice Couette described this arrangement in his thesis, which he presented in Paris in 1890. For this reason, modern investigators refer to flow between rotating cylinders as Taylor–Couette flow. In this article I trace the beginnings of the subject back to Isaac Newton and, by discussing the contributions of Newton, George Stokes, Max Margules, Arnulph Mallock, Couette, Taylor, S. Chandrasekhar and others, show how the study of this flow evolved to its place of prominence today.

Those not involved in rotating cylinder flow might well inquire what all the fun is about. That question is not hard to answer: Rotating cylinder flow is easy to produce, beautiful to observe and as profound a subject in fluid dynamics as there is. Characteristically, when meetings including this subject are held, they draw an astonishing group of participants ranging from lubrication engineers to pure mathematicians. These people mingle easily and enjoy each other's contributions. The pages of prestigious journals such as *Physics of Fluids, Journal of Fluid Mechanics, Physical Review* and *Physical Review Letters* abound with new observations and discoveries on the topic.

What is the nature and significance of a field such as Taylor–Couette flow, which has attracted the attention of giants in the past and continues to engage some of the best and brightest young investigators? To begin with, the flow produces fascinating patterns that vary in complicated ways with changes in the rotation rates of the cylinders. The instabilities and flow patterns challenge the most ingenious theorists to explain them. The challenge to experimenters is no less. A century ago early investigators, particularly Mallock and Couette, built state-of-the-art apparatus, and this tradition continues today as researchers bring the methods and techniques of modern condensed matter physics to bear on the problem. Since it is fundamentally a nonlinear subject, fluid dynamics has

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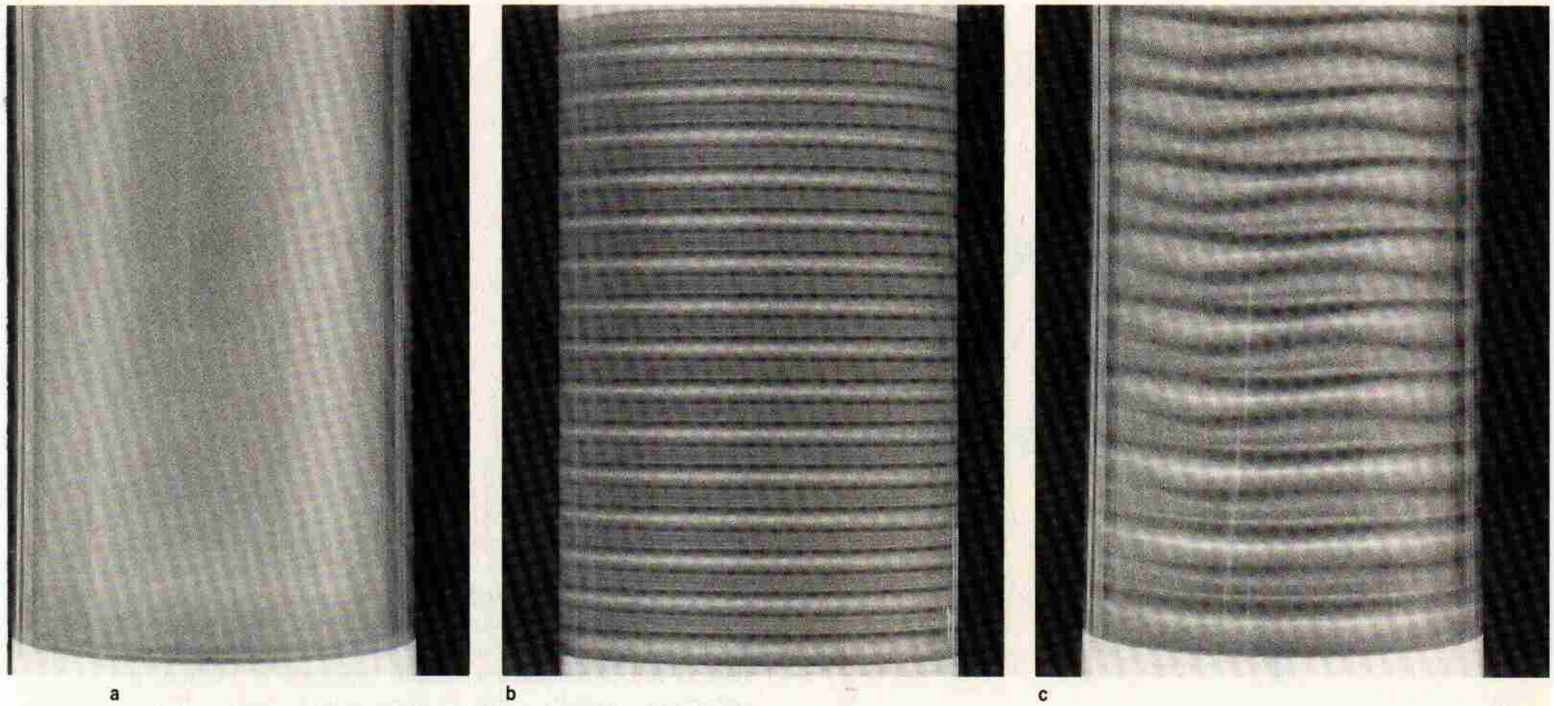


