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A non-oscillating solution technique for skew upwind and QUICK-family schemes

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Abstract. An assessment of implementing an oscillation free FRAM algorithm on the flow-oriented skew upwind and QUICK schemes is made in this study for convection dominated flow problem. The monotonically bounded FARM method, along with the attractive properties of flow-oriented QUICK and skew upwind schemes, can capture the nearly discontinuous phenomena in the flow field. The candidates of QUICK-family include original QUICK, QUICK for two dimensions (QUICK-2D), and QUICK with Estimated Streaming Terms (QUICKEST). The candidates in skew family include Skew Upwind Differencing (SUD), Skew Upwind Differencing with artificial Cutoff (SUD-Cut), Volume-Weighted Skew Upwind Differencing (VWSUD), Corner Skew Upwind Differencing (CSUD), and Directional Transportive Upwind Differencing (DTUD) schemes. The performance of filtering methodology on the investigated schemes is evaluated by testing the following three problems: (1) Pure convection of an obilque-step profile, (2) Transport of a cosine hill or rectangular block in a rotating flowfield, (3) Backward-facing step and driven cavity problems. The best schemes among the investigated two families are QUICKEST and DTUD respectively.

1 Introduction

Numerical prediction of hydrodynamic problems containing thin layer, species density jump, temperature discontinuity, and shock wave by a high-order scheme for convective terms may corrupt the flow with unphysical oscillations to a large extent. The lack of positive influence coefficient accounts for the spurious oscillations even for smooth flowfield.

Most upwind and hybrid schemes (Sheu 1989) for convection-diffusion problem suffer from severe false diffusion when flow direction is at a finite angle to the grid lines. These diffusion errors are usually produced from the local one-dimensional treatment for multi-dimensional problems. The resulting discretization error tends to augment the transport of dependent variables in the direction normal to the local streamline. The improvement on that subject, therefore, can be made by employing a flow-oriented scheme such that the real physics is not masked. The major approaches in the context of finite difference method for reducing false diffusion error include QUICK (Quadratic Upstream Interpolation for Convective Kinematics) scheme of Leonard (1979), SKEW upwind scheme of Raithby (1976), and LOAD (LOcally Analytic Differencing) scheme of Wong and Raithby (1979). The implementation of upwinding discretization on convective terms along the flow direction, unfortunately, cannot prevent the appearance of oscillations near the neighbor of discontinuity. None of the above schemes are entirely satisfactory as far as reduction of false diffusion error, and suppression of numerical oscillations in the cases of high Peclet and discontinuacs flows are concerned.

The elimination of oscillations near the discontinuities can be made by simply adding an artificial viscosity in an ad hoc manner. This approach may degrade the solution quality in terms of accuracy. Extensive studies have been made recently for obtaining non-oscillatory high resolution solutions for flow field containing discontinuities. FRAM (Filtering Remedy And Methodology) of Chapman (1981), SMART (Sharp and Monotonic Algorithm for Realistic Transport) of Gaskell and Lau (1988), and SHARP (Simple High-Accuracy Resolution Program) by Leonard (1988) have been successfully applied to suppress the discontinuous-oriented oscillations.

The present study aims to incorporate FRAM technique to MAC solution algorithm (1965) to evaluate its effectiveness of suppressing oscillations near the neighbor of discontinuities. The employed QUICK-family schemes for discretizing flux terms include original QUICK, QUICK-2D, and QUICKEST (Ou Yang 1990). The investigated skew upwind family includes Skew Upwind Differencing (SUD) scheme, Skew Upwind Differencing with artificial Cutoff (SUD-Cut) scheme (Miao et al. 1984), Volume-Weighted Skew Upwind Differencing (VWSUD) scheme (Miao et al. 1984), Corner Skew Upwind Differencing (CSUD) scheme (Chiou 1990), and Directional Transportive Upwind Differencing (DTUD) scheme (Eraslan et al. 1983).

2 MAC solution algorithm

The governing equations for incompressible laminar flows can be expressed by the following compact form:

$$\phi_t + (u\phi)_x + (v\phi)_y = \Gamma(\phi_{xx} + \phi_{yy}) + S^\phi, \tag{1}$$

where ϕ

	ϕ	Γ	S^ϕ
continuity	1	0	0
x-momentum	u	ν	$-p_x$
y-momentum	v	ν	$-p_y$

The momentum equations in Eq. (1) can be discretized to the following form:

$$\phi_{i,j}^{n+1} = \phi_{i,j}^n + \Delta t(S^\phi - F\phi X - F\phi Y - VIS\phi), \tag{2}$$

where u, v are velocity components in x, y directions, t is the time, p is the ratio of pressure to constant density, ν is the kinematic viscosity. Other variables in Eq. (2) are defined as follows:

$$F\phi X = (\phi u)_x, \quad VIS\phi = \phi_{xx} + \phi_{yy}$$

$$F\phi Y = (\phi v)_y, \qquad S^i = -\frac{\partial p}{\partial i} \quad (i = x, y).$$

The dependent variables are stored on the staggered mesh that possible pressure oscillations can be avoided. The staggered arrangements on vector quantities u, v and scalar quantity result in different cells shown in Fig. 1.

At each time step, the new velocities u, v on the entire mesh are computed explicitly from x, y momentum equations using the previous computed pressure and velocities. These velocities are then improved in an iterative manner by adjusting the cell pressure to satisfy the mass conservation. As dependent variables satisfy mass as well as momentum equations, the computed pressure and velocities are taken as the initial values for computing the solutions at next time step.

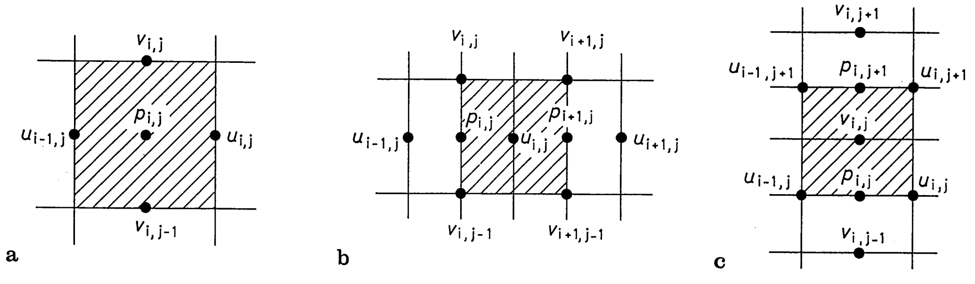

Fig. 1 a–c. Staggered cells in MAC formulation

The velocities on the no-slip, symmetric, inflow and outflow boundaries are imposed after using Eq. (2). The specifications of them are not required after each pass during the pressure iteration. After each real time step, boundary conditions for scalar quantities are imposed. The allowable time steps for advancing MAC computation are restricted by the heuristic stability analysis (Harlow et al. 1965).

3 Investigated skew upwind schemes

The source terms S^{ϕ} and viscous terms $VIS\phi$ are approximated by central differencing scheme. The advection terms $F\phi X$, $F\phi Y$ in transport Eq. (2) are approximated by the appropriate interpolated values of velocity components ϕLW, ϕRW, ϕBW, ϕTW at some upstream locations. The general expressions for convective terms are given by the following forms:

$$F\phi X = \frac{1}{\Delta x}(uR\cdot\phi RW - uL\cdot\phi LW), \quad F\phi Y = \frac{1}{\Delta y}(vT\cdot\phi TW - vB\cdot\phi BW),$$

where

$$FUX: uL = \tfrac{1}{2}(u_{i,j} + u_{i-1,j}), \qquad uR = \tfrac{1}{2}(u_{i,j} + u_{i+1,j});$$
$$FUY: vB = \tfrac{1}{2}(v_{i,j} + v_{i+1,j-1}), \qquad vT = \tfrac{1}{2}(v_{i,j} + v_{i+1,j});$$
$$FVX: uL = \tfrac{1}{2}(u_{i-1,j} + u_{i-1,j+1}), \qquad uR = \tfrac{1}{2}(u_{i,j} + u_{i,j+1});$$
$$FVY: vB = \tfrac{1}{2}(v_{i,j} + v_{i,j-1}), \qquad vT = \tfrac{1}{2}(v_{i,j} + v_{i,j+1}).$$

The investigated discretization schemes for convective terms are distinguished themselves by the expressions of ϕLW, ϕRW, ϕBW, ϕTW.

