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Coupons and Price Discrimination in Vertically-Correlated Markets

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This research analyzes the non-cooperative and cooperative strategies with respect to manufacturer and retailer coupons. In a model with one manufacturer selling its product to one retailer, it is found that the retailer can achieve third-degree price discrimination equilibrium in retail markets by issuing coupons to demanders with higher elasticity. Although facing only one retailer, the manufacturer can also achieve the same third-degree price discrimination equilibrium by issuing coupons directly to demanders of higher elasticity. However, when only one firm issues the coupon, both manufacturer and retailer coupons can help alleviate the channel profit loss due to double marginalization. If the manufacturer and the retailer non-cooperatively issue coupons, then the subgame-perfect Nash equilibrium outcomes are equivalent to those under the successive third-degree price discrimination. Moreover, cooperative strategies between the manufacturer and the retailer can eliminate double marginalization, achieve the vertical integration effect, and lead to higher profits, consumer surpluses, and social surpluses than non-cooperative coupon strategies. Copyright © 2004 John Wiley & Sons, Ltd.

INTRODUCTION

At first sight, coupons serve as a price promotion strategy which benefits consumers. However, it is well known in existing literature that coupons can be used by a monopolistic firm as a price discrimination instrument to exploit consumer surplus (see for example, Varian, 1980; Narasimhan, 1984; Salop and Stiglitz, 1982; Jeuland and Narasimhan, 1985; Lazear, 1986; Gerstner and Hess, 1991a, b, 1995; and so on). Since only specific groups of consumers receive coupons, the coupon can separate consumers into two groups and hence *de facto* separate one market into two markets. The firm can thus use coupons to price-discriminate against those consumers receiving no coupons so as to promote its profit.

Most countries have competition laws prohibiting firms from explicit price discrimination. However, firms can implicitly separate consumers into groups and then practice price discrimination through different promotion strategies. For example, seasonal sales separate consumers into two groups: One pays a higher price at the regular time and the other pays a lower price at the sales time. Firms can also separate demand into two groups according to certain identifiable characteristics. For example, there are many special offers for students and/or faculties in respect to magazine subscriptions, credit card applications, travel, haircuts, etc. These practices enable firms to effectively charge different prices for academic and non-academic groups.

As a price promotion strategy, coupons can also be a means for price discrimination. Supermarkets

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often offer consumers retail coupons that are valid for a limited period of time. However, not all consumers use coupons in their shopping, because consumers do not always come to a supermarket when there is a coupon for a specific product. Due to different opportunity costs of time, consumers spend different amounts of time searching for coupons. As a result, some consumers do not use coupons even when there is a coupon for a specific product. Coupons thus enable firms to separate consumers into groups and effectively charge them different prices for the same product.

There are plenty of existing studies on price discrimination effects through price promotion strategies. Varian (1980) pioneers the formal modeling of price promotion strategies. With price promotion strategies, the firm is able to segment consumers into informed and uninformed groups and achieve price discrimination. Varian assumes that each consumer in the model buys, at most, one unit of the product. A consumer decides to buy one unit of the product if, and only if, his reserved utility (maximum willingness to pay) is no less than the price. Therefore, Varian proposes a demand inelasticity framework to analyze the relationship between sales promotion and price discrimination.

Later research on sales promotion also followed Varian (1980) to assume the demand inelasticity framework (see for example, Salop and Stiglitz, 1982; Narasimhan, 1984; Lazear, 1986; etc). This demand inelasticity framework is then applied by the coupon and rebate literature (for instance, Jeuland and Narasimhan, 1985; Gerstner and Hess, 1991a, b, 1995). They further assume that only those with low demand will use coupons or rebates, because high- and low-demanders have different redemption costs. Consumers of the same type will make the same purchase decisions at equilibrium; all consumers of the same type will choose to buy either one unit of the product or none at all.

Narasimhan (1984) and Jeuland and Narasimhan (1985) assume that high- and low-demanders have inelastic demands and prove that the firm can achieve price discrimination by issuing coupons. Since the time cost of high-demanders is very high, they tend not to search for and redeem the coupon. Therefore, the coupons allow the firm to charge two distinctive prices on the high- and low-demanders so as to maximize its profit. Although coupons do not change the total demand quantity, they reduce the consumer and social surpluses.