Taylor–Couette flow with the inner cylinder rotating. The fluid is seeded with Kalliroscope, a material made from fish scales, whose microscopic platelets align in a shear flow and reflect light. In **a** the rotation rate is below critical and the flow is laminar. In **b** the critical velocity predicted by linear stability theory is exceeded and we see Taylor vortex flow. In **c** a further increase in speed has excited azimuthal traveling waves on the Taylor vortices. (Courtesy of Harry Swinney and Randall Tagg, University of Texas, Austin.) **Figure 1**

few generalizations of the kind to which physicists are accustomed—in electromagnetic theory, for example. Understanding a flow is taken to mean solving the Navier–Stokes equation for the flow. A solution is impossible to find in many instances in engineering and in nature, and so it is useful to have some experiments that are relatively easy to construct and that have sufficiently high symmetry to be amenable to theoretical treatment and numerical simulation. The two paradigms most often used today are Taylor–Couette flow and Rayleigh–Bénard convection. (An example of the latter is the flow induced by heating a layer of fluid from below.) Both systems are capable of many variations and are the subject of much current interest.

Isaac Newton

Our subject begins with Newton, who in 1687 considered the circular motion of fluids.[1] In Book II, Section IX, of the *Principia* he offers, in the form of the following "Hypothesis," the definition of what is now called a Newtonian fluid:

> The resistance, arising from the want of lubricity in the parts of a fluid, is *caeteris paribus*, proportional to the velocity with which the parts of the fluid are separated from each other.

Today we would say the viscous stresses are proportional to the velocity gradient for a Newtonian fluid.

In Proposition 51, Newton says:

> If a solid cylinder infinitely long, in an uniform and infinite fluid, revolve with an uniform motion about an axis given in position, and the fluid be forced round by only this impulse of the cylinder, and every part of the fluid continues uniformly in its motion, I say that the periodic times of the parts of the fluid are as their distances from the axis of the cylinder. [See figure 2.]

In Corollary 2, Newton continues:

> If a fluid be contained in a cylindric vessel of an infinite length, and contain another cylinder within, and both the cylinders revolve about one common axis, and the times of their revolutions be as their semidiameters, and every part of the fluid continues in its motion, the periodic times of the several parts will be as the distances from the axis of the cylinders.

The flow in figure 2 is about a centrally rotating cylinder. The flow imagined in Corollary 2 is that which results if this flow is bounded by a second concentric outer cylinder. (The second cylinder does not change the flow in this case because of the angular velocity that Newton has specified for it.) This must be one of the earliest references to flow in the annulus between rotating cylinders. One imagines that Newton, like many investigators who followed him, was attracted to this example of flow because of its simplicity and symmetry.