3.1 Skew upwind differencing (SUD) scheme

Skew upwind differencing scheme (Raithby 1976) was specifically designed to reduce numerical diffusion as the flow direction is oblique to the grid system. The reduction of artificial damping is accomplished by considering the direction of velocity at the cell of interest. The interpolated

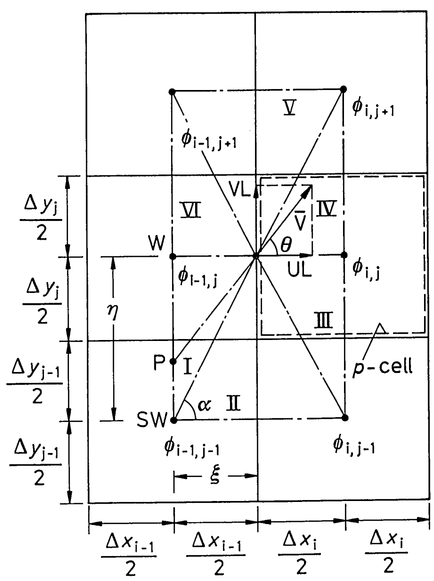

Fig. 2. Illustration of SUD scheme

convective quantity depends on the value at P which is determined by two appropriate nodes, W and SW for example, in Fig. 2. The location of P is at the intersection of WSW and a line with slope of velocity vector passing throught W.

There exist six possible pairs of nodes, between which convective quantity will be interpolated, as far as the direction of velocity at the cell face is considered. The interpolation expressions for convective terms are as follows:

$$\phi LW = \begin{cases} \dfrac{(\eta-\delta)\cdot\phi_{i-1+ia,j} + \delta\cdot\phi_{i-1+ia,j-js}}{\eta} & \alpha \geqq \theta \\[2ex] \dfrac{\left(\dfrac{\Delta\chi_i}{2}+\zeta\right)\cdot\phi_{i-1,j-js} + \left(\dfrac{\Delta\chi_{i-1}}{2}-\zeta\right)\cdot\phi_{i,j-js}}{\left(\dfrac{\Delta\chi_{i-1}+\Delta\chi_i}{2}\right)} & \alpha < \theta, \end{cases}$$

where

$$is = \frac{uL}{|uL|}, \qquad ia = \frac{1-is}{2},$$

$$js = \frac{vL}{|vL|}, \qquad ja = \frac{1-js}{2},$$

$$\xi = \frac{\Delta x_{i-1+ia}}{2}, \qquad \eta = \frac{\Delta y_j + \Delta y_{j-js}}{2},$$

$$\theta = \tan^{-1}\left|\frac{vL}{uL}\right|, \qquad \alpha = \tan^{-1}\left(\frac{\eta}{\xi}\right),$$

$$\delta = \xi\cdot\tan\theta, \qquad \zeta = is\cdot\eta\cdot\cot\theta,$$

$$\phi RW = \begin{cases} \dfrac{(\eta-\delta)\cdot\phi_{i+ia,j} + \delta\cdot\phi_{i+ia,j-js}}{\eta} & \alpha \geqq \theta \\[2ex] \dfrac{\left(\dfrac{\Delta\chi_{i+1}}{2}+\zeta\right)\phi_{i,j-js} + \left(\dfrac{\Delta\chi_i}{2}-\zeta\right)\phi_{i+1,j-js}}{\left(\dfrac{\Delta\chi_i+\Delta\chi_{i+1}}{2}\right)} & \alpha < \theta, \end{cases}$$

where

$$is = \frac{uR}{|uR|}, \qquad ia = \frac{1-is}{2},$$

$$js = \frac{vR}{|vR|}, \qquad ja = \frac{1-js}{2},$$

$$\xi = \frac{\Delta x_{i+ia}}{2}, \qquad \eta = \frac{\Delta y_j + \Delta y_{j-js}}{2},$$

$$\theta = \tan^{-1}\left|\frac{vR}{uR}\right|, \qquad \alpha = \tan^{-1}\left(\frac{\eta}{\xi}\right),$$

$$\delta = \xi\cdot\tan\theta, \qquad \zeta = is\cdot\eta\cdot\cot\theta,$$

$$\phi BW = \begin{cases} \dfrac{(\xi - \delta)\cdot\phi_{i,j-1+ja} + \delta\cdot\phi_{i-is,j-1+ja}}{\zeta} & \alpha \geqq \theta \\[2ex] \dfrac{\left(\dfrac{\Delta y_j}{2} + \zeta\right)\cdot\phi_{i-is,j-1} + \left(\dfrac{\Delta y_j}{2} - \zeta\right)\phi_{i-is,j}}{\left(\dfrac{\Delta y_{j-1} + \Delta y_j}{2}\right)} & \alpha < \theta, \end{cases}$$

where

$$is = \frac{uB}{|uB|}, \qquad ia = \frac{1-is}{2},$$

$$js = \frac{vB}{|vB|}, \qquad ja = \frac{1-js}{2},$$

$$\xi = \frac{\Delta\chi_i + \Delta\chi_{i-is}}{2}, \quad \eta = \frac{\Delta y_{j-1+ja}}{2},$$

$$\theta = \tan^{-1}\left|\frac{vB}{uB}\right|, \quad \alpha = \tan^{-1}\left(\frac{\xi}{\eta}\right),$$

$$\delta = \eta\cdot\tan\theta, \qquad \zeta = js\cdot\xi\cdot\cot\theta,$$

$$\phi TW = \begin{cases} \dfrac{(\xi - \delta)\cdot\phi_{i,j+ja} + \delta\cdot\phi_{i-is,j+ja}}{\xi} & \alpha \geqq \theta \\[2ex] \dfrac{\left(\dfrac{\Delta y_{j+1}}{2} + \zeta\right)\cdot\phi_{i-is,j} + \left(\dfrac{\Delta y_j}{2} - \zeta\right)\cdot\phi_{i-is,j-1}}{\left(\dfrac{\Delta y_j + \Delta y_{j+1}}{2}\right)} & \alpha < \theta, \end{cases}$$

where

$$is = \frac{uT}{|uT|}, \qquad ia = \frac{1-is}{2},$$

$$js = \frac{vT}{|vT|}, \qquad ja = \frac{1-js}{2},$$

$$\xi = \frac{\Delta\chi_i + \Delta\chi_{i-is}}{2}, \quad \eta = \frac{\Delta y_{j-1+ja}}{2},$$

$$\theta = \tan^{-1}\left|\frac{vT}{uT}\right|, \quad \alpha = \tan^{-1}\left(\frac{\xi}{\eta}\right),$$

$$\delta = \eta\cdot\tan\theta, \qquad \zeta = js\cdot\xi\cdot\cot\theta.$$

3.2 Skew upwind differencing with artificial cut off (SUD-Cut) scheme

The interpolated expressions for SUD-Cut (Miao et al. 1984) are the same as those in SUD as $\alpha \geqq \theta$. In the case of $\alpha < \theta$, the interpolated values are only contributed by ϕ at one of two nodal points on a line which is intersected with the upstream part of velocity vector. The resulting expressions are as follows:

$$\phi LW = \begin{cases} \dfrac{(\eta-\delta)\cdot\phi_{i-1+ia,j} + \delta\cdot\phi_{i-1+ia,j-js}}{\eta} & \alpha \geqq \theta \\ (1-ia)\cdot\phi_{i-1,j-js} + (ia)\cdot\phi_{i,j-js} & \alpha < \theta, \end{cases}$$

$$\phi RW = \begin{cases} \dfrac{(\eta-\delta)\cdot\phi_{i+ia,j} + \delta\cdot\phi_{i+ia,j-js}}{\eta} & \alpha \geqq \theta \\ (1-ia)\cdot\phi_{i,j-js} + (ia)\cdot\phi_{i+1,j-js} & \alpha < \theta, \end{cases}$$

$$\phi BW = \begin{cases} \dfrac{(\xi-\delta)\cdot\phi_{i,j-1+ja} + \delta\cdot\phi_{i-is,j-1+ja}}{\xi} & \alpha \geqq \theta \\ (1-ja)\cdot\phi_{i-is,j-1} + (ja)\cdot\phi_{i-is,j} & \alpha < \theta, \end{cases}$$

$$\phi TW = \begin{cases} \dfrac{(\xi-\delta)\cdot\phi_{i,j+ja} + \delta\cdot\phi_{i-is,j+ja}}{\xi} & \alpha \geqq \theta \\ (1-ja)\cdot\phi_{i-is,j} + (ja)\cdot\phi_{i-is,j+1} & \alpha < \theta. \end{cases}$$

The definitions of $is, ia, js, ja, \xi, \eta, \theta, \alpha$, and δ in above expressions are the same as those in SUD.

3.3 Volume-weighted skew upwind differencing (VWSUD) scheme

The convective quantities at the surface of cell can also be approximated by introducing two upstream volumes A_0, A_1 as weighting factors. A_0 and A_1 in Fig. 3 are bounded by a line connecting the centers of adjacent upstream cells and two lines parallel to velocity vector at the center of control surface. These parallel lines start from the corners of control volume and extend all the way back to a line connecting the centers of two adjacent cells.

