

# Modeling and Simulation of the Interstitial Medium Deformation Induced by the Needle Manipulation During Acupuncture

Yannick Deleuze<sup>1,2,\*</sup>, Marc Thiriet<sup>1,3,4</sup> and Tony Wen-Hann Sheu<sup>2,5</sup>

<sup>1</sup> Sorbonne Universités, UPMC Univ Paris 06, UMR 7598, Laboratoire Jacques-Louis Lions, F-75005, Paris, France.

<sup>2</sup> Department of Engineering Science and Ocean Engineering, National Taiwan University, No. 1, Sec. 4, Roosevelt Road, Taipei, Taiwan.

<sup>3</sup> CNRS, UMR 7598, Laboratoire Jacques-Louis Lions, F-75005, Paris, France.

<sup>4</sup> INRIA-Paris-Rocquencourt, EPC REO, Domaine de Voluceau, BP105, 78153 Le Chesnay Cedex.

<sup>5</sup> Center for Advanced Study in Theoretical Sciences, National Taiwan University, No. 1, Sec. 4, Roosevelt Road, Taipei, Taiwan.

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**Abstract.** In this paper, we study the effects of inserted needle on the subcutaneous interstitial flow. A goal is to describe the physical stress affecting cells during acupuncture treatment. The model consists of the convective Brinkman equations to describe the flow through a fibrous medium. Numerical studies in FreeFem++ are performed to illustrate the acute physical stress developed by the implantation of a needle that triggers the physiological reactions of acupuncture. We emphasize the importance of numerical experiments for advancing in modeling in acupuncture.

**AMS subject classifications:** 76M10, 76S05, 76Z05, 92C50

**Key words:** Finite element method, FreeFem++, acupuncture, Brinkman model, interstitial fluid flow.

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## 1 Introduction

Acupuncture is one of the oldest healing practices and alternative medicines. This minimally invasive procedure involves a penetration of skin with needles to stimulate specific points on the body, called acupoints, with the purpose of restoring balance in the flow of  $qi$  through meridians [1]. While this so-called traditional Chinese medicine has been

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\*Corresponding author. *Email addresses:* yannick.deleuze@ljl1.math.upmc.fr (Y. Deleuze), marc.thiriet@upmc.fr (M. Thiriet), twsheu@ntu.edu.tw (T. W.-H. Sheu)

recognized in 2010 on the Representative List of the Intangible Cultural Heritage of Humanity by UNESCO and was endorsed by the World Health Organization, there is a great demand to enlighten its underlying concepts such as *qi*, meridians, and acupoints. The absence of scientific background of acupuncture mechanisms certainly has motivated us to carry out modeling and numerical simulation of both macroscopic and microscopic aspects of the acupuncture process.

The classical technique involves inserting a hair-thin needle into acupoints. The needle is then manipulated to yield a local mechanical stress field through needle motions (lifting–thrusting cycle or rotation). After a short term of needle manipulation, the needle is retained until desired effects have been achieved. The mechanical interaction of needle with the skin and the subcutaneous tissue has been demonstrated [2]. This stimulation via mechanotransduction, that is activated by the stretch of mechanosensitive ion channels at the mastocyte surface, can lead to a cascade of biochemical reactions that drive acupuncture effects [3].

The same acupoint can be stimulated by acupressure, moxibustion, electroacupuncture [4], and, more recently, by laser acupuncture [5]. In the context of moxibustion, physical stimulation via the transfer of heat from burning moxa has been investigated [6–9]. Whatever the operation mode is, calcium entry in the mastocyte triggers degranulation and release of chemoattractants, neural stimulants, and endocrine substances. Such a process is sustained by the recruitment of mastocytes (chemotaxis). We have developed a model of chemotactic self-sustained response of mastocytes [10, 11]. The developed mathematical model constitutes a system of parabolic partial differential equations. Its simplest form describes the evolution of the density of mastocytes and the chemoattractant concentration subjected to a physical stress.

The present study is aimed at simulating the effects of an inserted needle on the interstitial flow. The reader is invited to read the accompanying paper [12] to get information on events occurring during the permanent regime, that is once the needle has been implanted and the stress field in the subcutaneous connective tissue is fully established.

This paper is organized in the following. Section 2 outlines the physiological mechanisms and the biological tissue involved during acupuncture treatment. Section 3 presents the mathematical modeling and the governing equations of flow in interstitium. The ALE finite element method is reviewed in Section 4. Numerical experiments are discussed in Section 5. In Section 6, concluding remarks are given.

## 2 Biological medium

### 2.1 Connective tissue

The skin consists of three layers of tissue known as the epidermis, dermis, and hypodermis lying above skeletal muscles. The hypodermis, a loose connective tissue, provides (1) structural and mechanical support, (2) transport of nutrients, metabolites, and waste

between the blood and tissues, (3) storage of energy, and (4) immunological defense. For a complete description of the extracellular matrix we refer to [13, 14].

Like other types of connective tissues, the loose connective tissue is constituted of scattered cells immersed in extracellular matrix. The extracellular matrix contains an abundant ground substance and relatively sparse fibers. The ground substance fills the space between cells and fibers and consists of water, proteoglycans, and other macromolecules, thereby forming a viscous hydrated gel that can stabilize fiber network. Proteoglycans control the level of hydration of connective tissues and thus can partially determine the physical properties of connective tissues.

There is ample evidence that acupoints are located mainly in the hypodermis close to neurovascular bundles [15]. Acupoints differentiate from nonacupoint locations by displaying high density of mastocytes and capillaries, high skin electrical conductance, and high ionic concentrations ( $K^+$ ,  $Ca^{++}$ ,  $Fe^{++}$ ,  $Mn^{++}$ ,  $Zn^{++}$ ,  $PO_4^{3-}$ ).

## 2.2 The interstitial fluid

The interstitial fluid contains water, ions and other small molecules. Such a fluid corresponds to plasma without macromolecules [16]. It interacts with the ground substance to form a gel-like medium.

A model taking into account individual fibers and cell adhesion complexes is already a falsification of the reality. Moreover, it is very costly from the computational sense. When considering an organized homogeneous matrix of fibers, computation of such a model shows the microscopic fluctuations of the fluid shear stress at the protein level [17].

