Navier: Blow-up and Collapse

Marco Cannone and Susan Friedlander

"All France knew of the disaster which happened in the heart of Paris to the first suspension bridge built by an engineer, a member of the Academy of Sciences; a melancholy collapse caused by blunders such as none of the ancient engineers—the man who cut the canal at Briare in Henri IV's time, or the monk who built the Pont Royal—would have made; but our administration consoled its engineer for his blunder by making him a member of the Council-general."

—Honoré de Balzac, from Le Curé de Village*, 1841*

In the 1820s Claude Louis Marie Henri Navier[1] was a professor at the École des Ponts et Chaussées in Paris, which was the Grande École that trained engineers in nineteenth-century France. At the same time as publishing his famous fluid equations (now known as the Navier-Stokes equations), he designed the first monumental suspension bridge to be built in Paris over the Seine. His bridge developed a crack just before it was to open, and political battles resulted in the removal of the bridge. Accusations were made that Navier was "too much of a theoretical mathematician" and not "practical" like the British bridge builders such as Brunel or Stevenson. This debate was a version of a more general dispute between the French and British approaches to mathematics, physics, and engineering. The different national approaches to science were in fact also reflected by the way Navier (in 1822) and Stokes (in 1845) derived their eponymous equations.

The word "blow-up" of the solutions for the Navier-Stokes equations is familiar to at least a subset of mathematicians, most of whom probably do not know of Navier's bridge. In 1824–6, however, Navier was best known for a two-volume treatise on bridges [13] and the impressive design for the Pont des Invalides over the Seine (Figure 1).

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[1]*Born in Dijon in 1785, he died in Paris in 1836.*

The controversy that followed the collapse of this bridge received wide coverage in the French press of the time, had repercussions for the Parisian world of finance, and was detrimental to Navier's reputation.

The Navier-Stokes Equations

The concept of "blow-up" for the Navier-Stokes equations has received considerable publicity recently in the context of one of the "million dollar" prize problems offered by the Clay Mathematics Institute. Briefly stated, an important problem in fluid dynamics is to answer the following question: In three dimensions does the velocity field of a fluid flow that starts smooth remain smooth for all time as the field evolves under the Navier-Stokes equations? A physical quantity such as the velocity, satisfying realistic boundary conditions, conceivably could develop a singularity in finite time, and this phenomenon is referred to as "blow-up". The partial differential equations known as the Navier-Stokes equations have proved to be among the most challenging to mathematicians of all the partial differential equations that arise from physics. More details can be found, for example, in the Clay prize description by Fefferman [5].

The first mathematical description of the motion of an "ideal" fluid was formulated by Euler [4] in 1755 as a statement of Newton's second law of motion applied to a fluid moving under an internal force known as the pressure gradient. The Euler equations governing the time evolution of the velocity vector field $\boldsymbol{v}(\boldsymbol{x}(t), t)$ and the (scalar) pressure $p(\boldsymbol{x}(t), t)$ of an incompressible fluid have the form