The expressions of interpolated values are as follows:

$$\phi LW = \frac{A_0\cdot\phi_{i-1+ia,j} + A_1\cdot\phi_{i-1+ia,j-js}}{A_0 + A_1}, \quad \phi RW = \frac{A_0\cdot\phi_{i+ia,j} + A_1\cdot\phi_{i+ia,j-js}}{A_0 + A_1},$$

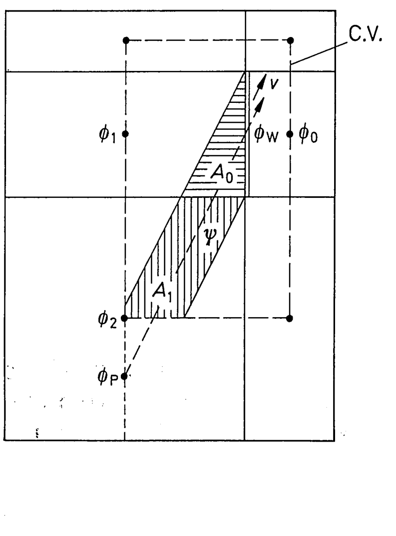

Fig. 3. Illustration of VWSUD scheme

where

$$A_0=\begin{cases}\frac{1}{2}\Delta x\Delta y\left(1-\frac{\beta}{4\alpha}\right) & \alpha\geqq\frac{\beta}{2}\\ \frac{1}{2}\Delta x\Delta y\frac{\alpha}{\beta} & \alpha<\frac{\beta}{2};\end{cases}\qquad A_1=\begin{cases}\frac{1}{4}\Delta x\Delta y\frac{\beta}{2\alpha} & \alpha\geqq\beta\\ \frac{1}{4}\Delta x\Delta y\left(1-\frac{\alpha}{2\beta}\right) & \alpha<\beta;\end{cases}$$

$$\alpha=\begin{cases}\frac{|uL|\cdot\Delta t}{\Delta x} & \text{for}\quad \phi=u \text{ at left surface}\\ \frac{|uR|\cdot\Delta t}{\Delta x} & \text{for}\quad \phi=u \text{ at right surface};\end{cases}\qquad \beta=\begin{cases}\frac{|vL|\cdot\Delta t}{\Delta y} & \text{for}\quad \phi=v \text{ at left surface}\\ \frac{|vR|\cdot\Delta t}{\Delta y} & \text{for}\quad \phi=v \text{ at right surface},\end{cases}$$

and

$$\phi BW=\frac{A_0\cdot\phi_{i,j-1+ja}+A_1\cdot\phi_{i-is,j-1+ja}}{A_0+A_1},\quad \phi TW=\frac{A_0\cdot\phi_{i,j+ja}+A_1\cdot\phi_{i-is,j+ja}}{A_0+A_1},$$

where

$$A_0=\begin{cases}\frac{1}{2}\Delta x\Delta y\left(1-\frac{\alpha}{4\beta}\right) & \beta\geqq\frac{\alpha}{2}\\ \frac{1}{2}\Delta x\Delta y\frac{\beta}{\alpha} & \beta<\frac{\alpha}{2},\end{cases}\qquad \alpha=\begin{cases}\frac{|uB|\cdot\Delta t}{\Delta x} & \text{for}\quad \phi=u \text{ at bottom surface}\\ \frac{|uT|\cdot\Delta t}{\Delta x} & \text{for}\quad \phi=u \text{ at top surface},\end{cases}$$

$$A_1=\begin{cases}\frac{1}{4}\Delta x\Delta y\frac{\alpha}{2\beta} & \beta\geqq\alpha\\ \frac{1}{4}\Delta x\Delta y\left(1-\frac{\beta}{2\alpha}\right) & \beta<\alpha,\end{cases}\qquad \beta=\begin{cases}\frac{|vB|\cdot\Delta t}{\Delta y} & \text{for}\quad \phi=v \text{ at bottom surface}\\ \frac{|vT|\cdot\Delta t}{\Delta y} & \text{for}\quad \phi=v \text{ at top surface}.\end{cases}$$

3.4 Corner skew upwind differencing (CSUD) scheme

The interpolated ϕ can also be determined differently (Chiou 1990) using volume area as weighting factor. The directions of two lines, passing through the corners in Fig. 4, are determined by the velocity angles at the corners. The values of A_0, A_1 at four corners can be similarly computed as

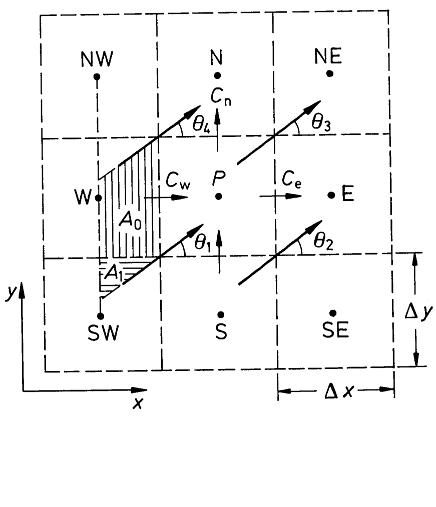

Fig. 4. Illustration of CSUD scheme

those in VWSUD using α_i, β_i defined by

$$\alpha_i = \frac{|u_i|\cdot\Delta t}{\Delta x}, \quad \beta_i = \frac{|v_i|\cdot\Delta t}{\Delta y} \quad (i = 1, 2, 3, 4).$$

The resulting expressions for ϕLW, ϕRW, ϕBW, ϕTW are as follows:

$$\phi LW = \frac{A_0\cdot\phi_{i-1+ia,j} + A_1\cdot\phi_{i-1+ia,j-1} + A_2\cdot\phi_{i-1+ia,j+1}}{A_0 + A_1 + A_2}, \quad \text{where} \quad is = \frac{|uL|}{uL}, \quad ia = \frac{1-is}{2},$$

$$\phi RW = \frac{A_0\cdot\phi_{i+ia,j} + A_1\cdot\phi_{i+ia,j-1} + A_2\cdot\phi_{i+ia,j+1}}{A_0 + A_1 + A_2}, \quad \text{where} \quad is = \frac{|uR|}{uR}, \quad ia = \frac{1-is}{2},$$

$$\phi BW = \frac{A_0\cdot\phi_{i,j-1+ja} + A_1\cdot\phi_{i-1,j-1+ja} + A_2\cdot\phi_{i+1,j-1+ja}}{A_0 + A_1 + A_2}, \quad \text{where} \quad js = \frac{|vB|}{vB}, \quad ja = \frac{1-js}{2},$$

$$\phi TW = \frac{A_0\cdot\phi_{i,j+ja} + A_1\cdot\phi_{i-1,j+ja} + A_2\cdot\phi_{i+1,j+ja}}{A_0 + A_1 + A_2}, \quad \text{where} \quad js = \frac{|vT|}{vT}, \quad ja = \frac{1-js}{2}.$$

3.5 Directional transportive upwind differencing (DTUD) scheme

This scheme (Eraslan et al. 1983) is a combination of transportive upwind differencing (TUD) scheme (Sharif et al. 1988) and skew upwind differencing scheme. The magnitude and direction of velocity vector at the cell faces are all taken into account. The advected values are interpolated by ϕ at four nodal points surrounding the point of interpolation:

$$\phi LW = (1-\xi_1)\cdot(1-\eta_1)\cdot\phi_{i,j} + \xi_1\cdot(1-\eta_1)\cdot\phi_{i-1,j} + (1-\xi_1)\cdot\eta_1\cdot\phi_{i,j-js} + \xi_1\cdot\eta_1\cdot\phi_{i-1,j-js},$$
$$\phi RW = (1-\xi_1)\cdot(1-\eta_1)\cdot\phi_{i+1,j} + \xi_1\cdot(1-\eta_1)\cdot\phi_{i,j} + (1-\xi_1)\cdot\eta_1\cdot\phi_{i+1,j-js} + \xi_1\cdot\eta_1\cdot\phi_{i,j-js},$$
$$\phi BW = (1-\eta_2)\cdot(1-\xi_2)\cdot\phi_{i,j} + \eta_2\cdot(1-\xi_2)\cdot\phi_{i,j-1} + (1-\eta_2)\cdot\xi_2\cdot\phi_{i-is,j} + \eta_2\cdot\xi_2\cdot\phi_{i-is,j-1},$$
$$\phi BW = (1-\eta_2)\cdot(1-\xi_2)\cdot\phi_{i,j+1} + \eta_2\cdot(1-\xi_2)\cdot\phi_{i,j} + (1-\eta_2)\cdot\xi_2\cdot\phi_{i-is,j+1} + \eta_2\cdot\xi_2\cdot\phi_{i-is,j},$$