On a microscopic scale, the interstitial is composed of fluid and solid fibers, thereby clearly forming two phases. Homogenized two-phase media coupling with a Newtonian fluid and an elastic matrix have been considered to model soft tissue [18]. This well-posed model exhibits both the fluid and the viscoelastic property of the fluid.

The interstitial tissue can be modeled as a porous medium [16]. The Darcy's law approximates fibers of the media as a continuum and allows to compute the actual microscopic flow phenomena that occur in the fibrous media. The phenomenological model cannot give information on unneeded microscopic events but the Darcy equation can describe macroscale flow patterns in porous media.

The Brinkman equation [19] is an extension of the Darcy equation. Introduction of a second order derivative into the Darcy equation allows the application of no-slip boundary conditions. The Brinkman equation can thus describe the flow field around solid bodies such as the embedded cells in extracellular matrix but the fibrous medium itself, e.g., the individual fibers, is still treated as a continuum.

The interstitial tissue can also be modeled by a poroviscoelastic material [20]. Biot developed a theory of poroelasticity for setting the resistance to flow through deformable fluid-saturated porous media [21–24]. The interstitial fluid, cells, and the extracellular matrix are viewed as a two-phase system consisting of a solid and a fluid phase, each of which is regarded as a continuum. Biot's equations for the linear theory of poroelasticity

are derived from the equations of linear elasticity for the solid matrix, the Navier–Stokes equations for the incompressible viscous fluid, and the Darcy’s law for the flow of a fluid through fibrous matrix.

In the context of acupuncture, the interstitial flow has been modeled by the Brinkman equations. The flow is driven by the difference in hydrostatic and osmotic pressures between the capillaries and the interstitial space [25,26].

Interstitial flow and deformed matrix fibers exert directly stress at the cell surface that can be sensed by membrane molecules. Mastocytes, among other cells, respond to fluidic stimuli caused by the fluid shear stress [27]. In this article, the transient convective Brinkman equations are applied to simulate the interstitial flow in a porous medium driven by a moving needle.

### 3 Mathematical modeling and methods

#### 3.1 Governing equations

In the present work, we study the effects of a moving needle on the interstitial fluid. We then consider a fluid motion governed by the incompressible convective Brinkman equations for a fluid-filled porous material. The Reynolds number and the Darcy number are normally small. As a result, the nonlinear drag such as the Forchheimer law [28] can be neglected. The derivation of the incompressible convective Brinkman equations was given in [29–31] in the context of heat transfer in porous media. The set of equations is as follows:

$$\frac{\rho}{\alpha_f} \left( \frac{\partial \bar{\mathbf{u}}}{\partial t} + \bar{\mathbf{u}} \cdot \nabla \left( \frac{\bar{\mathbf{u}}}{\alpha_f} \right) \right) - \mu \nabla^2 \bar{\mathbf{u}} + \frac{1}{\alpha_f} \nabla (\alpha_f p_f) = -\frac{\mu}{\mathcal{P}} \bar{\mathbf{u}} \quad \text{in } \Omega, \quad (3.1a)$$

$$\nabla \cdot \bar{\mathbf{u}} = 0 \quad \text{in } \Omega, \quad (3.1b)$$

where  $-\frac{\mu}{\mathcal{P}} \bar{\mathbf{u}}$  denotes the Darcy drag,  $\mu$  the fluid dynamic viscosity,  $\rho$  the fluid density,  $\mathcal{P}$  the Darcy permeability,  $\bar{\mathbf{u}}$  the averaged velocity and  $p_f$  the pressure. The averaged velocity is defined as

$$\bar{\mathbf{u}} = \alpha_f \mathbf{u}_f, \quad (3.2)$$

where  $\mathbf{u}_f$  is the fluid velocity and

$$\alpha_f = \frac{\text{fluid volume}}{\text{total volume}} \quad (3.3)$$

is the fluid volume fraction. This volume fraction corresponds to the effective porosity of the medium. The fluid fractional volume  $\alpha_f$  is taken as a space-dependent parameter to model the distinguished properties of an acupoint.

The system (3.1), subject to the initial conditions given below, are solved in the domain  $\Omega$

$$\bar{\mathbf{u}}(\mathbf{x}, 0) = \bar{\mathbf{u}}_0(\mathbf{x}). \quad (3.4)$$

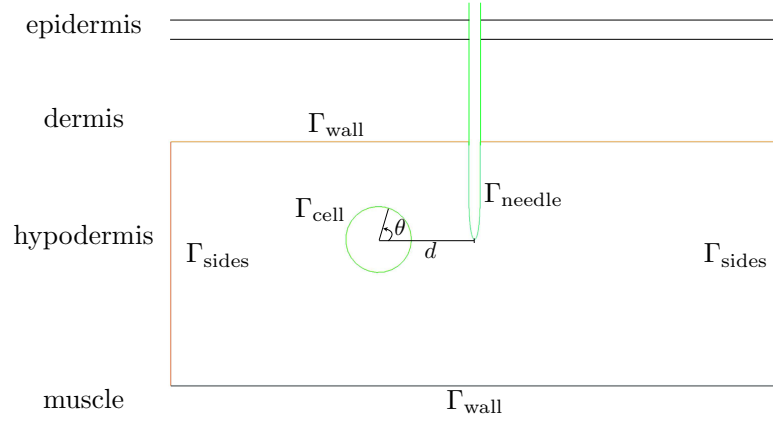


Figure 1: Schematic of the computational domain  $\Omega$ . The cell is mapped to the trigonometric unit circle where  $\theta$ , in radian, is the angle.  $d$  is the distance along the  $x$ -axis between the cell center and the needle tip.

The domain boundary (Fig. 1) can be decomposed into the needle boundary denoted by  $\Gamma_{needle}$ , an impervious boundary (wall) denoted by  $\Gamma_{wall}$ , and the open boundary on the sides denoted by  $\Gamma_{sides}$ .