Gerstner and Hess (1991a, b) theoretical models study promotional strategies of an upstream manufacturer and a downstream retailer with high- and low-demanders both having inelastic demand. They assume that initially, the retailer only serves high-demanders and that only the manufacturer can issue rebates (a 'pull' strategy). Since only the low-demanders will redeem the rebate, the manufacturer can thus achieve price discrimination by the rebates. When the manufacturer designs the promotional strategies to induce the retailer to serve all consumers, profits of the manufacturer and retailer will increase, along with the social surplus. Moreover, while the retailer prefers the manufacturer to use trade deals (a 'push' strategy), the manufacturer nevertheless prefers to simultaneously use 'pull' and 'push' strategies, in which case the channel profit is maximized.

Gerstner *et al.* (1994) use both theoretical and empirical approaches to prove that a higher retail markup percentage influences the manufacturer to use price discrimination in a less intensive way. They first establish a model similar to Gerstner and Hess (1991a, b) in which high- and low-demanders both have inelastic demand, taking the retailer's markup percentage into account. Their empirical model explicitly shows that the manufacturer coupon can achieve the price discrimination effect. In order to empirically test their theoretical proposition, the data of manufacturer coupons collected in ten US cities between April 1990 and October 1991 are used.

Gerstner and Hess (1995) further explore the cooperative and non-cooperative pricing strategies of the downstream retailer when the upstream wholesaler issues rebates. Under a non-cooperative pricing strategy, the retailer will increase the retail price to offset the profit-shifting effect of the manufacturer's rebate, causing a reduction in channel profit. The cooperative strategies by these two firms maximize the channel profit and consumer surplus.

Ali *et al.* (1994) take into account the effects on the rebate rate of the redemption rate, demand sensitivity, purchase acceleration effect, and the repurchase rate. They neglect the promotion strategy of the downstream firm and consider only the manufacturer's promotion strategy. Their simulation shows that the rebate rate will increase with

demand sensitivity and the re-purchase rate, while declining with redemption rate and purchase acceleration effect.

With the collected receipts from shoppers leaving five major North Carolina supermarkets, Hess and Gerstner (1993) empirically study double couponing. They find that double couponing is served as an instrument for price discrimination.

From the above literature review, we know that all existing literature examining coupon/rebate strategies assumes a perfectly inelastic demand, i.e., every consumer chooses whether or not to buy one unit of the product (for example, see Varian, 1980; Salop and Stiglitz, 1982; Narasimhan, 1984; Jeuland and Narasimhan, 1985; Lazear, 1986; Gerstner and Hess, 1991a, b, 1995; Gerstner *et al*., 1994). A consumer will purchase one unit of the product if, and only if, his reserved utility is no less than the product price. Due to this strong assumption, it can only be said that with the coupon, the firm charges different prices to two consumers groups. However, none of the existing literature is able to mathematically show that the coupon can be an instrument to achieve third-degree price discrimination. Moreover, these papers are unable to solve for equilibrium coupon values from the first-order conditions of the manufacturer's and the retailer's profit maximization problems, not to mention that they cannot express the coupon-made third-degree price discrimination in terms of demand elasticities.

As Schmalensee (1981), Varian (1985), and Carlton and Perloff (1994) show, in order to practice third-degree price discrimination, a monopolist should impose a higher price on consumers with lower demand elasticity, and a lower price on consumers with higher demand elasticity. The resulting effect on social welfare is ambiguous as it may increase or decrease the social surplus. If the monopolist initially serves two markets under uniform pricing, then third-degree price discrimination will normally reduce the social surplus. In contrast, if the monopolist initially supplies to only one market, then third-degree discrimination is likely to increase the social surplus.

La Croix (1983) tries to empirically prove that the coupon can always improve social welfare regardless of the shape of relevant demand curves. However, Levedahl (1984) immediately refutes La Croix's conclusion with empirical findings. Without formal modeling, Levedahl conjectures that through coupons, a firm can achieve third-degree price discrimination effect which, in most cases, reduces social welfare. In this paper we will mathematically prove that the firm can indeed achieve third-degree price discrimination effect by issuing coupons. In order to do so, we will instead use a continuous demand curve setup which is different to that in the existing literature.

When examining the issue of coupons in a model with one manufacturer and one retailer, most of the existing literature focuses on coupon strategies of the downstream retailer, while neglecting those of the upstream manufacturer, not to mention the case in which both firms issue coupons. We will compare the equilibria when coupons are issued by the manufacturer, the retailer, or both of them to third-degree price discrimination equilibrium.