where

$$is = \begin{bmatrix} \dfrac{|uB|}{uB} & \text{for} \quad \phi = u \text{ at bottom surface} \\ \dfrac{|uT|}{uT} & \text{for} \quad \phi = u \text{ at top surface} \end{bmatrix} \qquad ia = \frac{1-is}{2},$$

$$js = \begin{bmatrix} \dfrac{|vL|}{uL} & \text{for} \quad \phi = v \text{ at left surface} \\ \dfrac{|vR|}{uR} & \text{for} \quad \phi = u \text{ at right surface} \end{bmatrix} \qquad ja = \frac{1-js}{2},$$

$$\xi_1 = \frac{\dfrac{\Delta x_i}{2} + \beta\cdot(uL)\cdot\Delta t}{\left(\dfrac{\Delta x_{i-1} + \Delta x_i}{2}\right)}, \qquad \eta_1 = \frac{\beta\cdot|vL|\cdot\Delta t}{\left(\dfrac{\Delta y_j + \Delta y_{j-js}}{2}\right)},$$

$$\xi_2 = \frac{\beta\cdot|uB|\cdot\Delta t}{\left(\dfrac{\Delta x_i + \Delta x_{i-is}}{2}\right)}, \qquad \eta_1 = \frac{\dfrac{\Delta y_j}{2} + \beta\cdot(vB)\cdot\Delta t}{\left(\dfrac{\Delta y_{j-1} + \Delta y_j}{2}\right)},$$

and β is an adjustable parameter.

4 QUICK-family schemes

The corresponding discretization equations for Eq. (1) can be written as follows:

$$\phi^{n+1}=\phi^n+C_l\cdot\phi_l-C_r\cdot\phi_r+C_b\cdot\phi_b-C_t\cdot\phi_t+\gamma_x\cdot\Delta x\cdot[(\phi_x)_r-(\phi_x)_l]+\gamma_y\cdot\Delta y\cdot[(\phi_y)_t-(\phi_y)_b]+\Delta t\cdot S^{\phi}, \tag{3}$$

where

$$C_l=\frac{u_l\cdot\Delta t}{\Delta x},\quad C_r=\frac{u_r\cdot\Delta t}{\Delta x},\quad C_b=\frac{v_b\cdot\Delta t}{\Delta y},\quad C_t=\frac{v_t\cdot\Delta t}{\Delta y},\quad \gamma_x=\frac{\Gamma\cdot\Delta t}{(\Delta x)^2},\quad \gamma_y=\frac{\Gamma\cdot\Delta t}{(\Delta y)^2}.$$

The interpolated expressions for $\phi_l, \phi_r, \phi_t, \phi_b$ are set up by QUICK-family schemes. The essential idea of QUICK scheme is based on representing the convective quantities, along the direction normal to the control surfaces, by two adjacent nodes at each side and one at the next upstream node. This approximation on convective terms can considerably enhance the convective stability by its inherent upwind curvature terms. The following three QUICK schemes are investigated in this study.

4.1 Direction splitting QUICK-1D scheme

The approximations of (ϕ_l, ϕ_r) and (ϕ_t, ϕ_b) can be made using the local one-dimensional concept of QUICK scheme:

$$\phi_r=\frac{1}{2}(\phi_{i+1,j}+\phi_{i,j})-\frac{u_r+|u_r|}{16u_r}(\phi_{i+1,j}-2\phi_{i,j}+\phi_{i-1,j})-\frac{u_r-|u_r|}{16u_r}(\phi_{i+2,j}-2\phi_{i+1,j}+\phi_{i,j}),$$

$$\phi_l=\frac{1}{2}(\phi_{i,j}+\phi_{i-1,j})-\frac{u_l+|u_l|}{16u_l}(\phi_{i,j}-2\phi_{i-1,j}+\phi_{i-2,j})-\frac{u_l-|u_l|}{16u_l}(\phi_{i+1,j}-2\phi_{i,j}+\phi_{i-1,j}),$$

$$\phi_t=\frac{1}{2}(\phi_{i,j+1}+\phi_{i,j})-\frac{v_t+|v_t|}{16v_t}(\phi_{i,j+1}-2\phi_{i,j}+\phi_{i,j-1})-\frac{v_t-|v_t|}{16v_t}(\phi_{i,j+2}-2\phi_{i,j+1}+\phi_{i,j}),$$

$$\phi_b=\frac{1}{2}(\phi_{i,j}+\phi_{i,j-1})-\frac{v_b+|v_b|}{16v_b}(\phi_{i,j}-2\phi_{i,j-1}+\phi_{i,j-2})-\frac{v_b-|v_b|}{16v_b}(\phi_{i,j+1}-2\phi_{i,j}+\phi_{i,j-1}).$$

The diffusion terms $(\phi_x)_r, (\phi_x)_l, (\phi_y)_t, (\phi_y)_b$ in Eq. (3) are approximated by the center differencing.

4.2 QUICK-2D scheme

The expressions of $\phi_l, \phi_r, \phi_t, \phi_b$ can be more accurately approximated as follows by incorporating the adjacent nodal values in both directions:

$$\phi_r=\frac{1}{2}(\phi_{i+1,j}+\phi_{i,j})-\frac{u_r+|u_r|}{16u_r}(\phi_{i+1,j}-2\phi_{i,j}+\phi_{i-1,j})-\frac{u_r-|u_r|}{16u_r}(\phi_{i+2j}-2\phi_{i+1,j}+\phi_{i,j})$$
$$+\frac{u_r+|u_r|}{48u_r}(\phi_{i,j+1}-2\phi_{i,j}+\phi_{i,j-1})+\frac{u_r-|u_r|}{48u_r}(\phi_{i+1,j+1}-2\phi_{i+1,j}+\phi_{i+1,j-1}),$$

$$\phi_l=\frac{1}{2}(\phi_{i,j}+\phi_{i-1,j})-\frac{u_l+|u_l|}{16u_l}(\phi_{i,j}-2\phi_{i-1,j}+\phi_{i-2,j})-\frac{u_l-|u_l|}{16u_l}(\phi_{i+1,j}-2\phi_{i,j}+\phi_{i-1,j})$$
$$+\frac{u_l+|u_l|}{48u_l}(\phi_{i-1,j+1}-2\phi_{i-1,j}+\phi_{i-1,j-1})+\frac{u_l-|u_l|}{48u_l}(\phi_{i,j+1}-2\phi_{i,j}+\phi_{i,j-1}),$$

$$\phi_t = \frac{1}{2}(\phi_{i,j+1} + \phi_{i,j}) - \frac{v_t + |v_t|}{16v_t}(\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}) - \frac{v_t - |v_t|}{16v_t}(\phi_{i,j+2} - 2\phi_{i,j+1} + \phi_{i,j})$$

$$+ \frac{v_t + |v_t|}{48v_t}(\phi_{i-1,j} - 2\phi_{i,j} + \phi_{i+1,j}) + \frac{v_t - |v_t|}{48v_t}(\phi_{i-1,j+1} - 2\phi_{i,j+1} + \phi_{i+1,j+1}),$$

$$\phi_b = \frac{1}{2}(\phi_{i,j} + \phi_{i,j-1}) - \frac{v_b + |v_b|}{16v_b}(\phi_{i,j} - 2\phi_{i,j-1} + \phi_{i,j-2}) - \frac{v_b - |v_b|}{16v_b}(\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1})$$

$$+ \frac{v_t + |v_t|}{48v_t}(\phi_{i-1,j-1} - 2\phi_{i,j} + \phi_{i+1,j-1}) + \frac{v_t - |v_t|}{48v_t}(\phi_{i-1,j} - 2\phi_{i,j} + \phi_{i+1,j}).$$