On the top and bottom of the domain  $\Omega$ , the boundary condition on  $\Gamma_{wall}$  is prescribed as

$$\bar{\mathbf{u}} = 0. \quad (3.5)$$

On the needle boundary  $\Gamma_{needle}$ , the boundary condition is imposed as

$$\bar{\mathbf{u}} = \mathbf{v}_{needle}. \quad (3.6)$$

On both sides of the domain  $\Omega$ , the traction-free boundary condition is prescribed on  $\Gamma_{sides}$  as

$$-\mu \nabla \bar{\mathbf{u}} \cdot \mathbf{n} + p_f \mathbf{n} = 0. \quad (3.7)$$

## 4 Computational model

### 4.1 Scaling and setting for numerical simulations

$L$  denotes the characteristic length that is the needle width and  $V$  the characteristic velocity set to be the needle maximum velocity. Rescaling the variables thereby leads to

$$\mathbf{x}' = \frac{\mathbf{x}}{L}, \quad t' = \frac{t}{(L/V)}, \quad p' = \frac{p_f}{(\rho V^2)}, \quad \mathbf{u}' = \frac{\bar{\mathbf{u}}}{V}. \quad (4.1)$$

In the resulting dimensionless form, after removing the prime in the rescaled variables, the dimensionless incompressible convective Brinkman equations read as

$$\frac{1}{\alpha_f} \frac{\partial \mathbf{u}}{\partial t} + \frac{1}{\alpha_f} \mathbf{u} \cdot \nabla \left( \frac{\mathbf{u}}{\alpha_f} \right) - \frac{1}{\text{Re}} \nabla^2 \mathbf{u} + \frac{1}{\alpha_f} \nabla(\alpha_f p) = -\frac{1}{\text{DaRe}} \mathbf{u}, \quad (4.2)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (4.3)$$

where Re is the Reynolds number and Da is the Darcy number. The two dimensionless parameters are defined as

$$\text{Re} = \frac{\rho L V}{\mu}, \quad \text{Da} = \frac{\mathcal{P}}{L^2}. \quad (4.4)$$

Numerical simulations are carried out considering  $\text{Re} = 0.103$  and varying values of Da. The values for the non-dimensional variables of the fluid are computed from the data available in [16].

In considering the above dimensionless governing equations, the boundary condition on the domain boundary is prescribed as

$$\mathbf{u} = 0 \quad \text{on } \Gamma_{wall}, \quad (4.5)$$

$$\mathbf{u} = \mathbf{v} \quad \text{on } \Gamma_{needle}, \quad (4.6)$$

$$-\frac{1}{\text{Re}} \nabla \mathbf{u} \cdot \mathbf{n} + p \mathbf{n} = 0 \quad \text{on } \Gamma_{sides}. \quad (4.7)$$

## 4.2 Numerical methods

The governing equations in Section 4.1 are solved using the finite element software FreeFem++ [32]. This code programs the discrete equations derived from the finite element weak formulation of the problem presented in Section 4.2.2 using a characteristic/Galerkin model to stabilize the convection terms. A straightforward implementation of the mesh moving scheme is presented in Section 4.2.1.

### 4.2.1 ALE implementation on moving meshes

Different methods applied to predict flows in moving domains with the finite element method exist. Moving mesh methods use meshes following the domain occupied by the fluid. The mesh is then exactly given at the boundary. The ALE framework is mathematically rigorous to describe transport phenomena in time and allows some freedom in the description of the mesh motion. However, it raises some implementation questions on the interface tracking with time discretization. Implementation of the ALE method can be done in FreeFem++ [33].

In the present paper, the ALE framework in FreeFem++ is employed to compute the flow in the moving domain. In the current problem setting, the motion of needle is prescribed with respect to time. The boundary of the domain is thus exactly known at each time step so that an area preserving mesh can be precisely generated.

For a detailed description of the ALE approach, the readers can refer to [34]. The framework of the ALE approach is briefly described below. Let  $\Omega(t)$  be the domain at each time  $t$  with regular boundary  $\partial\Omega(t)$ . In the Eulerian representation, the fluid is described by

$$\mathbf{u}(\mathbf{x},t) \quad \text{and} \quad p(\mathbf{x},t), \quad \forall \mathbf{x} \in \Omega(t). \quad (4.8)$$

To follow a moving domain, one can define the ALE map as

$$\tilde{\mathcal{A}}: \tilde{\omega} \times \mathbb{R}^+ \rightarrow \mathbb{R}^2 \quad (\tilde{\mathbf{x}},t) \rightarrow \tilde{\mathcal{A}}(\tilde{\mathbf{x}},t) := \tilde{\mathcal{A}}_t, \quad (4.9)$$

such that  $\omega(t) = \tilde{\mathcal{A}}(\tilde{\omega},t)$ , where  $\tilde{\omega}$  is the reference computational domain. Given an ALE field  $\tilde{q}: \tilde{\omega} \times \mathbb{R}^+ \rightarrow \mathbb{R}$ , its Eulerian description is given by

$$\forall \mathbf{x} \in \Omega(t), q(\mathbf{x},t) = \tilde{q}(\tilde{\mathcal{A}}_t^{-1}(\mathbf{x}),t). \quad (4.10)$$

In ALE framework, the computational domain velocity (ALE velocity or grid velocity) is defined as

$$\tilde{\mathbf{a}}(\tilde{\mathbf{x}},t) = \frac{\partial \tilde{\mathcal{A}}}{\partial t}(\tilde{\mathbf{x}},t), \quad \forall \tilde{\mathbf{x}} \in \tilde{\omega}, \quad (4.11)$$

so that we can get

$$\mathbf{a}(\mathbf{x},t) = \tilde{\mathbf{a}}(\tilde{\mathcal{A}}_t^{-1},t). \quad (4.12)$$

The ALE time-derivative is defined as

$$\left. \frac{\partial q}{\partial t} \right|_{\tilde{\mathcal{A}}} = \frac{d}{dt} q(\tilde{\mathcal{A}}(\tilde{\mathbf{x}},t),t), \quad (4.13)$$

and the following identity holds

$$\left. \frac{\partial q}{\partial t} \right|_{\tilde{\mathcal{A}}} = (\mathbf{a} \cdot \nabla) q + \frac{\partial q}{\partial t}. \quad (4.14)$$