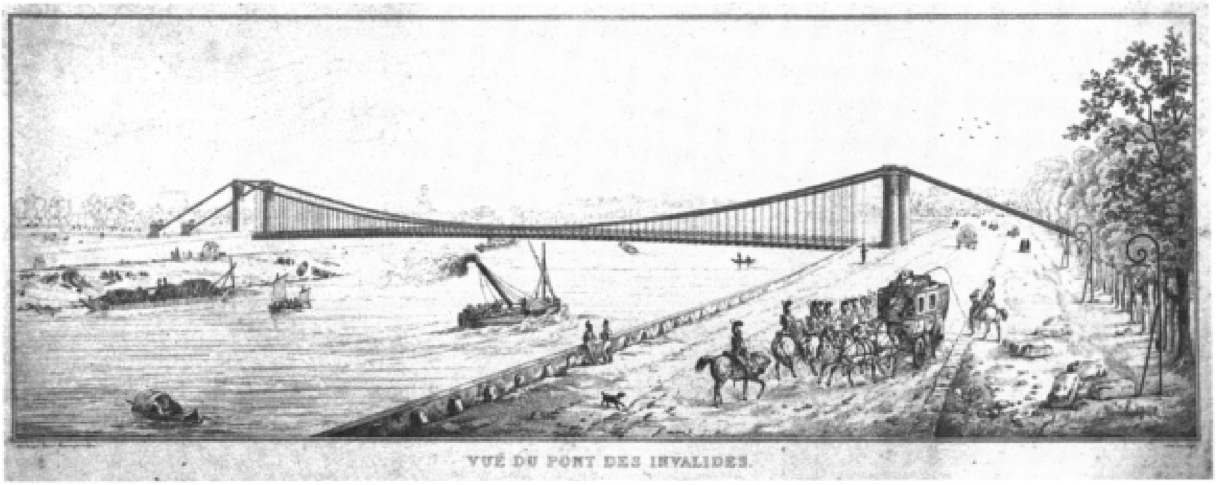


Figure 1. Pont des Invalides, taken from the original drawing by Navier in [13].

$$(1)\quad \begin{cases} \frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\,\boldsymbol{v} &= -\nabla p \quad (\boldsymbol{x}\in\mathbb{R}^n, t>0) \\ \nabla\cdot\boldsymbol{v} &= 0 \end{cases}$$

with initial condition

$$\boldsymbol{v}(\boldsymbol{x},0) = \boldsymbol{v}_0(\boldsymbol{x}),$$

where $\boldsymbol{v}_0(\boldsymbol{x})$ is a given divergence-free vector field. Here we restrict our attention to incompressible fluids filling all of $\mathbb{R}^n$, where n is the space dimension, which we take to be 2 or 3. These equations, while important theoretically, omit the effects of friction and bring about, as pointed out by D'Alembert, "a singular paradox which I leave to geometricians to explain" [1]. To incorporate friction, the French mathematician-engineer Navier (Figure 2) published in 1822 a paper [12] with the derivation of the equations of motion for a viscous fluid in which he included the effects of attraction and repulsion between neighboring molecules. From purely theoretical considerations he derived the following modification of the Euler equations:

$$(2)\quad \begin{cases} \frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\,\boldsymbol{v} &= \epsilon\Delta\boldsymbol{v} - \nabla p \quad (\boldsymbol{x}\in\mathbb{R}^n,\ t>0) \\ \nabla\cdot\boldsymbol{v} &= 0, \end{cases}$$

again with initial condition $\boldsymbol{v}(\boldsymbol{x},0) = \boldsymbol{v}_0(\boldsymbol{x})$ for a given divergence-free vector field $\boldsymbol{v}_0(\boldsymbol{x})$ in $\mathbb{R}^n$.

For Navier, ϵ was simply a function of the molecular spacing to which he attached no particular physical significance. His seminal paper [12] was presented at the French Académie des Sciences and was well received. He was elected a member of the Académie in the mechanics section in January 1824.

The equations for the motion of a viscous fluid were rederived by Cauchy in 1828 and by Poisson in 1829. In 1843 Barré de Saint-Venant published a derivation of the equations on a more physical basis that applied not only to the so-called laminar flows considered by Navier but also to turbulent flows. However, the person whose name is now attached with Navier's to the viscous equations is the British mathematician-physicist George Gabriel Stokes[2] (Figure 3). In 1845 he published a derivation of the viscous equations in a manner that is followed in most current texts. Unlike Navier, he made it clear that the parameter ϵ has an important physical meaning: namely, ϵ measures the magnitude of the viscosity (i.e., the friction of the fluid). Interesting details of the history of the fluid equations can be found in the books of Grattan-Guinness [8] and of Rouse and Ince [15].

Since the Navier-Stokes equations incorporate effects of friction, they are physically more realistic than the Euler equations. However, both systems of equations are important for physical and mathematical reasons. For example, Constantin [3] suggests that it is finite-time blow-up in the Euler equations that is the physically more important problem, since blow-up requires large gradients in the limit of zero viscosity (ϵ goes to zero). As Fefferman [5] remarks, finite-time blow-up in the Euler equations is an open and challenging mathematical problem, just as it is for the Navier-Stokes equations.

Two versus Three Dimensions

In this brief article we can give no details about the complexity of the systems of fluid equations (1) and (2), but we will make a few observations that may indicate a little about the challenges the systems pose to mathematicians. The Euler and the Navier-Stokes equations are nonlinear with the same nonlinear term, $(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}$. Exactly this "amount" of nonlinearity appears to be particularly subtle and could imply blow-up in finite time. A question intimately related to the possible loss of regularity of the solutions is given by the possible loss of their uniqueness. The solutions to the Euler and the Navier-Stokes equations are known to be locally regular

[2] *Born in Skreen, County Sligo, Ireland, in 1819, he died in Cambridge in 1903.*

and unique in time, but at the instant T when they cease to be regular (if such an instant exists) the uniqueness could also be lost. The following simple example illustrates blow-up and loss of uniqueness in an equation with a nonlinearity.

