NONCOMMUTATIVITY NONASSOCIATIVITY AND NONLOCALITY IN STRING THEORY

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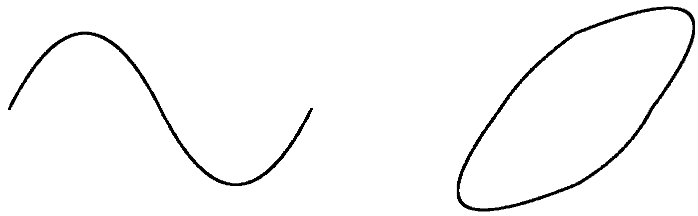
String theory is a promising candidate for quantum gravity (and everything else).



open string **closed string**

String $\simeq \infty$ particles

Nature of Space-Time

String Theory $\rightarrow$ 9+1 dimensions.

Geometry $\leftrightarrow$ Physics

Euclidean Geometry

(Newtonian Physics)

$$\downarrow$$

Riemannian Geometry

(General Relativity – Gravity)

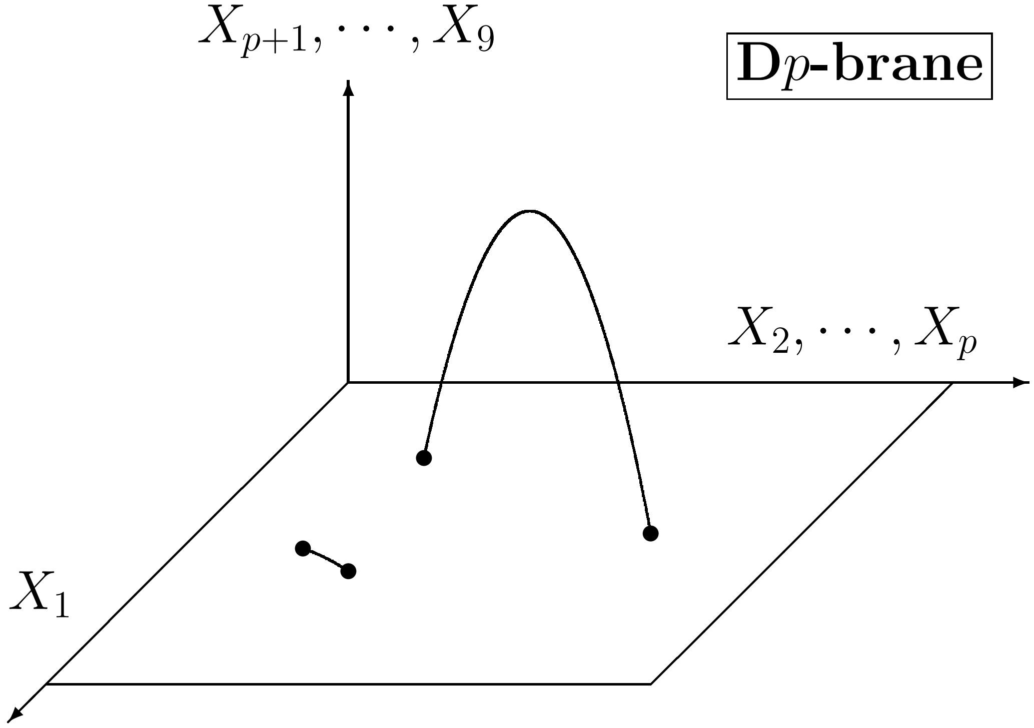
$$\downarrow$$

Non-Commutative Geometry

(Quantum Gravity, String Theory)

Quantum Spacetime:

[**Yang:** **On Quantized Spacetime, Phys. Rev. 72 (47) 874**]



Dp-brane $\simeq$
p-dim. subspace on which open strings end

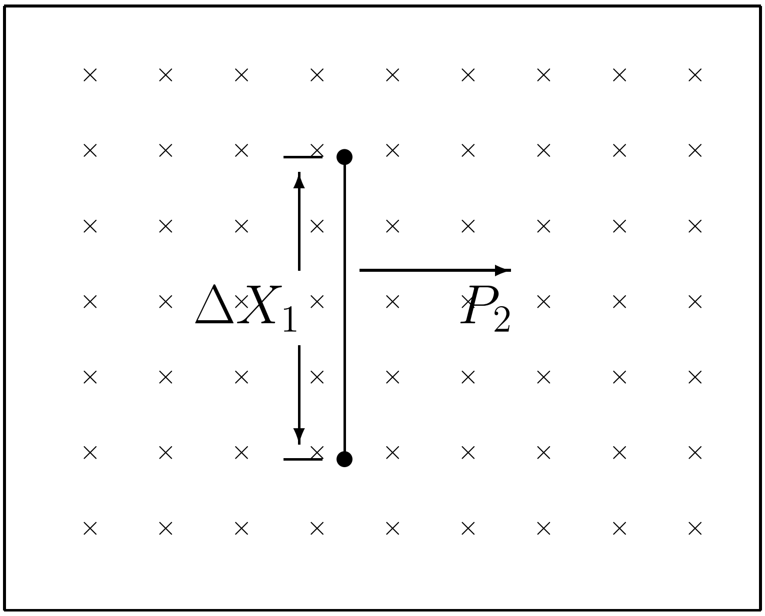
Physics on N coincident D-branes $\simeq$
$SU(N)$ Yang-Mills theory