A general method used to construct the mapping, or equivalently the domain velocity  $\mathbf{a}$ , consists of solving the following equations

$$-\nabla^2 \mathbf{a} + \nabla p = 0, \quad (4.15)$$

$$\nabla \cdot \mathbf{a} = 0, \quad (4.16)$$

$$\mathbf{a}|_{\partial\Omega} = \mathbf{v}. \quad (4.17)$$

The needle motion is prescribed by setting the vertical velocity of the tip to  $v_2 = v \cos(2\pi ft)$ , where the needle insertion speed  $v$  and the oscillation frequency  $f$  of the needle are equal to  $10 \text{ mm.s}^{-1}$  and  $0.5 \text{ s}^{-1}$ , respectively [2] (see Fig. 2). In the ALE framework, subject to a prescribed needle motion, the Eqs. (4.2)-(4.3) become

$$\left. \frac{1}{\alpha_f} \frac{\partial \mathbf{u}}{\partial t} \right|_{\tilde{\mathcal{A}}} - \frac{1}{\alpha_f} (\mathbf{a} \cdot \nabla) \mathbf{u} + \frac{1}{\alpha_f} \mathbf{u} \cdot \nabla \left( \frac{\mathbf{u}}{\alpha_f} \right) - \frac{1}{\text{Re}} \nabla^2 \mathbf{u} + \frac{1}{\alpha_f} \nabla(\alpha_f p) = -\frac{1}{\text{DaRe}} \mathbf{u}, \quad (4.18)$$

$$\nabla \cdot \mathbf{u} = 0. \quad (4.19)$$

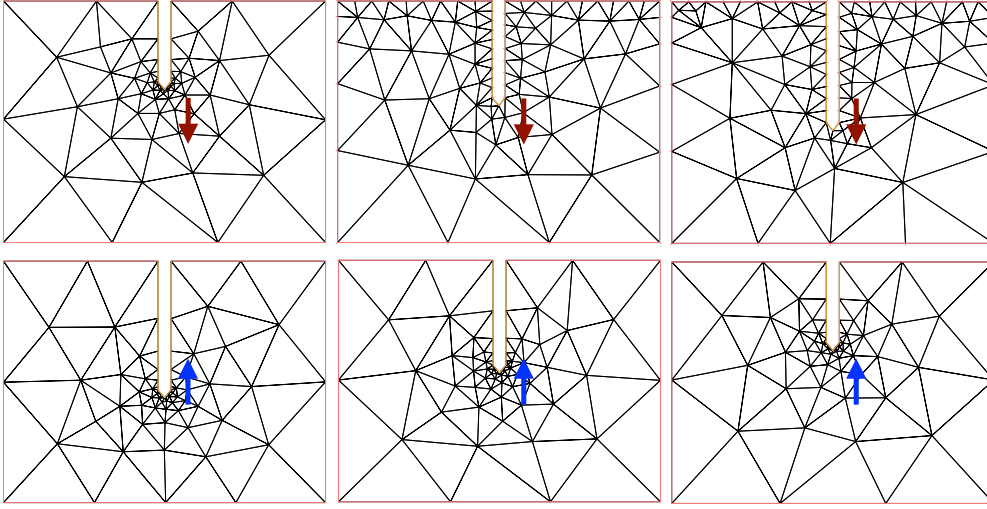


Figure 2: Illustration of the prescribed moving needle motion and the generated meshes (not actual meshes). At each time step, the new mesh is generated by the FreeFem++ `movemesh` function.

The solutions  $\mathbf{u}$  and  $p$  are sought subject to the initial (3.4) and boundary conditions (3.5)-(3.7) described in Section 4.1. The ALE time-derivative induces a correction term in the transport term by subtracting the computational domain velocity  $\mathbf{a}$  from the transport velocity  $\mathbf{u}$ .

#### 4.2.2 Finite element discretization

To solve for the convective Brinkman equations and to fix the pressure constant part, the convective Brinkman equations with the pseudo compressibility approximation are investigated

$$\frac{1}{\alpha_f} \frac{\partial \mathbf{u}}{\partial t} \Big|_{\mathcal{A}} - \frac{1}{\alpha_f} (\mathbf{a} \cdot \nabla) \mathbf{u} + \frac{1}{\alpha_f} \mathbf{u} \cdot \nabla \left( \frac{\mathbf{u}}{\alpha_f} \right) - \frac{1}{\text{Re}} \nabla^2 \mathbf{u} + \frac{1}{\alpha_f} \nabla (\alpha_f p) = -\frac{1}{\text{DaRe}} \mathbf{u} \quad \text{in } \Omega(t), \quad (4.20)$$

$$\nabla \cdot \mathbf{u} + \varepsilon p = 0 \quad \text{in } \Omega(t), \quad (4.21)$$

where  $\varepsilon$  is a small parameter. The formulation consisting of (4.20)-(4.21) follows the so called artificial compressibility method introduced in [35] and [36].

The convective Brinkman equations are approximated with the method of characteristics for the nonlinear convection term and a Galerkin method for the rest of the spatial



derivative terms. The time discretization of (4.20)-(4.21) gives

$$\begin{aligned} \frac{1}{\Delta t} \left( \frac{\mathbf{u}^{n+1}}{\alpha_f} - \left( \frac{\mathbf{u}^n}{\alpha_f} \right) \circ X^n(\mathbf{x}) \right) - \frac{1}{\alpha_f} (\mathbf{a} \cdot \nabla) \mathbf{u}^{n+1} - \frac{1}{\text{Re}} \nabla^2 \mathbf{u}^{n+1} + \frac{1}{\alpha_f} \nabla (\alpha_f p^{n+1}) \\ = -\frac{1}{\text{DaRe}} \mathbf{u}^{n+1} \quad \text{in } \Omega^{n+1}, \end{aligned} \quad (4.22)$$

$$\nabla \cdot \mathbf{u}^{n+1} + \varepsilon p^{n+1} = 0 \quad \text{in } \Omega^{n+1}. \quad (4.23)$$

Note that  $\mathbf{X}^n \approx \mathbf{x} - \frac{\mathbf{u}^n}{\alpha_f}(\mathbf{x}) \Delta t$ . For all  $\varphi \in H^{1/2}(\Gamma_{\text{needle}})$ , let us introduce the product space

$$V_\varphi = \left\{ (\mathbf{w}, q) \in [H^1(\Omega)]^2 \times L^2(\Omega), \mathbf{w} = \varphi \text{ on } \Gamma_{\text{needle}}, \mathbf{w} = 0 \text{ on } \Gamma_{\text{wall}} \right\}. \quad (4.24)$$