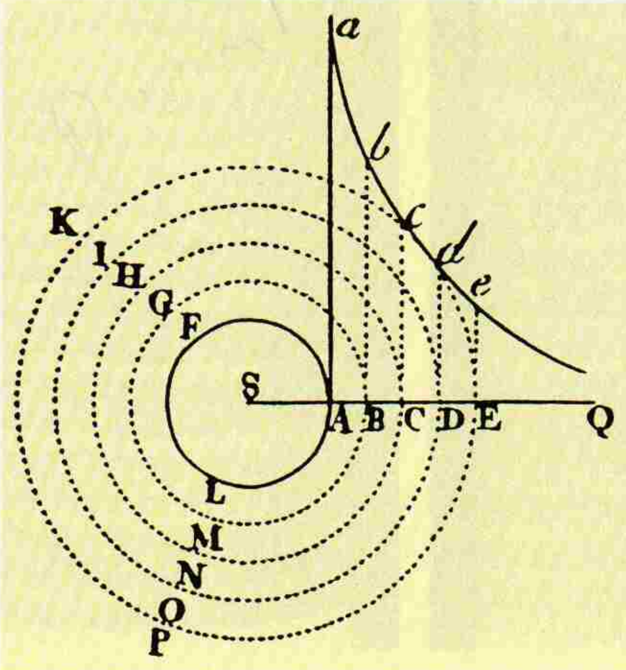
George Gabriel Stokes

A successor of Newton's in the Lucasian chair at Cambridge was one of the great pioneers of theoretical fluid dynamics—George Stokes (see figure 3). In a paper published in the *Transactions of the Cambridge Philosophical Society* in 1848, 161 years after Newton wrote on the subject, Stokes says:[2]

> Let us now consider the motion of a mass of uniform inelastic fluid comprised between two cylinders having a common axis, the cylinders revolving uniformly about their axis and the fluid being supposed to have altered its permanent state of motion.

He then solves for the velocity of the fluid and continues:

> The case of motion considered in this article may perhaps admit of being compared with experiment, without knowing the conditions which must be satisfied at the surface of a solid. A hollow, and a solid cylinder might be so mounted as to admit of being turned with different uniform angular velocities round their common axis, which is supposed to be vertical. If both cylinders are turned, they ought to be



Newton's figure describing rotating fluid motion about a cylinder, designated by the letters AFL. (From ref. 1.) **Figure 2**

turned in opposite directions, if only one, it ought to be the outer one; for if the inner one were made to revolve too fast, the fluid near it would have a tendency to fly outwards in consequence of the centrifugal force, and eddies would be produced. As long as the angular velocities are not great, so that the surface of the liquid is very nearly plane, it is not of much importance that the fluid is there terminated; for the conditions which must be satisfied at a free surface are satisfied for any section of the fluid made by a horizontal plane, so long as the motion about that section is supposed to be the same as it would be if the cylinders were infinite. The principal difficulty would probably be to measure accurately the time of revolution, and distance from the axis, of the different annuli. This would probably be best done by observing motes [dust particles] in the fluid. It might be possible also to discover in this way the conditions to be satisfied at the surface of the cylinders; or at least a law might be suggested, which could be afterwards compared more accurately with experiment by means of the discharge of pipes and canals.

Several points in the discussion are worth noting. First, we see that Stokes is concerned that the boundary conditions at the solid surfaces are unknown. This is hardly trivial—nearly a century would elapse before the no-slip condition for a fluid at a solid wall was universally accepted. Indeed, it was Taylor's analysis of rotating cylinder flow that settled the matter. Second, the realization that rotating the inner cylinder would produce the least stable flow and lead to eddies such as we see in figure 1 is surely the intuition of genius. It would be another 75 years before the flow in figure 1 was noted (by Taylor). Third, Stokes is concerned with the boundary conditions at the free surface of partially filled cylinders. Fourth, he is remarking on the use of a tracer to mark the flow—something we do easily today with a laser Doppler velocimeter, which measures fluid velocity by observing the Doppler shift of scattered light from small particles seeded in the flow.

When the equations of motion for a viscous fluid were formulated by Claude Navier (1823) and Stokes (1845), a considerable debate arose on how to best measure viscosity. As we shall see, it did not take long before experimenters interested in this question realized that there are two forms of fluid motion, which today we would term (roughly) laminar and turbulent. Because turbulent flow is not described by simple integrals of motion, its occurrence usually leads to anomalously high values of viscosity.

Max Margules

Margules (see figure 3) was born in Brody, Galicia, then part of the Austrian Empire. He was trained in Vienna in theoretical physics but became perhaps the first theoretical meteorologist. He began meteorological studies at the end of the 1880s and worked in the subject until 1906. Several important equations are named for him.[3]

Margules appears to have been the first person to seriously propose constructing a rotating cylinder viscometer. In 1881 he wrote:[4]

> Suppose a cylinder hangs vertically on a vertical axis which rotates uniformly. Suppose the cylinder is immersed in a coaxial cylindric container, which contains the fluid to be investigated. Then, due to the friction of the fluid, the relative position of the cylinder with respect to the axis during the rotation will be different from the one in the state of rest. Now one can measure the torque by means of a simple apparatus which results in a torsion angle of equal magnitude; this way one measures the resistance of the fluid against the rotation of the cylinder. The latter motion we assume to be stationary. (Any oscillations of the cylinder about the axis of rotation are strongly damped in a very viscous fluid; the same is true in a less viscous one if the container is relatively narrow.) Therefore the motion of the fluid between the two faces of the cylinder will become stationary.