4.3 QUICKEST (QUICK with estimated streaming terms) scheme

The basic idea behind QUICKEST algorithm (Leonard 1979) is to estimate the temporal term on the right-hand side of Eq. (3) so that more accurate time average between two consecutive time steps can be computed. The interpolated quantities can be written as follows:

$$\phi_l = \frac{1}{2}(\phi_{i,j} + \phi_{i-1,j}) - C_l \cdot GRAD_l \cdot \frac{\Delta x}{2} + \left[\frac{\gamma_x}{2} - \frac{1}{6}(1 - C_l^2)\right] \cdot (\Delta x)^2 \cdot CURV_l,$$

$$\phi_r = \frac{1}{2}(\phi_{i,j} + \phi_{i+1,j}) - C_r \cdot GRAD_r \cdot \frac{\Delta x}{2} + \left[\frac{\gamma_x}{2} - \frac{1}{6}(1 - C_r^2)\right] \cdot (\Delta x)^2 \cdot CURV_r,$$

$$\phi_t = \frac{1}{2}(\phi_{i,j} + \phi_{i,j+1}) - C_t \cdot GRAD_t \cdot \frac{\Delta y}{2} + \left[\frac{\gamma_y}{2} - \frac{1}{6}(1 - C_t^2)\right] (\Delta y)^2 \cdot CURV_t,$$

$$\phi_b = \frac{1}{2}(\phi_{i,j} + \phi_{i,j-1}) - C_b \cdot GRAD_b \cdot \frac{\Delta y}{2} + \left[\frac{\gamma_y}{2} - \frac{1}{6}(1 - C_b^2)\right] \cdot (\Delta y)^2 \cdot CURV_b.$$

where

$$GRAD_l = \frac{\phi_{i,j} - \phi_{i-1,j}}{\Delta x}, \quad GRAD_t = \frac{\phi_{i,j+1} - \phi_{i,j}}{\Delta y},$$

$$GRAD_r = \frac{\phi_{i+1,j} - \phi_{i,j}}{\Delta x}, \quad GRAD_b = \frac{\phi_{i,j} - \phi_{i,j-1}}{\Delta y},$$

$$CURV_l = \frac{u_l + |u_l|}{2u_l(\Delta x)^2} \cdot (\phi_{i,j} + 2\phi_{i-1,j}\phi_{i-2,j}) + \frac{u_l - |u_l|}{2u_l(\Delta x^2)} \cdot (\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}),$$

$$CURV_r = \frac{u_r + |u_r|}{2u_r(\Delta x)^2} \cdot (\phi_{i+1,j} + 2\phi_{i,j} + \phi_{i-1,j}) + \frac{u_r - |u_r|}{2u_r(\Delta x)^2} \cdot (\phi_{i+2,j} - 2\phi_{i+1,j} + \phi_{i,j}),$$

$$CURV_t = \frac{v_t + |v_t|}{2v_t(\Delta x)^2} \cdot (\phi_{i,j+1} + 2\phi_{i,j} + \phi_{i,j-1}) + \frac{v_t - |v_t|}{2v_t(\Delta x)^2} \cdot (\phi_{i,j+2} - 2\phi_{i,j+1} + \phi_{i,j}),$$

$$CURV_b = \frac{v_b + |v_b|}{2v_b(\Delta x)^2} \cdot (\phi_{i,j} + 2\phi_{i,j-1} + \phi_{i,j-2}) + \frac{v_b - |v_b|}{2v_b(\Delta x)^2} \cdot (\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}).$$

The first two terms in above expressions represent the linear interpolation and the gradient across their respective surfaces l, r, t, and b. The last term represents the upstream-biased curvature term which is a function of Courant number C as well as diffusion number γ.

The expressions of $(\phi_x)_l, (\phi_x)_r, (\phi_y)_t, (\phi_y)_b$ in Eq. (3) are as follows:

$$(\phi_x)_l = GRAD_l - C_l \cdot \frac{\Delta x}{2} \cdot CURV_l, \quad (\phi_y)_t = GRAD_t - C_t \cdot \frac{\Delta y}{2} \cdot CURV_t,$$

$$(\phi_x)_r = GRAD_r - C_r \cdot \frac{\Delta x}{2} \cdot CURV_r, \quad (\phi_y)_b = GRAD_b - C_b \cdot \frac{\Delta y}{2} \cdot CURV_b.$$

5 Nonlinear filtering FRAM algorithm

If a flow field contains certain initial discontinuous data or regions of high gradients, the conventional solution procedures are inadequate to compute the non-oscillating solution. A brief review of approaches which can remove as much of artificial diffusion as possible and still retain high accuracy and boundness is as follows.

Boris and Book (1973) developed an anti-diffusion flux limiter in their FCT algorithm to dampen the non-physical oscillations. Zalesak (1979) extended this idea to a generalized multi-dimensional flux corrected transport scheme which can eliminate the operator splitting requirement. A noise detection filtering scheme developed by Forester (1977) can also remedy the clipping problem in Boris's FCT scheme by using several problem dependent free parameters. A nonlinear damping algorithm FRAM of M. Chapman (1981) will be employed in the present study to check if it can effectively suppress the numerical noise due to the high-order discretization of convective terms. The damping of oscillations by FRAM is achieved by selectively adding a strong local dissipation flux to the equations as local bounds computed in the provisional step are not within the specified limit. This added dissipation flux is a function of local velocities, space as well as temporal increments, and Lagrangian solutions of dependent variables.

The FRAM solution algorithm for Eq. (1) are computed by the following three steps:

(1) *High-order solution step.* The solutions $\phi_{i,j}^{n+1}$ for Eq. (1) are computed by the investigated high-order QUICK-family or SKEW-family schemes.

(2) *Lagrangian solution step.* Compute four surrounding Lagrangian solutions $\phi_{i+1,j}^*, \phi_{i-1,j}^*, \phi_{i,j+1}^*$, and $\phi_{i,j-1}^*$ with respect to $\phi_{i,j}$ shown in Fig. 5. The Lagrangian solution $\phi_{p,q}^*$, for example, is computed by

$$\phi_{p,q}^* = \phi_{p,q}^n + \Gamma \cdot \Delta t \cdot \left(\frac{\phi_{p+1,q}^n - 2\phi_{p,q}^n + \phi_{p-1,q}^n}{\Delta x^2} + \frac{\phi_{p,q+1}^n - 2\phi_{p,q}^n + \phi_{p,q-1}^n}{\Delta y^2} \right)$$

(3) *Corrector step.* Compare the computed high-order solution $\phi_{i,j}^{n+1}$ in step (1) with $(\phi_{i,j}^*)_{\max}$ and $(\phi_{i,j}^*)_{\min}$ which are defined as follows:

$$(\phi_{i,j}^*)_{\max} = \max(\phi_{i+1,j}^*, \phi_{i-1,j}^*, \phi_{i,j+1}^*, \phi_{i,j-1}^*),$$
$$(\phi_{i,j}^*)_{\min} = \min(\phi_{i+1,j}^*, \phi_{i-1,j}^*, \phi_{i,j+1}^*, \phi_{i,j-1}^*).$$

If $\phi_{i,j}^{n+1} < (\phi_{i,j}^*)_{\max}$ or $\phi_{i,j}^{n+1} > (\phi_{i,j}^*)_{\min}$ one uses high-order solution as the update solution.

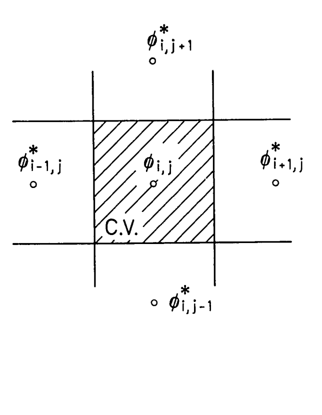

Fig. 5. Control volume for $\phi_{i,j}$ and its surrounding Lagrangian values

Otherwise, the first-order upwind scheme is switched to compute the interpolated values:

$$\phi RW = \phi_r = \frac{\phi_{i+1,j}+\phi_{i,j}}{2} + \frac{|u_r|}{u_r}.\frac{\phi_{i,j}-\phi_{i+1,j}}{2}, \quad \phi LW = \phi_l = \frac{\phi_{i,j}+\phi_{i-1,j}}{2} + \frac{|u_l|}{u_l}.\frac{\phi_{i-1,j}-\phi_{i,j}}{2},$$

$$\phi TW = \phi_t = \frac{\phi_{i,j+1}+\phi_{i,j}}{2} + \frac{|v_t|}{v_t}.\frac{\phi_{i,j}-\phi_{i,j+1}}{2}, \quad \phi BW = \phi_b = \frac{\phi_{i,j-1}+\phi_{i,j}}{2} + \frac{|v_b|}{v_b}.\frac{\phi_{i,j-1}-\phi_{i,j}}{2}.$$

6 Test problems and results

The main purpose of investigating the following two inviscid problems in 6.1 and 6.2 is to demonstrate how the proposed filtering procedure works for solving flowfield containing large gradients. Two laminar flow problems without internal discontinuity will be tested to demonstrate the incorporation of filtering procedure to MACUP code (Sheu 1989) is not necessary.