Let

$$(a, b) = \int_{\Omega^{n+1}} ab \, dx. \quad (4.25)$$

From the weak form of (4.22)-(4.23) subject to the boundary conditions in (3.5)-(3.7), the time discretization gives the following scheme

find  $(\mathbf{u}^{n+1}, p^{n+1}) \in V_g$  such that

$$\begin{aligned} \frac{1}{\Delta t} \left( \frac{\mathbf{u}^{n+1}}{\alpha_f} - \left( \frac{\mathbf{u}^n}{\alpha_f} \right) \circ X^n(\mathbf{x}), \mathbf{w} \right) - \left( \frac{1}{\alpha_f} (\mathbf{a} \cdot \nabla) \mathbf{u}^{n+1}, \mathbf{w} \right) \\ + \frac{1}{\text{Re}} \left( \frac{1}{\alpha_f} \nabla \mathbf{u}^{n+1}, \nabla \mathbf{w} \right) - \left( \alpha_f p^{n+1}, \nabla \cdot \left( \frac{\mathbf{w}}{\alpha_f} \right) \right) + \frac{1}{\text{DaRe}} (\mathbf{u}^{n+1}, \mathbf{w}) = 0, \end{aligned} \quad (4.26)$$

$$\left( \nabla \cdot \mathbf{u}^{n+1}, q \right) + \varepsilon (p^{n+1}, q) = 0, \quad (4.27)$$

$$\forall (\mathbf{w}, q) \in V_0.$$

The Taylor-Hood P2-P1 elements are adopted to ensure satisfaction of the LBB (Ladyžhenskaya [37]–Babuška [38]–Brezzi [39]) stability condition. Note that temporal accuracy order of the presented characteristic/Galerkin method is one. Meshes are generated within FreeFem++ so that the solution is independent of the mesh. Mesh adaptation is performed prior to simulations so as to enhance the mesh quality around the needle and the cell.

## 5 Results and discussion

### 5.1 Effect of the needle motion on the interstitial flow

In this study, the needling direction is perpendicular to the skin surface. In practice, it is possible that the needling direction is oblique to the skin surface. The simulation results

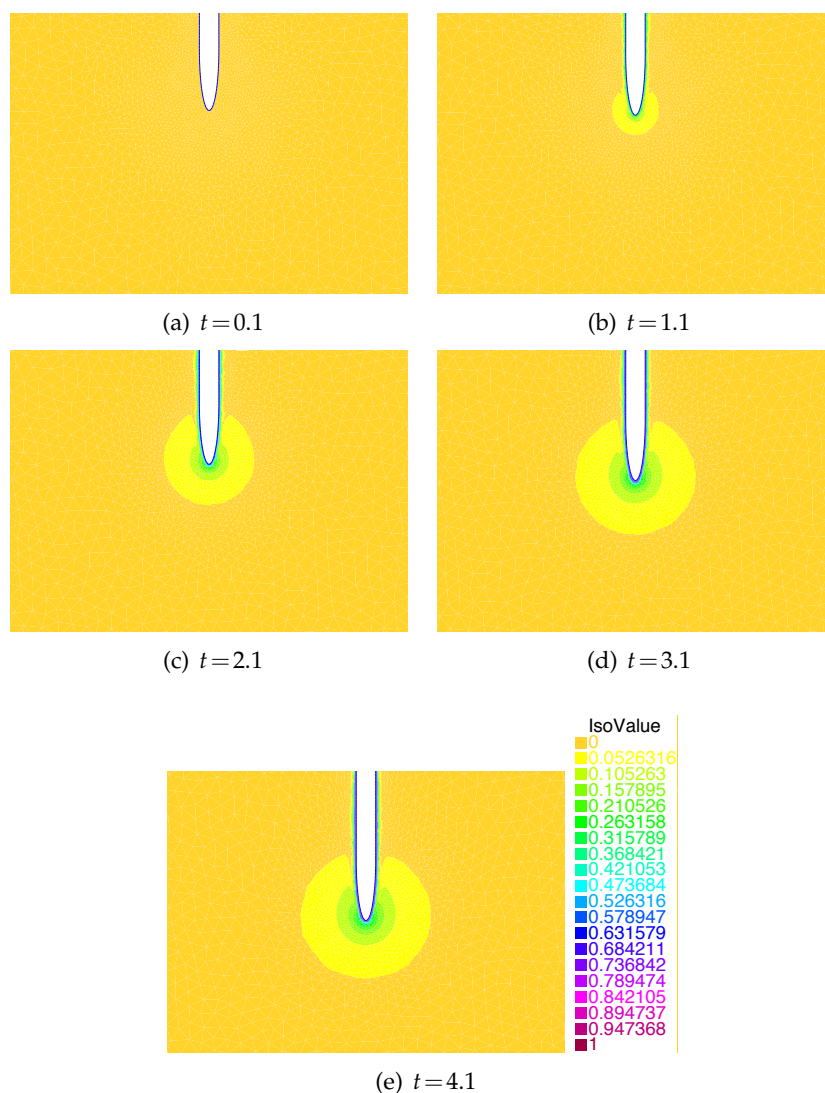


Figure 3: The predicted time varying velocity magnitude  $|\mathbf{u}|$  resulting from the needle motion in interstitial fluid.

show that the insertion of an acupuncture needle can influence interstitial fluid flow. Indeed, under a stress field, the meshwork constituted by hydrated proteoglycans and glycoproteins can be degraded, hence undergoing fluidization. The computed velocity field shows that the magnitude of the velocity is higher in regions close to the needle tip (Fig. 3). At a location away from the needle, where the effect of the stress field vanishes, the meshwork is not affected. After a certain delay, when the matrix reaches a new equilibrium state, that is in the permanent regime before the following needle manipulation, proteoglycans and glycoproteins can repolymerize (gelification).