In order to analyze coupon strategies of the manufacturer and the retailer and their associated social welfare implications, we will follow the above literature to build up a successive monopoly model. We define non-cooperative strategies as those in which both firms maximize their own profits. Cooperative strategies in contrast are those in which both firms maximize the channel profit.

Double marginalization takes place in a successive monopoly model. This is because the demand curve faced by the upstream firm is the marginal revenue curve of the downstream firm. The marginal revenue curve faced by the upstream firm then shifts further inward from that faced by the downstream firm. As a result, the total output and corresponding consumer and producer surpluses under a successive monopoly are all less than those under a vertically integrated monopoly (Spengler, 1950).

Double marginalization reduces the channel profit. It is then an important issue for vertical channel participants to achieve efficient agreements, in order to increase the channel profit (Jeuland and Shugan, 1983, 1988; Shugan, 1985; Ingene and Parry, 1995). Most previous literature on vertically correlated markets with coupons (Gerstner and Hess, 1991a, b, 1995; Gerstner *et al*., 1994) finds that manufacturer coupons can achieve price discrimination and therefore alleviate the channel profit loss due to double marginalization. In this paper it can further be shown that not only manufacturer but also retailer coupons can accomplish these effects. Moreover, a cooperative coupon strategy can achieve the vertical

integration effect and eliminate double marginalization.

This paper is organized as follows: Following introduction, the next section discusses non-cooperative coupon strategies of the manufacturer and the retailer. The following section studies cooperative coupon strategies of the manufacturer and the retailer. The penultimate section contains the social welfare analysis. The final section concludes this paper.

NON-COOPERATIVE STRATEGIES OF UPSTREAM AND DOWNSTREAM FIRMS

The Basic Model

In the basic model we assume that there is a monopolistic manufacturer selling its product through a retailer. The retailer orders the product from the manufacturer at wholesale price w and then re-sells it to consumers at retail price p. There are two groups of consumers: high-demanders and low-demanders. Their demand functions are, respectively, $D_H = H - p$, $D_L = L - p$, with $H > L$. High-demanders have a higher willingness to pay than low-demanders. In order to ensure that both kinds of consumers have positive consumption, the condition $5L > 3H$ is assumed; that is, the difference between the high and low demand curves is not too large. To simplify the calculation without losing generality, we assume that the manufacturer's marginal production cost and the retailer's sales cost are zero.[1]

Both the manufacturer and the retailer are allowed to issue coupons, but only the low-demanders will use coupons, while high-demanders will neither search for, nor use, coupons. The manufacturer can issue manufacturer coupons, offering consumers a discount of m dollars per product. The retailer can also issue retailer coupons, giving consumers a discount of r dollars per product.

There are two stages in this game. In the first stage, the manufacturer chooses the wholesale price (w) and value of the manufacturer coupon (m) to maximize his own profit. In the second stage, the retailer, given the manufacturer's decision, chooses the retail price (p) and retailer coupon value (r) to maximize his own profit. That is, the upstream manufacturer and downstream retailer follow a Stackelberg mode with the upstream manufacturer acting as a Stackelberg leader, while the downstream retailer acts as a follower. The decision flow chart for this pricing game is depicted in Figure 1. We apply the solution concept of the sub-game perfect Nash equilibrium (SPNE) and solve this game by backward induction. We therefore first solve the second stage equilibrium conditions and then substitute them into the first stage optimization problems.[2]

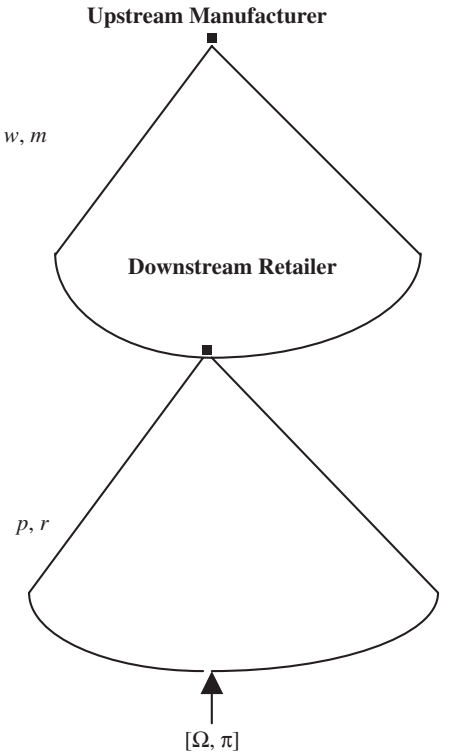

Figure 1. Pricing Game of the Manufacturer and the Retailer.