Even in 1930, papers in the *Physical Review* refer to the "Margules rotating cylinder type viscometer."[5]

Seven years after Margules published this paper, two young men, Mallock and Couette, began to build rotating cylinder viscometers and made preliminary announcements in London and Paris. It appears that they were unaware of each other's work and that only Couette knew of Margules's paper.

Henry Reginald Arnulph Mallock

On 30 November 1888, Lord Rayleigh, secretary of the Royal Society of London, communicated a paper by Mallock (see figure 3) titled "Determination of the Viscosity of Water."[6] Mallock was a nephew of William

Pioneers of Taylor–Couette flow. Top: George Stokes, 1819–1903. (From ref. 2, volume V.) Middle: Max Margules, 1856–1930. (From ref. 24.) Bottom: Arnulph Mallock, 1851–1933, in a picture taken by C. V. Boys at a party in London for R. W. Wood of Johns Hopkins University. (Used with permission of the President and Council of the Royal Society.) **Figure 3**

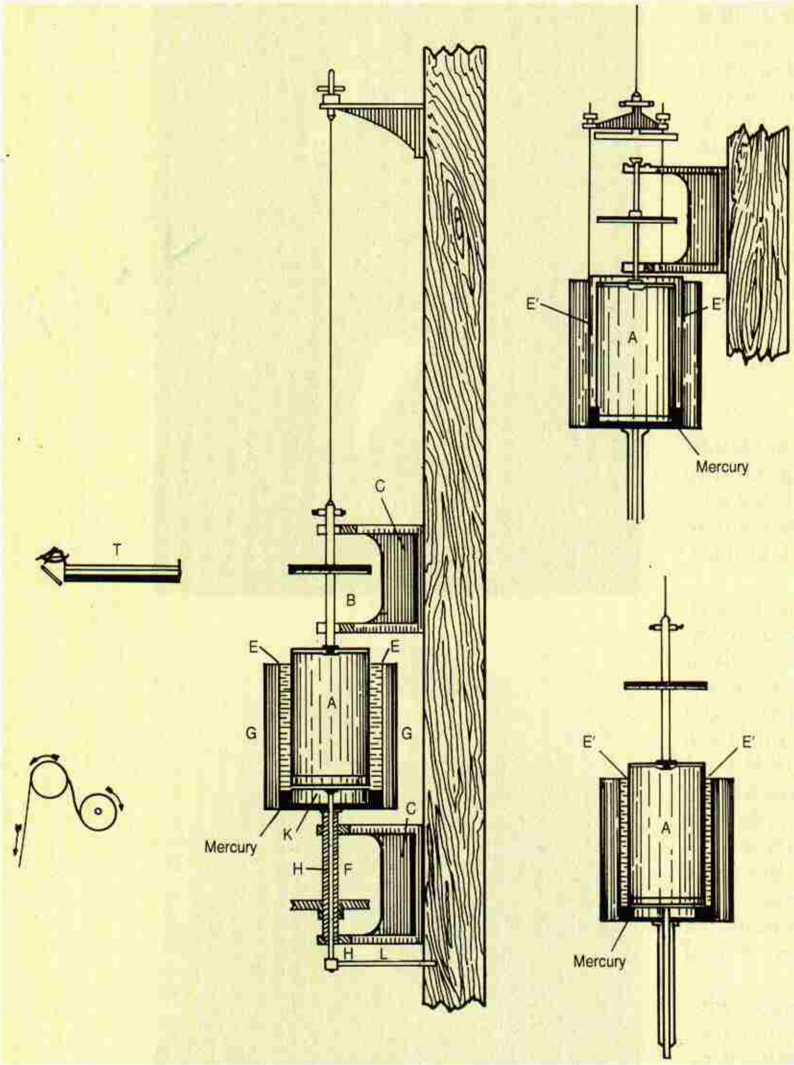
Froude, the famous naval architect. He studied at Oxford and after graduating helped Froude build the original ship model tank: a trough of water used to test ship models by towing. In 1876 Mallock went to work as an assistant to Rayleigh; because he was a skilled instrument builder, Mallock was especially valuable.