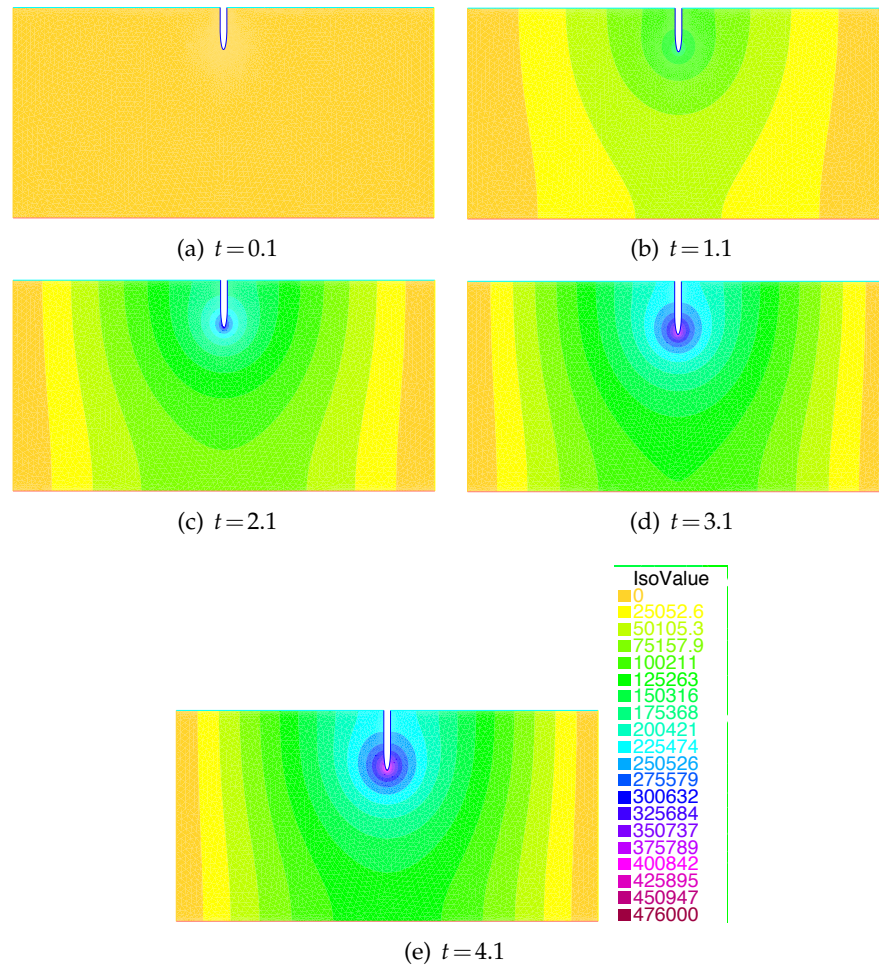


Figure 4: The predicted time varying pressure contours resulting from the needle motion in interstitial fluid.

Furthermore, when the needle reaches its maximum speed, the interstitial pressure gradient becomes higher at a location close to the needle tip (Fig. 4). The changes in the interstitial fluid flow and the high pressure gradient can affect the activities of the mastocyte pools in the stimulated area.

## 5.2 Effects of the fractional fluid volume and the Darcy permeability on the interstitial flow

The fractional fluid volume  $\alpha_f$ , defined in Eq. (3.3), and the Darcy permeability  $\mathcal{P}$  are chosen to model the fibrous tissue matrix. It is noted that the fractional fluid volume and the permeability alone cannot fully describe the microscopic behavior of the fiber matrix. Acupoints are situated in regions, close to neurovascular bundles, in which fi-

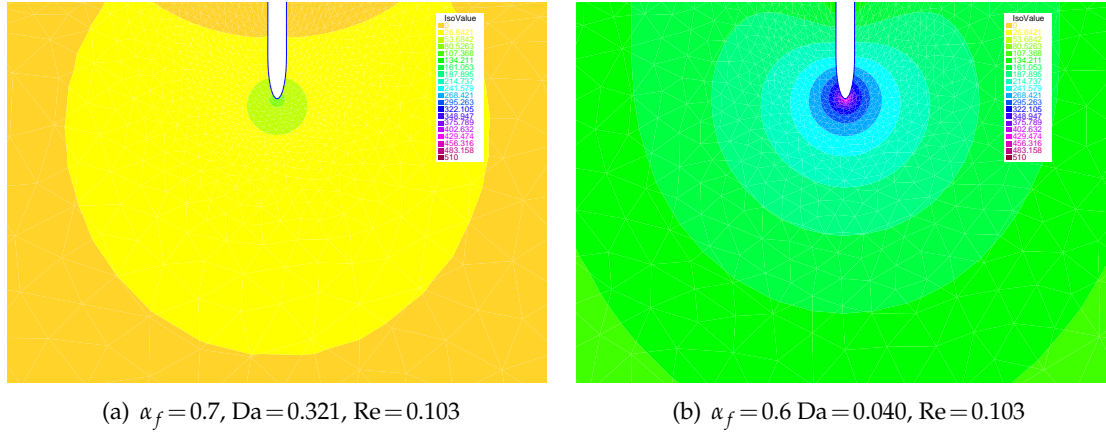


Figure 5: The predicted pressure contours resulting from the motion of a needle in interstitial fluid of the subcutaneous connective tissue at acupoint (b) and in a non-acupoint (a). High interstitial pressure is only expected to occur at acupoint.

brous matrix, capillaries, lymphatic vessels, nervous structures, and cells abound. It is then reasonable to make the assumption that the fractional fluid volume  $\alpha_f$  is lower at the acupoint loci. In soft tissues, the permeability of the tissue  $\mathcal{P}$  varies with the fluid volume fraction  $\alpha_f$  [16]. Amongst the laws for the permeability of the tissue reported in [40], the Karman-Kozeny relation given in [41] is adopted. The Karman-Kozeny equation relates the permeability  $\mathcal{P}$  to the fractional fluid volume  $\alpha_f$  and the extracellular matrix fiber properties. With a simplified and structured network of cylindrical parallel and perpendicular fibers, the Karman-Kozeny equation reads as follows [16]

$$\mathcal{P} = \frac{\alpha_f f_r^2}{f_k}, \quad (5.1)$$

where  $f_r = (f_s^2 - f_d^2)/4f_d$  and  $f_k = (2/3)k_+ + (1/3)k_-$ . Note that  $f_d$  and  $f_s$  correspond to the fiber diameter and the fiber spacing, respectively.