6.1 Pure convection of an oblique-step profile

A pure convection of oblique-step profile is attempted to evaluate the effectiveness of FRAM technique for suppressing oscillatory high-order SKEW or QUICK-family solutions. The compu-

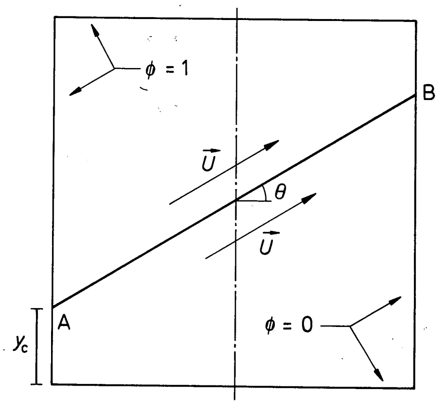

Fig. 6. Transport of a step profile of ϕ in a uniform velocity region

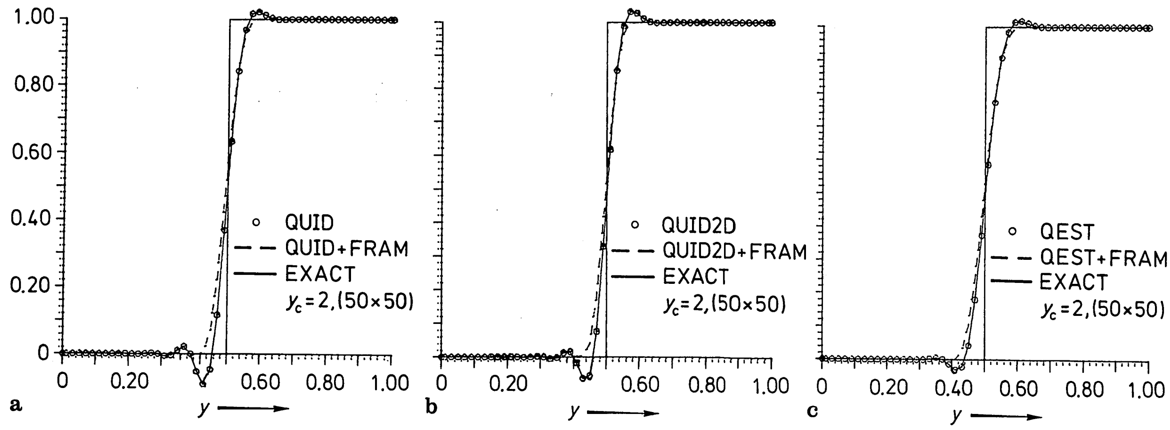

Fig. 7 a–c. Computed results at the mid-section for $y_c = 2$. **a** QUICK-1D; **b** QUICK-2D; **c** QUICKEST

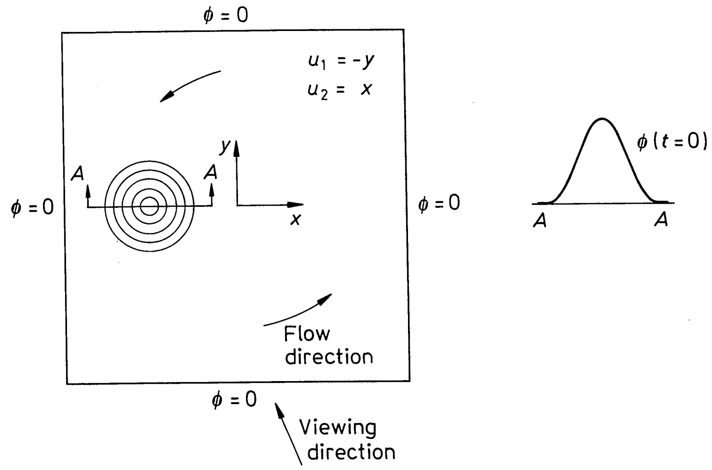

Fig. 12. Transport of a cosine hill in a rotating flowfiled

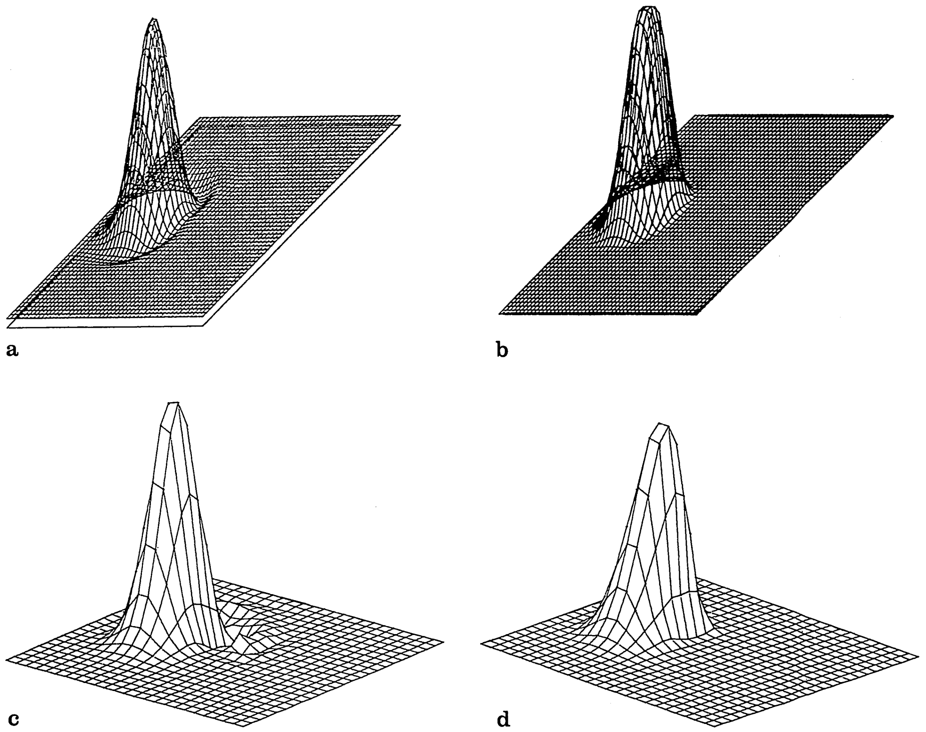

Fig. 13 a–d. Computed profiles or a cosine hill after rotating 360°. **a** QUICKEST; **b** QUICKEST + FRAM; **c** DTUD; **d** DTUD + FRAM

defined as follows for evaluating the accuracy of investigated schemes.

$$\text{ERROR} = \left[\sum_{i=1}^{N} (\phi_e - \phi_i)^2 / N \right]^{1/2}$$

ϕ_e and N in above expression represent exact solution and total grid number respectively. Typical convergence histories are shown in Fig. 11 for $y_c = 2$.

6.2 Transport of a cosine hill or rectangular block in a rotating flowfield

The transport of a cosine or square block in a rigid rotating flowfield has emerged recently as another standard test problem. A given cosine hill is initially located in a rigid rotating flowfield, $u = -y$, $v = x$, shown in Fig. 12. The computations are conducted on a 50 × 50 uniform mesh.

The computed results after rotating 360° using two best investigated schemes (Chiou 1990; Ou Yang 1990), namely QUICKEST and DTUD, are shown in Fig. 13. The wiggles are shown near the root of hill if FRAM is not used. As FRAM is turned on, the computed solutions become monotonic. The corresponding projections of cosine hill on the flat plane are shown in Figs. 14, 15 without any spurious bubbles. An even harder problem, namely transport of a square block, was also investigated by DTUD in this study. The computed results are shown in Fig. 16.

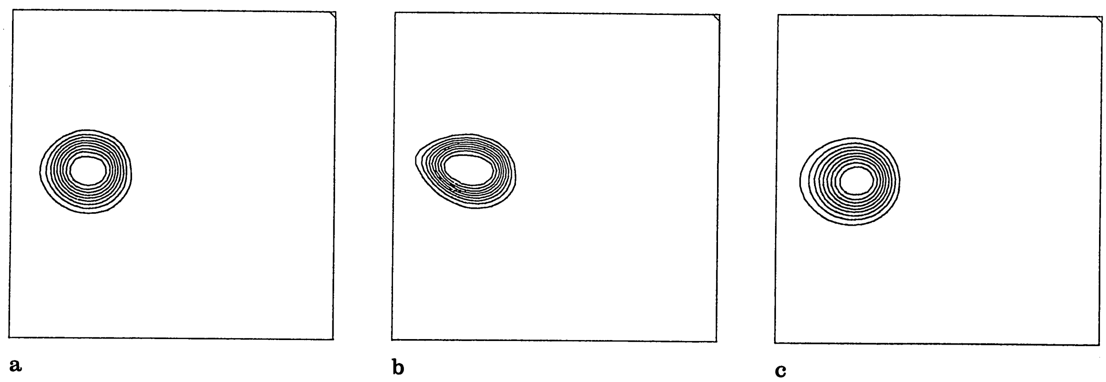

Fig. 14a–c. Computed projections of cosine hill. **a** QUICK-1D + FRAM; **b** QUICK-2D + FRAM; **c** QUICKEST + FRAM