Mallock's 1888 paper describes water viscosity experiments conducted during April and May of that year using a pair of concentric cylinders with the outer one driven and the inner one suspended on a torsion fiber, as shown on the left side of figure 4. Mallock ventured that the experiments might be "of some interest on account of the newness of the method employed." In July of 1895 Lord Kelvin, president of the Royal Society, communicated a full paper by Mallock.[7] In this pioneering paper, Mallock describes various precautions he took so that "the water in the annulus between E and A is very nearly in the same condition it would be if E and A were infinitely long." This statement is a precursor of much current discussion in classical Couette flow concerning the influence of end conditions on the flow.

Mallock's apparatus was designed to operate with three different cylinder arrangements. In the first one, shown on the left side of figure 4, the outer cylinder E rotates and produces torque on the inner cylinder A. Another cylinder G surrounds E. The gap between E and G is filled with water, as is the inner cylinder A, and thermometers are placed in the water. The temperature in the annulus between A and E is taken to be the mean of the two thermometer readings. Air is trapped in a region at the bottom of A so that fluid torque is exerted only on the cylindrical wall of A. The short cylinder K is stationary. Mercury is placed between K and E in an attempt to produce end conditions with the same velocity distribution as occurs in the water being measured. The telescope T reads the displacement of the calibrated circular disk attached to the upper stem B, which supports cylinder A.

The upper right part of the figure shows an arrangement to rotate the inner cylinder and measure torque on a suspended outer cylinder. The arrangement shown at the lower right was used to repeat the experiments in which the outer cylinder rotates, using cylinders of different sizes. Overall, the cylinders are sizable, with diameters ranging from 15 to 20 cm and heights of about 25 cm. At the lower left of the figure is a paper recorder for data.

Mallock found that when the inner cylinder is rotating, the torque and angular velocity are not linearly related, and he concluded (incorrectly) that such a flow is always unstable. In retrospect we see that Mallock's lowest speed of rotation, about 2 rpm, is larger than the critical value calculated to produce Taylor vortices for the size of the cylinders he used. With the outer cylinder rotating, he found the flow to be stable at low rotation



Mallock's diagram of the apparatus he used in his pioneering investigations of the flow of fluids between concentric cylinders. (Redrawn from ref. 7.) **Figure 4**

rates and unstable at high rates.

Mallock's experiments were watched with great interest by Kelvin, who was thinking about stability theory at the time. In a 10 July 1895 letter to Rayleigh, Kelvin wrote:[8]

> On Saturday I saw a splendid illustration by Arnulph Mallock of our ideas regarding instability of water between two parallel planes, one kept moving and the other fixed. Coaxial cylinders, nearly enough planes for our illustration[, were used]. The rotation of the outer can was kept very accurately uniform at whatever speed the governor was set for, when left to itself. At one of the speeds he shewed me, the water came to a regular regime, *quite smooth*. I dipped a disturbing rod an inch or two down into the water and immediately the torque increased largely. *Smooth* regime would only be reestablished by slowing down and bringing up to speed again, gradually enough.
>
> Without the disturbing rod at all, I found that by resisting the outer can by hand somewhat suddenly, but not very much so, the torque increased suddenly and the motion became visibly turbulent at the lower speed and remained so.

It is worth noting here that in 1920, Rayleigh made the first step toward understanding the stability of the flow by calculating the stability in the absence of viscosity. He showed that the flow is stable provided the square of the angular momentum per unit mass of the fluid increases monotonically outward.[9] This means, in particular, that the motion with only the inner cylinder rotating is unstable, while the motion with only the outer cylinder rotating is stable to infinitesimal perturbations. Viscosity modifies these conclusions, a subject taken up by Taylor in 1923.

M. Maurice Couette

Couette was born in Tours, France, 9 January 1858 and was a professor at the university at Angers, France, when he died 18 August 1943. Little is known about his career. In Paris in 1888 Couette announced the first experiments with his viscometer.[10] His most important conclusion was that there are two forms of fluid motion, one given by exact integrals of the equations of motion and one, at higher speeds, that does not conform to the integrals of motion. Couette was aware of Osborne Reynolds's pioneering studies, published in 1883, on turbulence in flow through pipes. The circular geometry of the pipes, however, is a fundamentally different flow.

In 1890 Couette published his thesis, which was a lengthy study of viscosity using a pair of cylinders with the outer one rotating and the inner one suspended on a fiber to measure torque.[11] The paper also contained a study of the use of flow through tubes to determine viscosity. Today such rotating cylinder viscometers are known as Couette viscometers, even though Mallock's clearly was developed independently at about the same time.