Consider the ordinary differential equation

$$\begin{cases} \frac{dy}{dt} &= y^{\alpha} \\ y(0) &= y_0 \end{cases} \tag{3}$$

for different values of α and y_0. When the nonlinearity is quadratic ($\alpha = 2$) and the initial condition is $y_0 = 1$, the solution is $y = 1/(1-t)$, which blows up at $t = 1$. On the other hand, if $\alpha = 1/2$ and $y_0 = 0$, then this differential equation has infinitely many regular solutions, $y_C = ((t-C)/2)^2$ for $t \geq C$ and $y_C = 0$ otherwise, C being an arbitrary constant such that $0 \leq C \leq \infty$ (with the convention that $y_\infty \equiv 0$). Finally, in the case $\alpha = 1/2$ and $y_0 = 1$, the differential equation has a regular solution, $y = ((t+2)/2)^2$ for $t \geq -2$ and $y = 0$ otherwise, that is unique and that exists for all time.

The effects of nonlinearity in the fluid equations are strikingly different in two dimensions ($2D$) and in three dimensions ($3D$). In fact, existence and uniqueness of regular solutions for all time for the $2D$ Navier-Stokes equations are classical results proved in 1933 by Jean Leray [7], whereas the analog in $3D$ is a Clay prize problem. One crucial difference between $2D$ and $3D$ is the constraint that equations (1) and (2) impose in $2D$ on the evolution of the vorticity, an important physical attribute of fluid motion. The vorticity, which we denote by $\boldsymbol{\omega}(\boldsymbol{x})$, is $\nabla \times \boldsymbol{v}$. Taking the curl of (1) and (2) gives the equations for the evolution of the vorticity in an inviscid fluid:

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla)\boldsymbol{v} \tag{4}$$

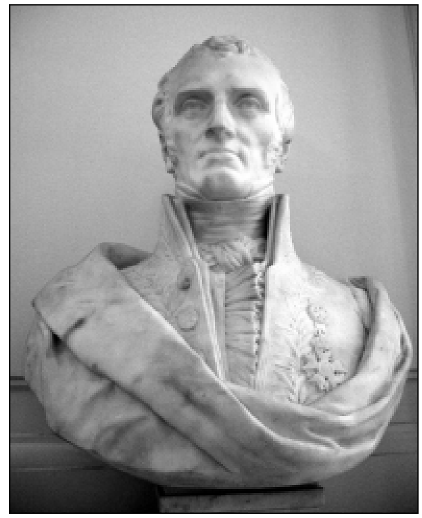
and in a viscous fluid:

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla)\boldsymbol{v} + \epsilon \Delta \boldsymbol{\omega}. \tag{5}$$

In $2D$ the vorticity is a scalar field multiplied by a unit vector perpendicular to the $2D$ plane of motion. Hence the term $(\boldsymbol{\omega} \cdot \nabla)\boldsymbol{v}$ vanishes in $2D$, and although (4) and (5) remain nonlinear, they are significantly simpler than the $3D$ equations. In $2D$ equation (4) becomes

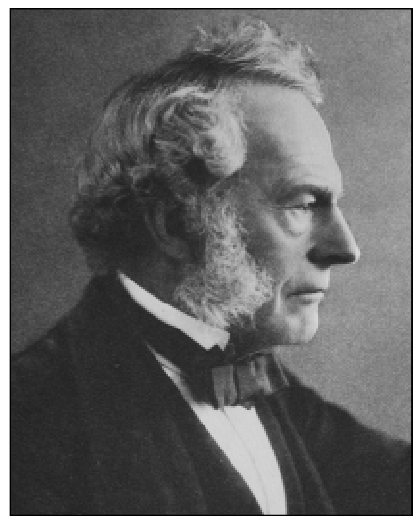
$$\frac{\partial \boldsymbol{\omega}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{\omega} \equiv \frac{d\boldsymbol{\omega}}{dt} = \mathbf{0}. \tag{6}$$

Thus in $2D$ the vorticity is a scalar quantity that is conserved along the trajectories of the fluid particles. Conservation of vorticity is a strong constraint on the complexity of the motions governed by the Euler equations. Only a weaker constraint known as Kelvin's circulation theorem exists for $3D$ flows where the term $(\boldsymbol{\omega} \cdot \nabla)\boldsymbol{v}$ does not vanish and may perhaps be instrumental in creating blow-up for the Euler equations.