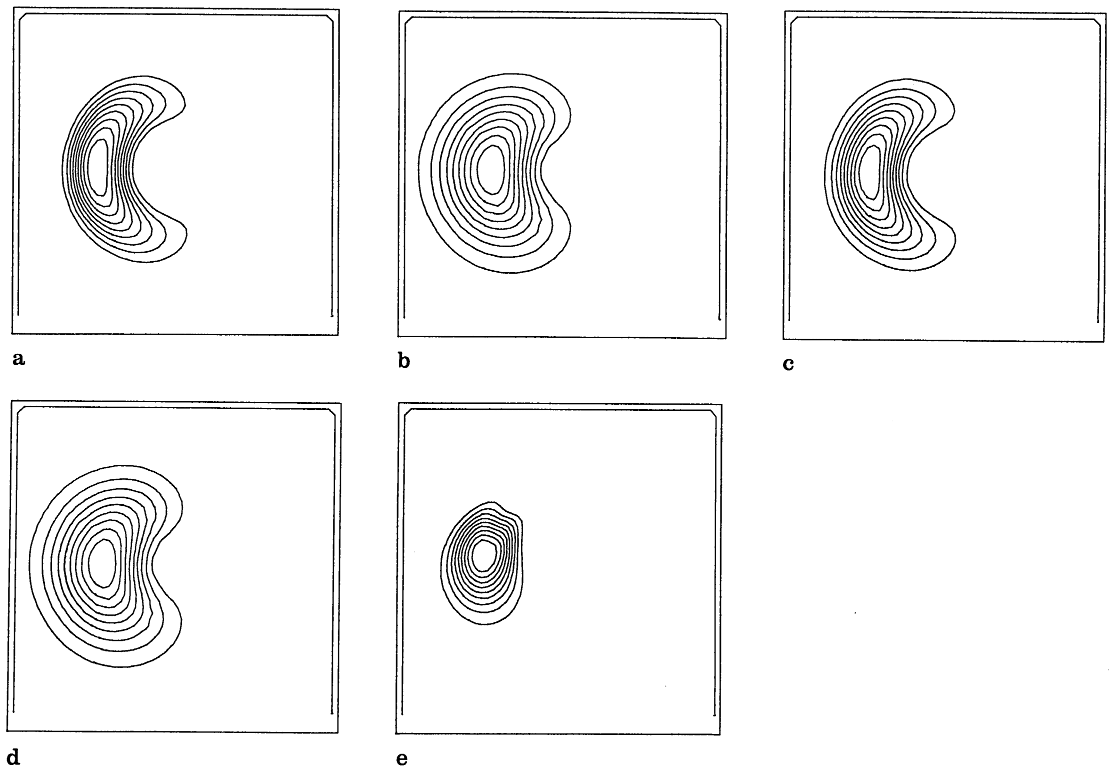

Fig. 15a–e. Computed projections of a cosine hill after rotating 360°. **a** SUD + FRAM; **b** VWSUD + FRAM; **c** SUD + FRAM; **d** CSUD + FRAM; **e** DTUD + FRAM

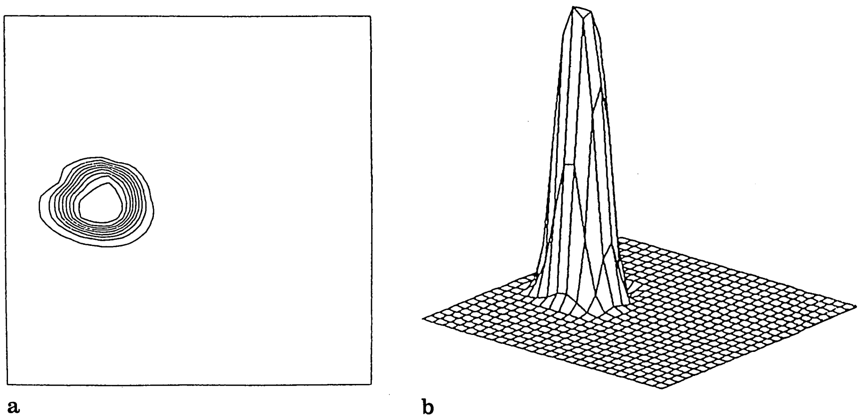

Fig. 16 a and b. Computed square block after rotating 360°. **a** Computed projection; **b** computed profile

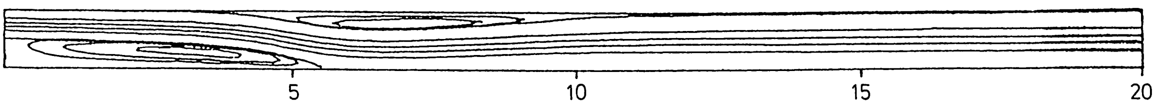

Fig. 17. Computed result of the backward-facing step for $R_e = 800$ using DTUD

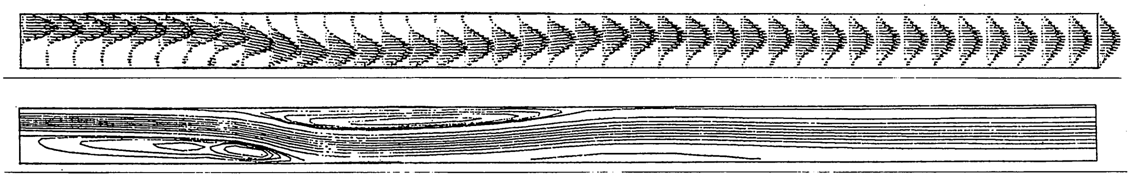

Fig. 18. Computed results for backward-facing step for $R_e = 1200$ using QUICKEST

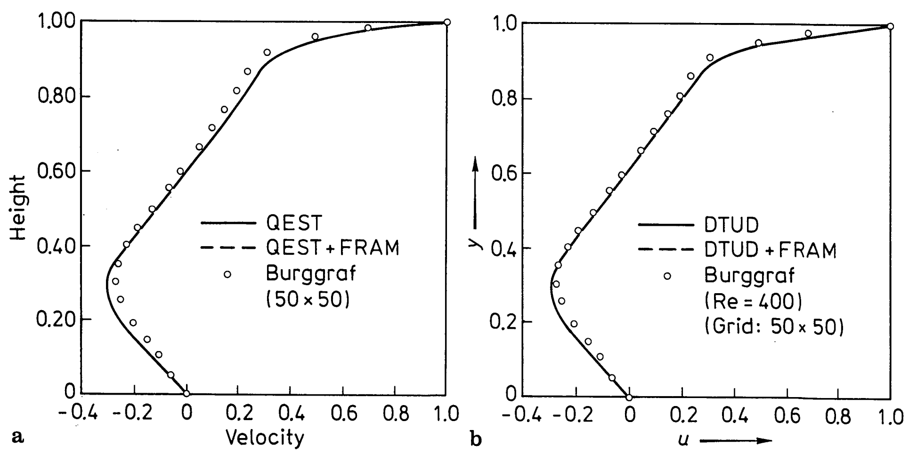

Fig. 19 a and b. Computed results of driven cavity for $R_e = 400$. **a** QUICKEST; **b** DTUD

6.3 Backward-facing step and driven cavity problems

The backward-facing step problem was tested at $R_e = 800, 1200$ and their computed results are shown in Figs. 17, 18. The computed solutions are similar to each other no matter filtering procedure is used or not.

The computed results of driven cavity problem at $R_e = 400$ are shown in Fig. 19. It indicates that the filtering procedure still has no effect.

7 Conclusions

The QUICK and SKEW-family schemes are incorporated in MACUP code to analyze inviscid as well as laminar flowfield. The suppression of those wiggles near the local high gradient region can be accomplished by incorporating the proposed filtering procedure FRAM into the formulation. The best quality of the investigated schemes in QUICK-family for the present test problems is QUICKEST no matter filtering procedure is used or not. The best quality of the investigated schemes in SKEW-family is DTUD.

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Communicated by S. N. Atluri, March 12, 1991

6.3 Backward-facing step and driven cavity problems

The backward-facing step problem was tested at $R_e = 800, 1200$ and their computed results are shown in Figs. 17, 18. The computed solutions are similar to each other no matter filtering procedure is used or not.

The computed results of driven cavity problem at $R_e = 400$ are shown in Fig. 19. It indicates that the filtering procedure still has no effect.

7 Conclusions

The QUICK and SKEW-family schemes are incorporated in MACUP code to analyze inviscid as well as laminar flowfield. The suppression of those wiggles near the local high gradient region can be accomplished by incorporating the proposed filtering procedure FRAM into the formulation. The best quality of the investigated schemes in QUICK-family for the present test problems is QUICKEST no matter filtering procedure is used or not. The best quality of the investigated schemes in SKEW-family is DTUD.