Figure 5 shows a cross section of Couette's large and impressive apparatus. His inner cylinder s, suspended by a steel torsion fiber, had a radius of about 14 cm and a height of about 8 cm. Short guard cylinders g at each end of the suspended cylinder were fixed to a tripod M. The tripod rested on three heavy piers. A 2.5-mm gap separated the inner cylinder s from the outer cylinder v, which was rotated by means of a pulley. The base of the apparatus was a square of cast iron 50 cm on a side.

Small torques were measured by the deflection of the inner cylinder; larger torques were balanced by means of an Atwood's machine attached to a pulley on the suspension mechanism. Couette showed that the viscosity of water is apparently constant up to some critical rotation rate, which corresponds to a Reynolds number R_c of about 2000. (In modern notation $R_c = \omega rd/\nu$, where ω is the angular velocity of the cylinder, r the radius of the outer cylinder, d the gap between cylinders and ν the kinematic viscosity.) Couette used his instrument to measure the viscosity of air and reported a value of 179 micropoise at 20 °C. (Half a century later Joyce Alvin Bearden[12] obtained a value of 182 micropoise.) Couette was also able to record the onset of turbulence in air, in much the same way he did in the experiment with water.

Both Mallock and Couette were great instrument builders. Their skills enabled them to build some of the most precise instruments seen to that date. The lengthy description of the construction of Couette's viscometer is impressive even today. Couette was also a competent theorist: He was the first to consider the eccentric cylinder problem in an effort to estimate the errors in viscosity that would result from misplacement of the suspended cylinder.

Couette's name soon came to be associated with the flow that he studied. The well-known book *Hydrodynamics*, which is a reprint of a 1932 National Research Council report, shows that Mallock, Couette and Margules were cited in those days.[13] The literature gives frequent reference to "plane Couette flow" as the flow between two planes, with one in motion. This is also true in the gas dynamic literature.[14]

Geoffrey Ingram Taylor

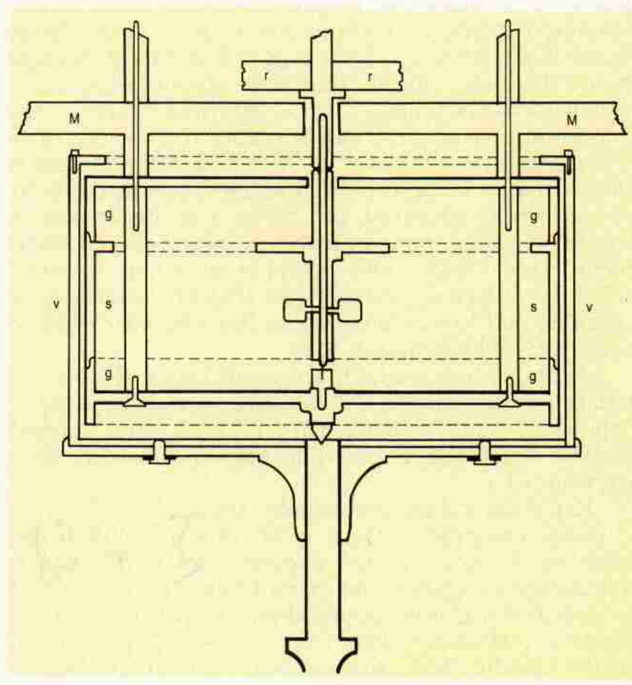
After these enterprising beginnings, the field became quiescent for almost 30 years, until Taylor (see figure 6) took up the problem. Taylor's 1923 paper contains an examination of linear stability theory for the general cases of viscous flow with both cylinders rotating in the same direction and in opposite directions.[15] Taylor's theoretical stability diagram for the flow was a *tour de force* considering the lack of computers, which are so much a part of today's research. His paper also contains an account of his experimental apparatus, which used ink visualization, and presents for the first time photographs and measurements of patterns in the unstable flow (see figure 1).

Taylor's paper, published in the *Philosophical Transactions of the Royal Society of London*, can fairly be called one of the most influential investigations of 20th-century physics. The correspondence that Taylor obtained between theory and experiment for the stability rested in an important way on the no-slip boundary condition for the flow at the solid surfaces. This success was taken by many as perhaps the most convincing proof of the correctness of the Navier–Stokes equations and of the no-slip boundary condition for the fluid at the cylinder walls. Such use of Taylor–Couette flow to confirm fundamental ideas in fluid dynamics has become a tradition. Most recently, the modern equations of motion for superfluid helium had the same success when Chris Swanson and I found[16] the temperature-dependent onset of Taylor vortices in helium II, which had been predicted by Chris Jones and Carlo Barenghi of the University of Newcastle upon Tyne, in England.