Photograph by the authors.

Figure 2. Bust of Claude Louis Marie Henri Navier, from the collection at the École Nationale des Ponts et Chaussées.



Photograph from *Mathematical and Physical Papers*, 1883-1910.

Figure 3. George Gabriel Stokes.

An important advance in the theory of partial differential equations was the concept of "weak" solutions introduced by Leray [7], particularly for the Navier-Stokes equations. This permits objects in much larger classes than the space of classical functions to be used to describe the motion of a fluid. It is easier to prove existence of a solution (regular or singular) in a larger class, but such a solution may not be unique! Leray's theory gives the existence of weak, possibly irregular, and possibly nonunique solutions to the Navier-Stokes equations. His approach is based on so-called energy estimates (i.e., bounds on the integral of the square of the velocity) and thus requires the initial data to be in $L^2(\mathbb{R}^n)$ in n dimensions. On the other hand, a completely different approach based on semigroup theory was introduced by Tosio Kato [9] and provides the existence of a global unique regular solution under the restrictive assumption of small initial data. Kato's method is based on scaling-invariance arguments related to the fractal geometric nature of the equations. This theory requires the initial data to be in $L^n(\mathbb{R}^n)$, because this is the only Lebesgue space $L^p(\mathbb{R}^n)$ that is invariant under the appropriate scaling, i.e.,

$$\|f(\boldsymbol{x})\|_{L^p(\mathbb{R}^n)} = \|\lambda f(\lambda \boldsymbol{x})\|_{L^p(\mathbb{R}^n)}.$$

Hence the function spaces for Leray's theory and for Kato's theory coincide when $n = 2$, but not when $n = 3$. Thus in $2D$ the theories complement each other, and therefore the solution to the Navier-Stokes equations with initial data in $L^2(\mathbb{R}^2)$ is regular and unique. In $3D$, however, the problem remains mysterious!

"De l'Entreprise du Pont des Invalides"

We now turn away from the aspects of Navier's work that are most familiar to the mathematical community and start our discussion about the background and events related to Navier the engineer and his ill-fated bridge.

Navier at the École Nationale des Ponts et Chaussées

The beginning of Navier's student career was not stellar. In a typically French procedure, there is a linear ordering of the list of admission at the École Polytechnique, and in 1802–3 Navier was placed 116th out of 117 in the order of merit! Navier's career improved dramatically, however, and by the end of his first year he was one of the top ten students. After a couple of years he joined the Corps at the École des Ponts et Chaussées in Paris, where his great-uncle Emiland Gauthey worked as one of the leading civil engineers in France. During Navier's studies at the École Polytechnique he was taught by and came under the influence of Fourier. Throughout his career Navier was a notable proponent of the important mathematical techniques developed by Fourier. This was not the case for most other engineers of this period. Furthermore, the textbooks that Navier wrote for practicing engineers introduced the basic principles of engineering science to a field that previously had been almost completely empirical.

The main areas of Navier's work concerned hydrodynamics, elasticity theory, and the design and construction of bridges. In 1819 he was teaching mechanics at the École des Ponts et Chaussées, working as a practical engineer, and carrying out theoretical research. His seminal derivation of the viscous fluid equations, written in the early 1820s, was work that he kept somewhat secret from his chief, Becquey, who considered Navier's primary task to be the design of bridges. Even today it seems that there is a tendency in France, not only at the Grandes Écoles, to classify scientists as either theoretical or practical and to discourage the melding of the two attributes. Navier was able to counter this tendency and achieved a synthesis of the practical and the theoretical. He obtained distinction and recognition both from scientists of the French Académie des Sciences and from engineers. As we will describe, however, his theoretical achievements were held against him in the "affair of the collapsing bridge".