D-brane with Background Field

B field background: *[Chu+Ho(98)]*

$$S = S_0 + \int B$$

$B =$ **2-form gauge field**



Nonlocality:

$$\Delta X_1 = B_{12} P_2$$

$$\Rightarrow [X_1, X_2] \sim [\Delta X_1, X_2] = i B_{12}$$

Dipole → Nonlocal → NC

Quantization of open string *[Chu+Ho(98,99)]*

$$\Rightarrow [X_i, X_j] = \left(\frac{B}{1+B^2}\right)_{ij} \simeq iB^{-1}_{ij}$$

Noncommutativity effectively accounts for the interactions with the background B field.

Correlation functions for NC theory $\simeq$ Corr. fx's for B field background

[Schomerus (99)] [Seiberg+Witten(99)]

$H = dB = 0 \rightarrow$ **Associativity**

$$(fg)h = f(gh)$$

For $H = dB \neq 0$:

<u>ASSOCIATIVE or NON-ASSOCIATIVE?</u>

Correlation functions ***[Cornalba+Shiappa(01)]*** $\rightarrow$

(indices omitted)

$$(f \bullet g) \bullet h - f \bullet (g \bullet h) = \frac{1}{6} B^{-1} B^{-1} B^{-1} H (\partial f)(\partial g)(\partial h) + \cdots$$

open string quantization: ***[Ho+Yeh(00)]***

$$[X, X] = iB^{-1} + \frac{1}{3} B^{-1} B^{-1} B^{-1} H P + \cdots$$

$\Rightarrow$ **associative algebra** ***[Ho(01)]***

The algebra of X and P is mixed,
like the quantized spacetime of Yang (47).

How to define gauge transformations?

[Ho(01)]

Non-Commutative Spacetime

In the neighborhood of N D-branes, quantize a probing D-brane ⇒

[Ho(00)]

$$[X_i, X_j] = iL_{ij}$$

same as Yang (47) for 3+1 dim.

For $AdS_2 \times S^2$**,** ***[Ho+Li(00)]***

Noncommutativity →

uncertainty relation →

Bekenstein-Hawking entropy for charged black hole.

NC → UNCERTAINTY → NONLOCALITY

Fluctuations of background field

[Chu+Ho+Kao(99)]

→ Uncertainty principle of string theory

[Yoneya(97), Li+Yoneya(98)]

$$\Delta x \Delta t \geq 1/T_s$$

→ Nonlocality

a salient feature of string theory

Perturbation theory of higher derivative or nonlocal theories

[Yang + Feldman:

The S-Matrix in the Heisenberg Representation,

Phys. Rev. 79, 972 (50)]

applied to string field theory ***[Cheng+Ho+Yeh(01,02)]***

Nonlocality $\sim$ limit of higher derivatives

Higher derivatives $\rightarrow$
more degrees of freedom

Nonlocality $\rightarrow \infty$ degrees of freedom

String $\simeq \infty$ ***particles***

$\cdots \cdots \cdots$

<u>**String $\simeq$ Nonlocal particle ?**</u>

Reparametrization symmetry of
(*nonlocal*) particle worldline $\simeq$
Conformal symmetry of string worldsheet

[Ho(02)]

particle worldline Lagrangian

$$L = L(x, \dot{x}, \ddot{x}, \cdots, x^{(n)})$$

$\boxed{n = 1\text{:}}$

Phase space: x **and its conjugate variable**

Reparametrization:

$$\delta x = \epsilon \dot{x}$$

is generated by the Hamiltonian

$$[H, x] = i\dot{x}$$

$\boxed{n = 2\text{:}}$

Phase space: x**,** $\dot{x}$ **and their conj. var.s**

Reparametrization: (in addition to the one above)

$$\delta \dot{x} = (\epsilon \dot{x})^{\cdot} = \epsilon \ddot{x} + \dot{\epsilon} \dot{x}$$

H **only generates the term for** ϵ**.**

New generator needed for $\dot{\epsilon}$**.**

Theories with higher order time derivatives:

Phase space is larger.