Taylor was one of the most influential figures of all time in fluid dynamics. A contribution by George Batchelor to a recent fluid mechanics symposium contains an excellent, illustrated account of Taylor's career.[17]

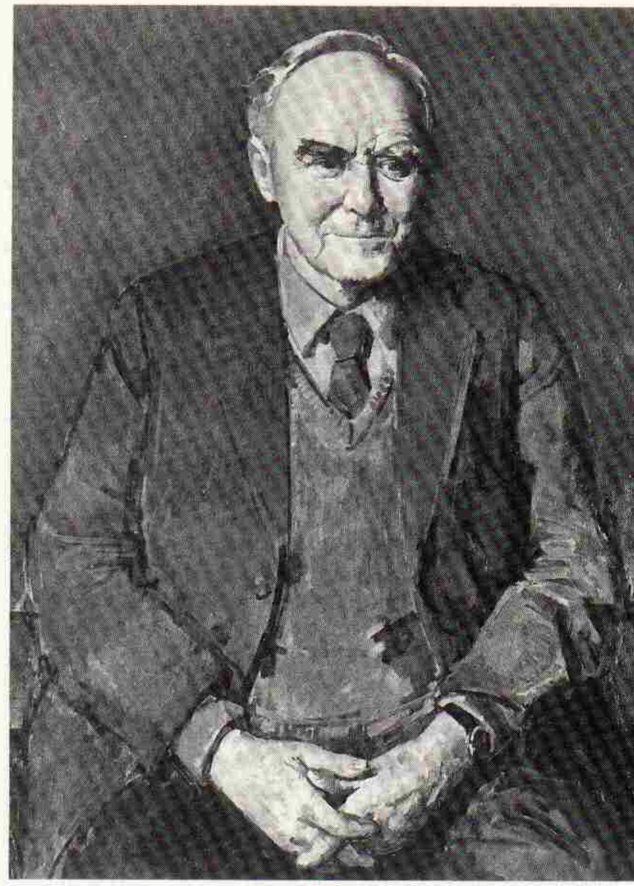
Subrahmanyan Chandrasekhar

In the 1950s the great astrophysicist Chandrasekhar (see figure 7) undertook a comprehensive study of hydrodynamic stability and, in his typical fashion, made many new contributions to the field. He synthesized what was known in a massive treatise called *Hydrodynamic and Hydromagnetic Stability*.[9] His book included basic discus-



Couette's cylinder scheme. This cross-sectional diagram of Couette's apparatus is taken from his 1890 thesis. The fixed cylinders g are guards, and s is the suspended cylinder. (Redrawn from ref. 11.) **Figure 5**

G. I. Taylor (1886–1975) in a 1966 portrait by Ruskin Spear. (Used with permission of the Master and Fellows of Trinity College, Cambridge.) **Figure 6**



sions of hydrodynamic stability and a major treatment of Rayleigh–Bénard convection and Taylor–Couette flow, in each case discussing the effect of a magnetic field if a conducting fluid were used. He addressed a number of generalizations of Taylor–Couette flow. Chandrasekhar's book brought our experimental and theoretical understanding of Taylor–Couette flow up to date and made possible the next generation of experiments and theories, though these did not follow until some years later.

Chandrasekhar, a good friend of Taylor's and now a Nobel laureate, continues his work at the University of Chicago. He also is the subject of a recent biography.[18]

Modern investigations

New experiments and theories have stressed the understanding of flows well beyond the onset of instability, where finite-amplitude flows, further bifurcations, chaos and turbulence occur with ever changing flow patterns. Visualization has allowed experimenters to gather a great deal of information about flow patterns. Various means have been used to achieve this. Taylor used ink ports to inject dye into the flow. The difficulty with this method is that after a while the fluid becomes too dark to use. Aluminum pigment powder from a paint store marks vortex flow with traces having good reflectance, because the particles align in the shear flow. Recently fish scales have been used the same way, as shown in figure 1. The flow patterns so obtained can be observed photoelectrically to produce time series of the flow, and this technique is now common. Time series and patterns can be correctly and sensitively observed, but there is no direct way to deduce velocities from Kalliroscope (fish scale) measurements. Laser Doppler velocimeter observations, however, tell us flow velocities at a point, but they are less useful for revealing patterns because of the large number of point measurements required to do so.