The Birth of Modern Suspension Bridges

Although suspension bridges date to antiquity in several ancient cultures,[3] the "modern" iron chain suspension bridge sees its direct forerunner in the constructions of James Finley, an American inventor and builder. Finley was a skilled engineer who worked in an environment very different from that of Navier at a Grande École, namely rural America in 1800. Finley was practical and empirical, successfully seeking to produce workable, simple, and generic designs that could be constructed by rural blacksmiths. An indication of his success is that by 1820 he had a number of patents, and over forty chain suspension bridges had been built in the U.S. The innovations of Finley were taken up and extended in Britain by Samuel Brown, a retired naval captain, who determined through experiments the most efficient shape for the iron links in a suspension bridge cable. He became a leading builder of suspension bridges, including in 1820 the Union Bridge over the river Tweed, whose span of 436 feet was nearly twice the longest span of any bridge that had been built in the U.S. The Union Bridge was shortly followed by other engineering feats in Britain, including the bridges built by such pioneers of the industrial revolution as Brunel, Stevenson, and Telford. Many more details concerning the history of the suspension bridge can be found in the book of Kranakis [10] and the paper of Picon [14].

Navier "Mathematicising" the Topic

When the French government called upon the École des Ponts et Chaussées to assist in the development of suspension bridge technology, it was natural that they looked to Britain. At the request of his superiors, Navier paid two visits to Britain in 1821 and 1823 to study suspension bridges.[4] His investigations resulted in a major book [13] that was published in 1823. In this book Navier brought to the subject for the first time the analytical and abstract approach of the mathematician. He sought to illustrate the power and challenge of abstraction versus an empirical ad hoc approach. He did what applied mathematicians today seek so often to do in constructing a mathematical model. As he remarked, modeling requires "a particular art which consists of replacing the very questions to be resolved by other questions that differ as little as possible and to which mathematics may apply." Navier suggested that results be reformulated so as to specify the theoretical limits within which they should be relevant and that mathematical analysis be used to determine the relationship between important parameters.

The main issues confronting a bridge designer were the equilibrium shape that could be achieved by a balance of the forces acting on components of the bridge and the stability of such an equilibrium to perturbations. Significant sources of such perturbations include traffic over the bridge, thermal expansion/contraction from solar heating, wind-driven oscillations, and forces due to the flow of water

[3] *Navier himself gave charming illustrations of antique bridges in his book [13].*

[4] *Between his visits Navier derived the Navier-Stokes equations.*

in the river. The expertise that Navier brought to these issues was considerable. His book on bridges contained well-known calculations needed to describe the equilibrium of chains and are based on simple differential equations whose approximate solutions are either catenary or parabolic. More sophisticated and original mathematics in his work included the use of Fourier's series solutions. For example, this occurs in the context of the displacement from equilibrium of a perfectly flexible chain (the so-called vibrating string problem), which is governed by the wave equation. Other typical mathematical arguments that can be found in Navier's book are connected with elasticity theory. There was a considerable similarity between the equations describing the distribution of the load in chains and those studied by Navier in his pioneering work on curved elastic rods and elastic rectangles. Again, the primary mathematical tools used by Navier were Fourier series. Cauchy, who is credited as the founder of modern elasticity theory, acknowledged that his research in this field had been inspired by a memoir of Navier published in 1820.

One very important aspect of suspension bridges is their susceptibility to destruction by drastic wind-induced oscillations. Such destruction occurred in Navier's time and also more recently (for example, the spectacular collapse of the Tacoma Narrows bridge in 1940). This topic, however, received little attention from Navier, possibly because it was too difficult to analyze with the tools he had available. In fact, it remains to this day a very subtle problem involving resonant nonlinear oscillations and turbulence in the wind, a problem that challenges the expertise of modern fluid dynamics.

The Collapse of Navier's Dream: Politics and Controversies

Not only did Navier present the theory of the suspension bridge in his major treatise [13], he also put forward a design for a monumental suspension bridge to be built across the Seine, connecting the Hôtel des Invalides (Napoleon's tomb) with the Champs Elysées. This bridge had a span of 155 meters and incorporated all of Navier's theory and knowledge acquired from his studies in Britain. It was to be a state-of-the-art achievement in both technology and artistic design, with a fashionable Egyptian[5] motif. The bridge was to be called the Pont des Invalides.[6] In choosing this site for his bridge, Navier was not motivated by practical issues: in fact, there was no great need for a bridge in this position, as there was in the east of Paris. Rather, he considered the grandeur and esthetics of a construction that would add to the "gloire" of France and the Corps des Ponts et Chaussées. He wished to demonstrate both that a bridge of beauty could be made from iron (rather than stone) and that it could be made following the dictates of theoretical analysis. To illustrate this, we juxtapose two quotations of Navier (from the translations in [10]):

> There exists no urgent necessity to construct a bridge to the Champs Elysées: there is no obligation to build a suspension bridge in Paris. But if it is desired that one be built, let it be made into a monument; let the character of grandeur be given to this work that the style of the construction admits of; let its disposition be calculated with the idea of forming an edifice approved by artists, agreeable to the public, and honorable to the administration.