Market Equilibrium without Coupons: The Benchmark Case

In order to identity the effects of coupons, we start with the case without coupons. When neither the upstream manufacturer nor the downstream retailer issues coupons, their profit maximization problems are, respectively:

$$\underset{w}{Max}\ \Omega = w(D_H + D_L), \tag{1}$$

$$\underset{p}{Max}\ \pi = (p - w)(D_H + D_L), \tag{2}$$

where Ω is the manufacturer's profit and π is the retailer's profit. According to the SPNE solution concept, we first solve the retailer's profit maximization problem in the second stage. The first-order condition for the retailer's profit maximization is:

$$\frac{\partial \pi}{\partial p} = H + L + 2w - 4p = 0. \tag{3}$$

From the above first-order condition, we know that in the second stage the retailer's best response function is $p = \frac{1}{4}(H + L + 2w)$, where $0 < \partial p/\partial w < 1$. That is, a rise in the wholesale price will increase the retail price. However, the retailer only transfers part of his cost to consumers. Substituting Equation (3) into Equation (1), we then obtain the first-order condition for the manufacturer's profit maximization in the first stage:

$$\frac{\partial \Omega}{\partial w} = \frac{1}{2}(H + L) - 2w = 0. \tag{4}$$

We hence obtain the SPNE wholesale price, retail price, high-demanders' demand quantity, low-demanders' demand quantity, consumer surplus (CS), manufacturer's profit, retailer's profit, and social surplus (SS) under the case without coupons:

$$\bar{w} = \tfrac{1}{4}(H + L), \quad \bar{p} = \tfrac{3}{8}(H + L),$$

$$\bar{D}_H = \tfrac{1}{8}(5H - 3L), \quad \bar{D}_L = \tfrac{1}{8}(5L - 3H),$$

$$\bar{D}_H + \bar{D}_L = \tfrac{1}{4}(H + L),$$

$$\overline{CS}_H = \tfrac{1}{128}(5H - 3L)^2, \quad \overline{CS}_L = \tfrac{1}{128}(5L - 3H)^2,$$

$$\bar{\Omega} = \tfrac{1}{16}(H + L)^2, \quad \bar{\pi} = \tfrac{1}{32}(H + L)^2,$$

$$\overline{SS} = \tfrac{1}{64}[23(H - L)^2 + 28HL]. \tag{5}$$

We can infer from (5) that the upstream manufacturer's profit is definitely higher than the downstream retailer's in the case without coupons. This is because the upstream manufacturer plays as a Stackelberg leader and the downstream retailer plays as a Stackelberg follower in this pricing game, which gives the upstream manufacturer 'first-mover advantage'. When both firms do not issue any coupons and adopt non-cooperative strategies, then the market structure is that of a successive monopoly which generates 'double marginalization' (Spengler, 1950) and lower consumer surplus.

Without coupons, both firms engage in uniform pricing. At the equilibrium without coupons, demand elasticities of high- and low-demanders are, respectively, $\bar{\varepsilon}_H = 3(H + L)/5H - 3L$ and $\bar{\varepsilon}_L = 3(H + L)/5L - 3H$. Note that $\bar{\varepsilon}_H < \bar{\varepsilon}_L$; that is, low-demanders have a higher demand elasticity than high-demanders.

Market Equilibrium with Retailer Coupon

Retailer coupons are popular in supermarkets. To illustrate how a retailer can achieve third-degree price discrimination through coupons, this subsection examines the case in which only the retailer is allowed to issue coupons. Denote the retailer coupon value by r. The effective low-demanders' demand function then becomes $D_L = L - p + r$ as the effective price paid by consumers with the coupon is $p - r$. The high-demanders' demand function remains the same as before, $D_H = H - p$. Therefore, the manufacturer and retailer profit maximization problems are, respectively,

$$\underset{w}{Max}\ \Omega = w(D_H + D_L), \tag{6}$$

$$\underset{p,r}{Max}\ \pi = (p - w)D_H + (p - w - r)D_L. \tag{7}$$

The game structure is similar to that in the previous subsection. In the first stage, the manufacturer chooses the wholesale price (w). Given the wholesale price, the retailer chooses the retail price (p) and the coupon value (r) in the second stage. We again apply backward induction to solve this game and obtain the SPNE coupon value, retail price, wholesale price, demand quantities, consumer surplus, and profits (the superscript d denotes the case when only the downstream retailer issues the coupon):