As Stokes imagined, the end conditions at the top and bottom are important. For the case in which the inner cylinder is rotating, fixed ends, rotating ends, tapered annulus ends and a mercury bottom have all been investigated.

Experimental protocol matters a great deal. The rate at which one approaches a given rate of rotation can influence the pattern that appears, and if the rate is approached too quickly, one can introduce hysteresis that is absent for slower approaches. A paper reporting dramatic instances of the nonuniqueness of patterns in Taylor–Couette flow was written in 1965 by Donald Coles.[19] In many respects this paper was the beginning of the modern era in our subject, even though some years were to pass before new activity picked up.[20]

An important characteristic of Taylor–Couette flow is the ratio of the radii of the cylinders. The aspect ratio is the ratio of the length of the cylinders to the gap between them. If the apparatus has a small aspect ratio, some chaotic flow may be observed; in apparatus of higher aspect ratio, turbulence is quite easily generated.

There are analogies to other flows in various limits. For example, the linear stability problem for rotation of the inner cylinder is closely analogous to the simple Rayleigh–Bénard problem. Flow between eccentric rotating cylinders is analogous in the narrow-gap limit to the geometry used in discussions of the hydrodynamics of journal bearings.

Variations on the flow first investigated by Mallock and Couette have been developed by many investigators over the years. Viscometers have been constructed, as we have seen, with either the inner or the outer cylinder rotating. Designs have been developed for ordinary fluids, for non-Newtonian fluids, for mercury in a magnetic field and for liquid helium.[16, 21] Torque measurements not only determine the viscosity of the fluid; they also can locate the onset of instability (by a sudden jump in apparent viscosity) and give information on the subsequent finite-amplitude flow. More generally, one can rotate the cylinders in the same or opposite directions, add axial or azimuthal flow and even introduce a radial temperature gradient between the cylinders.

External fields can also be applied to the fluid. For example, a conducting fluid such as mercury can be used in the presence of a magnetic field.[9] Recently at the University of Oregon, we have been studying the effects of Coriolis forces by placing the apparatus horizontally on a rotating table.[22] Generally, such external fields tend to stabilize the flow.

Modulated stability experiments can be done by varying the angular velocity of either cylinder in a time-dependent way.[23] Depending on which cylinder is modulated, the result can be either stabilizing or destabilizing.

None of these variations is trivial. In most cases

S. Chandrasekhar (b. 1910) in 1961, on the occasion of the publication of his book *Hydrodynamic and Hydromagnetic Stability*. He is sitting with the book in front of a large Taylor–Couette apparatus built by Dave Fultz and Russell Donnelly at the University of Chicago. (Photograph courtesy of Fultz.) **Figure 7**

simple analogies have failed to predict the experimental results, and careful experimental work has had to proceed in tandem with theoretical and numerical work.

Many concentric cylinder arrangements offer controlled stirring of solutions under predictable rates of shear. These are now finding applications in the preparation of solutions of biological materials. Spatial patterns in the flow evolve as the speed of rotation increases. The dynamics become very complicated, as do the conditions necessary to achieve a reproducible flow, because they depend on the past history in a special way.

When the outer cylinder is rotating, the fluid exhibits a direct transition to turbulence (skipping the patterned flows such as those in figure 1); this phenomenon merits far more attention than it has received. Interest in turbulent Taylor–Couette flow is reviving. Harry Swinney's group at the University of Texas, Austin, has recently been active in this area. Liquid helium will allow very high Reynolds numbers to be generated because of its low kinematic viscosity. Its use should greatly extend the range of Reynolds numbers that can be achieved in turbulence studies.

These and undoubtedly other phenomena and opportunities make the Taylor–Couette problem one of continuous interest and great intellectual breadth. I believe this breadth, as well as the stature of past and present investigators, attracts young people to the field. In addition there is the beauty of the flow, which can be produced with relatively modest resources. There is the challenge of trying to understand a very difficult nonlinear subject with endless variations. There is the constant interplay between theory and experiment. Newcomers with new ideas can usually find theorists and numerical analysts willing to collaborate with them.

It is now three centuries since Newton first considered rotational flow, one century since Mallock and Couette read their first papers on the subject and two-thirds of a century since Taylor's monumental paper. The pace of discovery is increasing, and I see no reason to believe that Taylor–Couette flow will cease to attract the attention of succeeding generations of physicists, mathematicians and engineers.

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