> …this study (of suspension bridges) would not have been possible without the progress made in mathematical analysis in recent times, and without the institutions by means of which those charged with the direction of public works are initiated into the most advanced ideas of mathematics.

In 1823 Navier presented meticulous plans to his superiors in the Corps des Ponts et Chaussées. Every detail was designed on the basis of a theoretical analysis in which Navier had sufficient confidence that he did not resort to the usual engineering practice of "overbuilding" (i.e., deciding what was needed in terms of strength and then multiplying the result by a number considerably greater than one). A committee of experts reported very favorably on Navier's project, convinced that "theory everywhere illuminates practice." They decided that because of its novel and nonessential nature, the bridge should be privately rather than publicly funded. Investors were sought in a company that would finance the building of the bridge in return for the rights to collect all the tolls for fifty-five years. Final approval was given by Becquey and the minister of the interior, the project was

[5] *In 1822 Navier derived the Navier-Stokes equations, Fourier published his fundamental research on the heat equation, and Champollion deciphered the Egyptian hieroglyphics.*

[6] *Today the site is occupied by Pont Alexandre III, constructed in 1890, whereas Pont des Invalides is the name given to the next bridge downstream, a stone arch constructed in 1850. Beautiful pictures and descriptions of the bridges of Paris can be found in [11].*

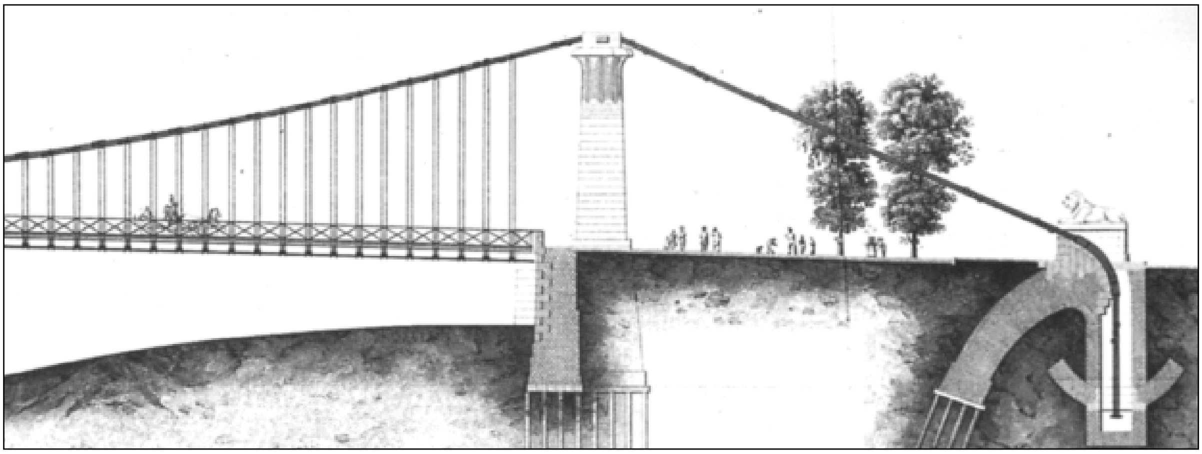


Figure 4. A portion of the bridge, as designed by Navier, showing the anchorage (see [13]).

open to bidding in April 1824, and in August building began.

Progress on the construction was good, and there was much general interest in this novel edifice connecting the two most fashionable districts in Paris. By September 1826 the bridge was almost complete. As Grattan-Guinness [8] reports, the *Moniteur Universel* told its readers: "…there is still about 5 weeks of work. But soon, stripped of its scaffolding, it will be possible to appreciate it quite complete, and we do not doubt that this first sight will sharply excite the interest of Parisians." However, just as this news bulletin was to appear, disaster struck. On the night of 6–7 September 1826 the buttress of the bridge in the right bank "cracked" when the surrounding earth was flooded due to a broken water pipe from a nearby pumping station. Now the strength of these buttresses in Navier's design had in fact already been questioned by the committee of experts. Even in July when the roadway was attached to the suspended chains, small cracks had appeared in the anchorages where the cables changed direction to descend vertically to their anchors (Figure 4). It was agreed that the buttresses would have to be strengthened. After the flooding this became imperative.