Repara. has more independent parameters:

$$\epsilon, \dot{\epsilon}, \ddot{\epsilon}, \cdots$$

$\rightarrow$ **More symmetry generators.**

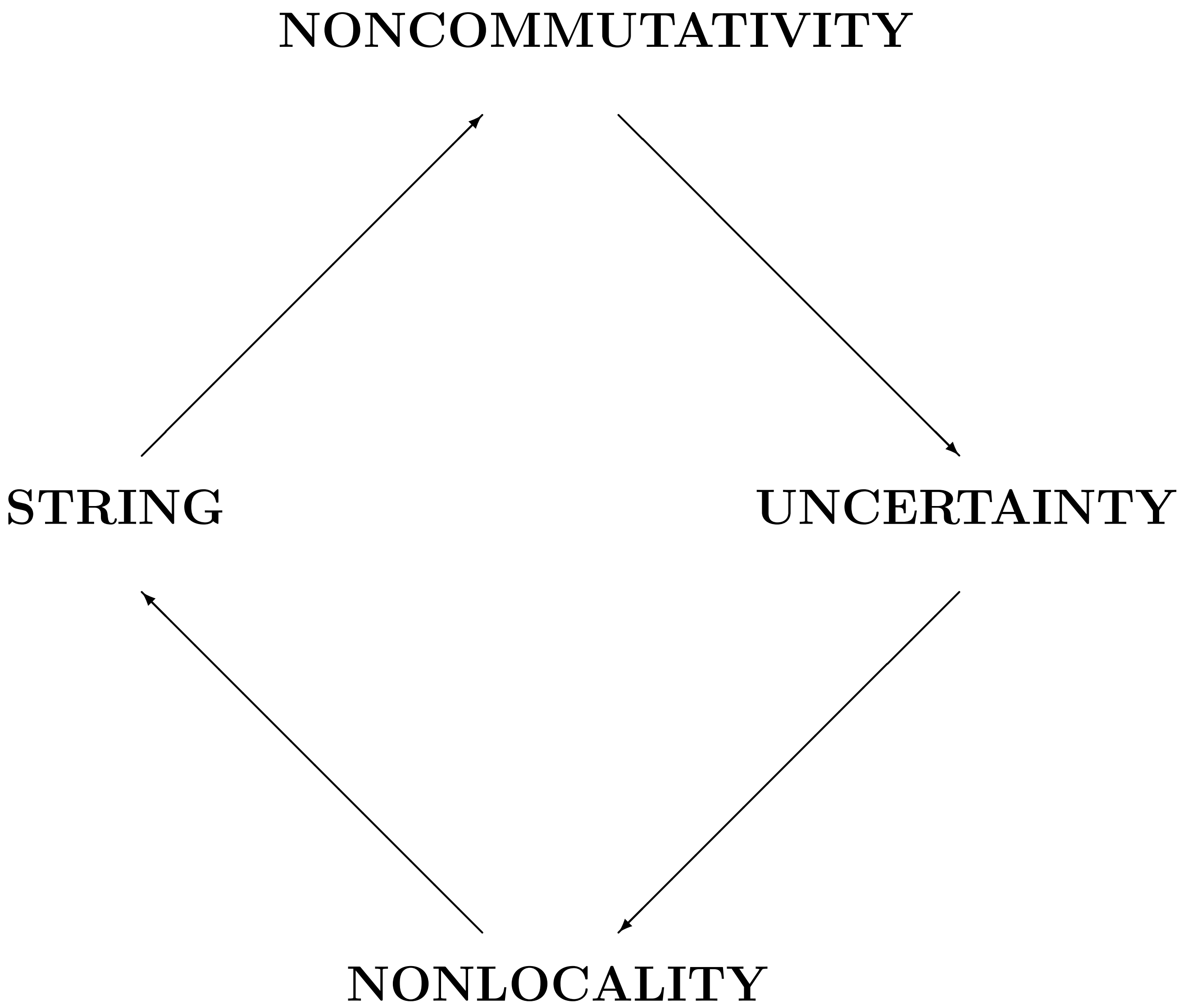
$$
\begin{aligned}
L = &\\
e\Big[&A^{(0)}(x) + A^{(1)}_{\mu}(x)Dx^{\mu}\\
&+A^{(01)}_{\mu}(x)D^2x^{\mu} + A^{(20)}_{\mu\nu}(x)Dx^{\mu}Dx^{\nu}\\
&+A^{(001)}_{\mu}D^3x^{\mu} + A^{(110)}_{\mu\nu}D^2x^{\mu}Dx^{\nu} + A^{(300)}_{\mu\nu\lambda}Dx^{\mu}Dx^{\nu}Dx^{\lambda}\\
&+\cdots\Big],
\end{aligned}
$$

$$
D = \frac{1}{e}\frac{d}{d\tau} = \textbf{covar. derivative}
$$

nonlocal particle $\simeq$
open string with spacetime metric $g_{\mu\nu} \to 0$

If analogous to String Field Theory,
Conformal Symmetry $\to$
theory of the background fields $A^{(\cdot)}(x)$

String Theory $\simeq$
theory of all theories of particles



NONASSOCIATIVITY?