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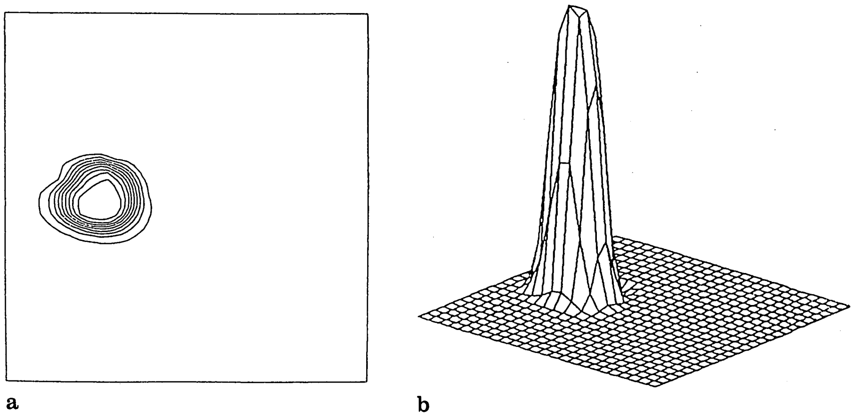


Fig. 16 a and b. Computed square block after rotating 360°. **a** Computed projection; **b** computed profile

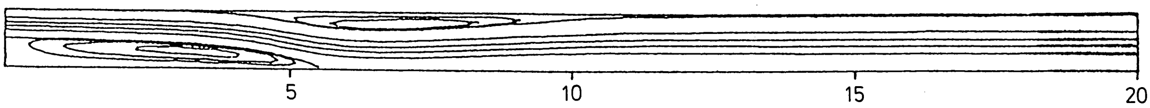


Fig. 17. Computed result of the backward-facing step for $R_e = 800$ using DTUD

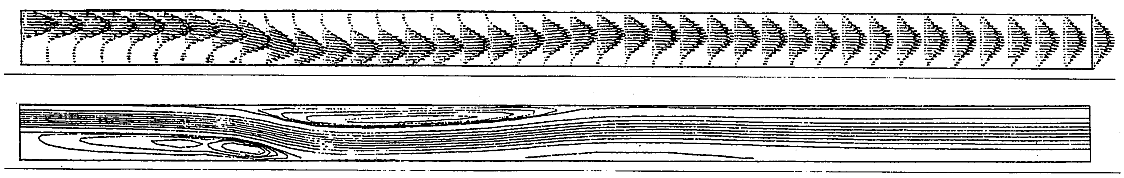


Fig. 18. Computed results for backward-facing step for $R_e = 1200$ using QUICKEST

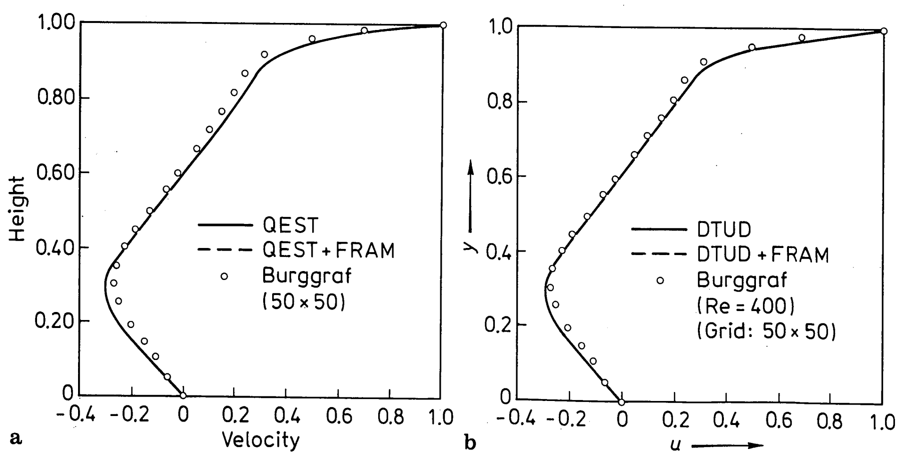
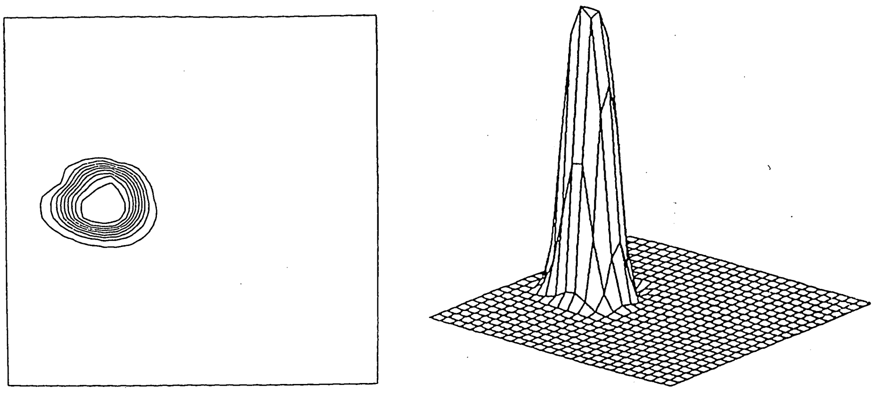


Fig. 19 a and b. Computed results of driven cavity for $R_e = 400$. **a** QUICKEST; **b** DTUD



a b

Fig. 16 a and b. Computed square block after rotating 360°. **a** Computed projection; **b** computed profile

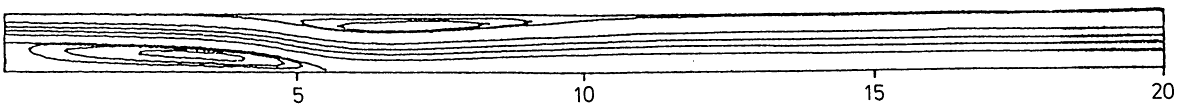


Fig. 17. Computed result of the backward-facing step for $R_e = 800$ using DTUD

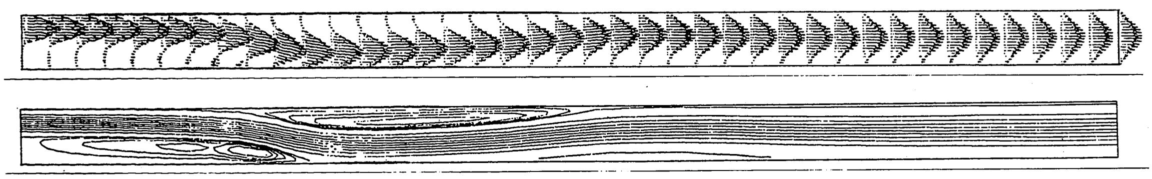


Fig. 18. Computed results for backward-facing step for $R_e = 1200$ using QUICKEST

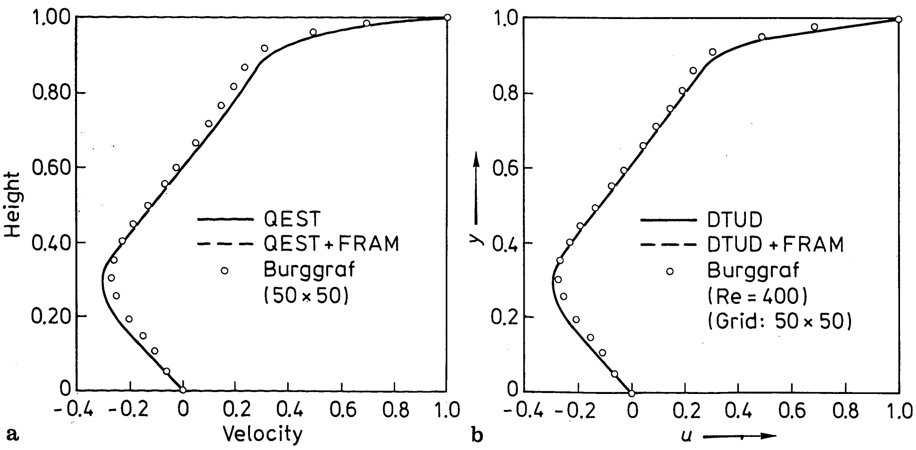


Fig. 19 a and b. Computed results of driven cavity for $R_e = 400$. **a** QUICKEST; **b** DTUD

6.3 *Backward-facing step and driven cavity problems*

The backward-facing step problem was tested at $R_e = 800, 1200$ and their computed results are shown in Figs. 17, 18. The computed solutions are similar to each other no matter filtering procedure is used or not.

The computed results of driven cavity problem at $R_e = 400$ are shown in Fig. 19. It indicates that the filtering procedure still has no effect.

7 Conclusions

The QUICK and SKEW-family schemes are incorporated in MACUP code to analyze inviscid as well as laminar flowfield. The suppression of those wiggles near the local high gradient region can be accomplished by incorporating the proposed filtering procedure FRAM into the formulation. The best quality of the investigated schemes in QUICK-family for the present test problems is QUICKEST no matter filtering procedure is used or not. The best quality of the investigated schemes in SKEW-family is DTUD.

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