At first it was assumed that repairs would be carried out and that the bridge would be completed, but politics and financial issues intervened. A dispute developed between the Corps and the contractor over who had financial responsibility for the repairs. A financier threatened to sue the Corps. Sarcastic articles appeared in the press against the Corps, and Navier was referred to as "that eminent man of science whose calculations fail in Paris." The Paris City Council, which for various political reasons had opposed the project from the beginning, seized the opportunity to attack it. The accident caused panic among the investors. The principal financial backer died, and no one could be found to replace him. Navier pleaded in public and in private to complete his project. But he was unsuccessful, and after many acrimonious discussions and accusations the bridge was eventually dismantled.

A government committee was formed to investigate the technical and financial ramifications of the accident to the Pont des Invalides. Criticism was levied at the Corps for being too attached to theory and for being too autocratic and elitist, as compared to the British and American engineering establishments. It was suggested that the chains must have been too heavy for the buttresses if they could not withstand a "slight accident". In contrast, the superior achievements of British bridge builders were praised for not "mathematicising" the problem.[7]

Navier wrote numerous letters and a report,[8] vigorously defending all engineering aspects of the design. However, he became the scapegoat for the public relations disaster for the Corps, which later (unfairly) passed him over for promotion.

The most comprehensive recent book on Navier's bridge is that of Kranakis [10]. In this excellent work she examines Navier's design for the buttresses and asks why Navier did not see a need for extra stonework or earthwork, despite the fact that each cable would bear a huge tension of over 1,000,000 pounds. Navier calculated the resultant of a sum of forces exerted by the cables and concluded that a comparatively slim buttress positioned at exactly the correct point could provide the necessary

[7] *"In all other countries, in Germany, England, Italy, where institutions like ours do not exist, works of this character are better done and far less costly than in France. Those three nations are remarkable for new and useful inventions in this line. I know it is the fashion to say, in speaking of our Écoles, that all Europe envies them; but for the last fifteen years Europe, which closely observes us, has not established others like them. England, that clever calculator, has better schools among her working population, from which come practical men who show their genius the moment they rise from practice to theory." H. de Balzac [2].*

[8] *This report appears as an appendix to the second edition of Navier's book on suspension bridges [13].*

resistance. However, even though his design was novel and untried, Navier did not appear to have tested it on a scale model, nor did he "overbuild" to compensate for possible error. His own statements suggest that this was rash. In particular, he noted that the resistance of the earth could only be calculated accurately for the vertical forces. For the horizontal forces the resistance depended significantly on the cohesion of the earth, "the evaluation of which is subject to great uncertainty."[9] Hindsight suggests that Navier's concept, in which the cables descend vertically to the anchorage, was probably workable (for example, this was successfully used in the Brooklyn Bridge). However, his implementation in the design for the Pont des Invalides may indeed have relied on a theoretical model that approximated reality but with insufficient accuracy for what was demanded of it.

Although the affair of the Pont des Invalides was a major setback in Navier's career and must have caused him personal distress, he continued to be a prominent scientist consulted by the French government on issues of science and technology. In 1831 he became a Chevalier de la Legion d'Honneur. He was a man of strong political views, following an ideology based on society taking advantage of science and technology. After a lifetime of what must have been very hard work producing remarkable and diverse achievements, he died at the relatively early age of fifty-one.

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Photographic Credit

Pont des Invalides (DG 750 (D.12.320), fol 4684 (planche XII), 4684 (page de titre)). All the original records and plans of Navier's bridge are at the École des Ponts et Chaussées, which moved from the original building at Rue des Saints-Pères in Paris to the campus at Champs-sur-Marne, where the first author's Université de Marne-la-Vallée is located.

[9] *This is particularly ironic because the movement of a liquid-particle suspension (e.g., the flooded earth) receives attention from modern fluid dynamicists, who use the Navier-Stokes equations with modifications of Navier's frictional term.*

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