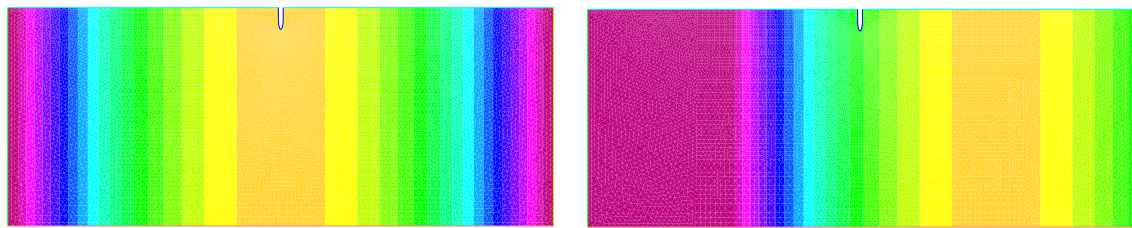
$$k_+ = 2\alpha_f^3 / (1 - \alpha_f) [\ln(1/(1 - \alpha_f)) - (1 - (1 - \alpha_f)^2) / (1 + (1 - \alpha_f)^2)]$$

and

$$k_- = 2\alpha_f^3 / (1 - \alpha_f) [2\ln(1/(1 - \alpha_f)) - 3 + 4(1 - \alpha_f) - (1 - \alpha_f)^2]$$

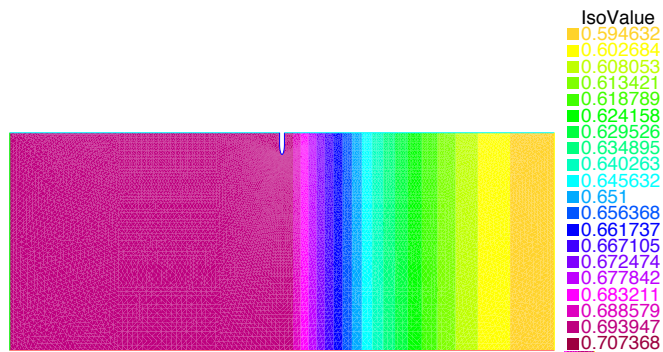
model the resistance to a flow perpendicular and parallel to the fibers, respectively. The flow through interstitium is computed according to the data given in [40]. The simulation results shown in Fig. 6 suggest that the effects of the needle on the interstitial pressure are more effective in a fiber richer tissue. In particular, the expected high interstitial pressure is achieved only at acupoint.

Another case to consider is the local decrease of the values  $\alpha_f$  and  $\mathcal{P}$  such that they reach their minimum precisely at the acupoint. The simulation results reveal that a

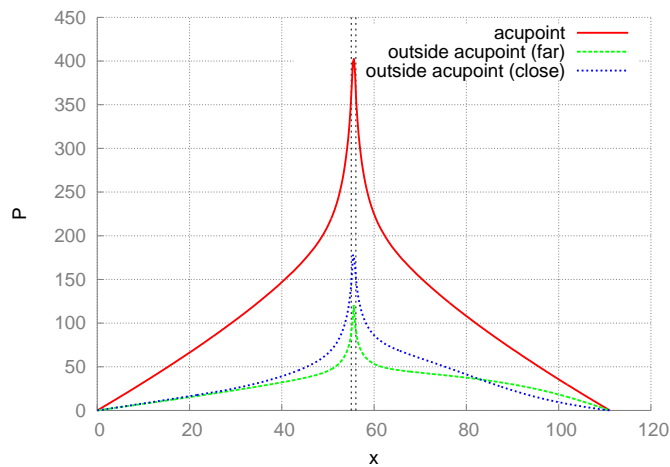


(a)  $\alpha_f$  distribution with needle at acupoint.

(b)  $\alpha_f$  distribution with needle close to acupoint.



(c)  $\alpha_f$  distribution with needle far from acupoint.



(d) Pressure  $p$  resulting from the motion of the needle at different locations.

Figure 6: The predicted contours of  $\alpha_f$ . Regions with a lower magnitude of  $\alpha_f$  correspond to regions where the fibrous medium is denser. The needle is inserted (a) at the acupoint, (b) close to the acupoint, and (c) far from the acupoint. On each numerical experiment, the needle is inserted at the same coordinate while the  $\alpha_f$  distribution is shifted. (d) High interstitial pressure is only expected to occur when the needle, displayed by the dot lines, is inserted at acupoint.

proper location of the insertion of the needle is relevant. The effects of the needle on the interstitial flow are greater when the needle is inserted exactly at the acupoint. According to the  $\alpha_f$  and  $\mathcal{P}$  distributions on the one hand and to the needle position with respect to the location of the peak density of fibers (Figs. 6(a), 6(b), and 6(c)) on the other hand, the expected high pressure solution is reached only when the needle is inserted at the acupoint (Fig. 6).

### 5.3 Shear stress and pressure distribution along the cell membrane

In this section we are interested in the effects of the fluidic stimuli on an interstitial cell. Simulations are carried out by considering no-slip boundary condition prescribed at the cell surface. Fig. 7 shows the velocity magnitude and the pressure contours with a cell added closely to the needle.

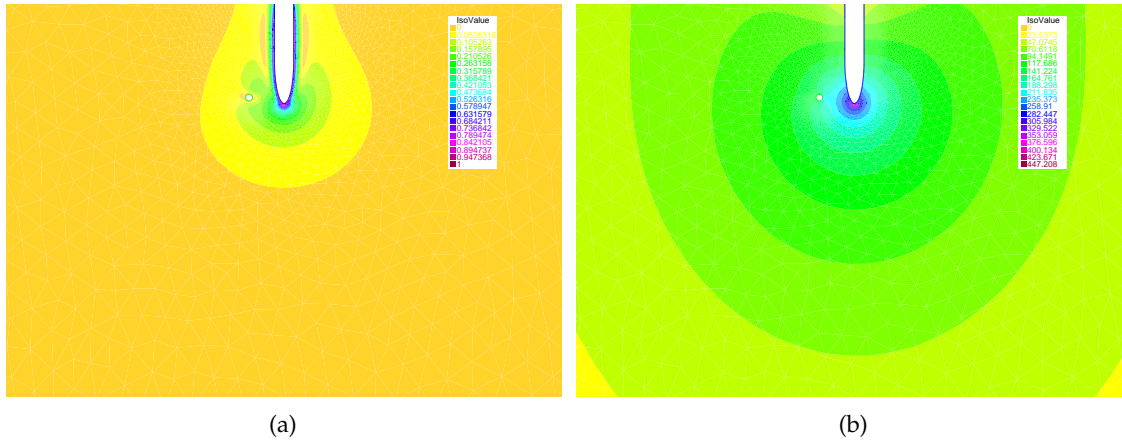


Figure 7: The predicted contours of (a) the velocity and (b) the pressure with an interstitial cell inserted at a location near the needle.

Fig. 8 shows the distribution of the shear stress and the pressure along the cell surface. The cell is mapped to the trigonometric unit circle, where  $\theta$  denotes the angle in radian. The pressure on the cell surface is higher at a point closest to the needle tip ( $\theta \in [\frac{\pi}{2}, \frac{3\pi}{2}]$ ), whereas the shear stress can be higher on the side furthest from the needle ( $\theta \in [-\frac{3\pi}{2}, \frac{\pi}{2}]$ ). The interstitial flow induced by needle motion leads to a phase shift around the cell surface of the maximum shear stress and pressure value. This shows that all the cell surface could be stimulated. Fig. 9 shows that high shear stress appears at a location close to the needle.

Mastocytes have been shown to respond to fluid shear stress [42]. These local mechanical forces can trigger the activation of mechanoresponsive proteins on the cell surface [11] so that  $\text{Ca}^{++}$  is allowed to enter the cytosol via pressure and shear stress gated ion channels.

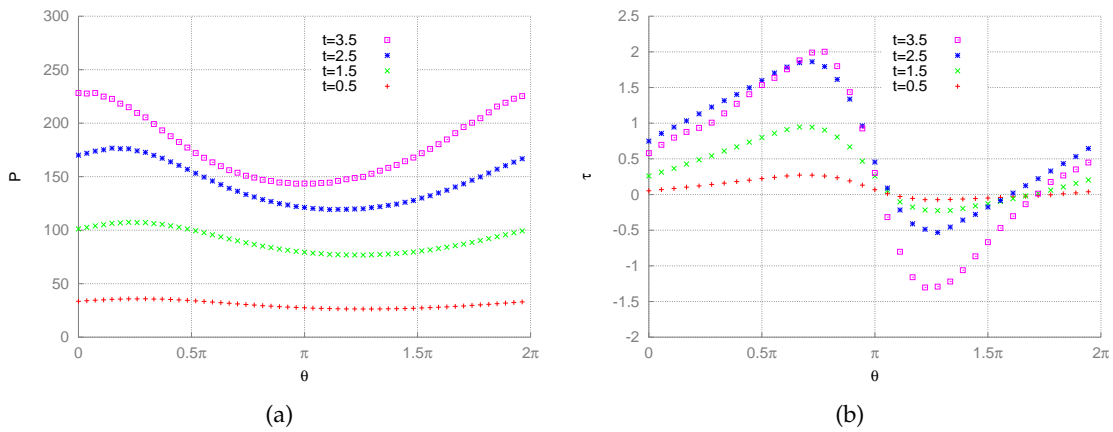


Figure 8: Evolution of (a) the pressure and (b) the shear stress distributions along the cell surface. The cell is mapped to the trigonometric unit circle where  $\theta$ , in radian, is the angle defined in Fig. 1.

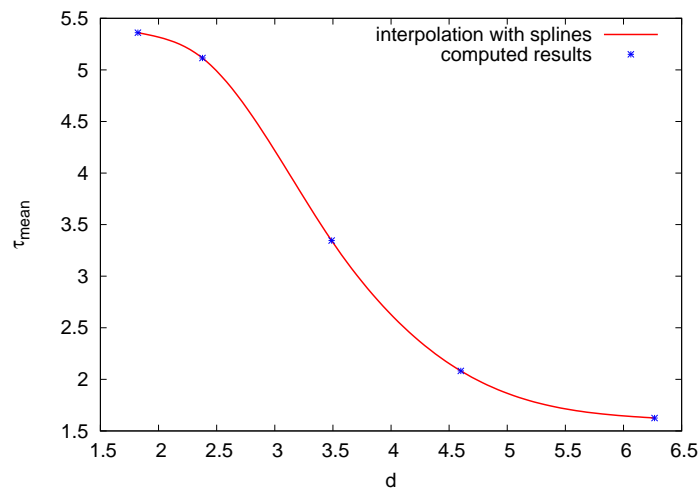


Figure 9: The predicted mean shear stress  $\tau_{\text{mean}}$  on the cell surface with respect to the distance  $d$  measured from the needle (see Fig. 1). A higher shear stress is expected to be observed at a location close to the needle.

## 6 Concluding remarks

Insertion into the skin of a thin needle is the most common acupuncture technique. The insertion of a needle generates a mechanical stress field in the interstitium. In the present study, the deformation of the extracellular matrix is neglected and only the effect of interstitial flow is considered, which leads to the following general remarks.

The proposed model for the interstitial flow is able to describe the shear stress on a given mastocyte and the interstitial pressure from the macroscopic point of view. The simulation results suggest that the needling effects are most effective when an accurate

insertion and a manipulation are exerted. High local fluid pressure and shear stress on cells are most likely to appear near the needling region.

The numerical results presented in this study seem to be sensitive to the fractional fluid volume and Darcy permeability. These two parameters describe the macroscopic property of the fiber matrix. The choice for the values of these parameters is a limitation of the model due to the lack of complete biological data. Thus, it is not possible to build a quantitative model for the acupoint yet. Furthermore, the proposed method does not allow the rotation of the needle to be taken into account. When considering the rotation of the needle, a large deformation of the tissue is observed with the twisting of the fibers around the needle, that in turn makes the corresponding change in interstitial flow. A 3D fluid/structure model taking into account the mechanics of the fibers should then be considered. However, numerical prediction of the interstitial pressure and shear stress is clearly an essential tool to gain a better understanding of the mechanism involved in the acupuncture needling.

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