$$r^d = \tfrac{1}{2}(H - L), \quad p^d = \tfrac{1}{8}(5H + L),$$

$$p^d - r^d = \tfrac{1}{8}(H + 5L), \quad w^d = \tfrac{1}{4}(H + L),$$

$$D_H^d = \tfrac{1}{8}(3H - L), \quad D_L^d = \tfrac{1}{8}(3L - H),$$

$$D_H^d + D_L^d = \tfrac{1}{4}(H + L),$$

$$CS_H^d = \tfrac{1}{128}(3H - L)^2, \quad CS_L^d = \tfrac{1}{128}(3L - H)^2,$$

$$\Omega^d = \tfrac{1}{16}(H + L)^2, \quad \pi^d = \tfrac{1}{32}(5H^2 - 6HL + 5L^2),$$

$$SS^d = \tfrac{1}{64}[19(H - L)^2 + 28HL]. \tag{8}$$

Comparing Equations (5) and (8), we find that the retailer will increase his price upon issuing the

coupon ($p^d = \bar{p} + \frac{1}{2} r^d > \bar{p}$), implying that the price paid by the high-demanders increases and their demand quantity decreases ($D_H^d < \bar{D}_H$). The effective price paid by the low-demanders, on the other hand, decreases ($p^d - r^d = \bar{p} - \frac{1}{2} r^d$) and their demand quantity increases ($D_L^d > \bar{D}_L$).

Given the assumption of the linear demand function, the increased demand quantity of low-demanders is equal to the decreased demand quantity of high-demanders, keeping the total demand quantity unchanged. This outcome is the same as that derived in Narasimhan (1984) and Jeuland and Narasimhan (1985) and can be verified by third-degree price discrimination. The wholesale price also remains the same ($w^d = \bar{w}$). Since neither the quantity nor the price changes, the manufacturer's profit stays the same ($\Omega^d = \bar{\Omega}$). The retailer, however, makes more profits ($\pi^d > \bar{\pi}$), because of the higher retail price and the fall in social surplus due to the price discrimination effect caused by the coupon ($SS^d - \overline{SS} = -\frac{1}{16}(H - L)^2 < 0$).

At the SPNE, the demand elasticity of high-demanders $\varepsilon_H^d = (5H + L)/(3H - L)$ is strictly less than that of low-demanders $\varepsilon_L^d = (H + 5L)/(3L - H)$. The effective retail price of high-demanders $p^d = \frac{1}{8}(5H + L)$ is strictly higher than that of low-demanders $p^d - r^d = \frac{1}{8}(H + 5L)$. This is exactly the outcome in which the retailer engages in third-degree price discrimination. In third-degree price discrimination equilibrium, the retailer would charge a higher price to high-demanders, as their demand elasticity is lower, and a lower price to low-demanders, whose demand elasticity is higher. The monopolistic retailer can thus use coupons to achieve third-degree price discrimination and circumvent the competition law. Compared to the outcome without coupons, the retailer coupon expands the difference in demand elasticities; that is, $\varepsilon_H^d > \bar{\varepsilon}_H$ and $\varepsilon_L^d < \bar{\varepsilon}_L$.

In order to verify that the retailer can achieve a third-degree price discrimination outcome by issuing retailer coupons, we define $p_H = p$ and $p_L = p - r$ and substitute them into (7) to yield:

$$\underset{p_H, p_L}{Max}\, \pi = (p_H - w)D_H + (p_L - w)D_L. \tag{7'}$$

This is exactly the objective function of a retailer charging high- and low-demanders two different prices p_H and p_L.

Note that the retailer coupon can lead to third-degree price discrimination and therefore alleviate the channel profit loss due to double marginalization . This can be easily proven by $\Omega^d + \pi^d - (\bar{\Omega} + \bar{\pi}) = \frac{1}{32}[8(H-L)^2 + 6HL] > 0$. That is, the channel profit under the retailer coupon is strictly higher than that without coupon.

Market Equilibrium with Manufacturer Coupon

Many manufacturers offer coupons to consumers by free-standing insert (FSI), run of press (ROP) newspapers, direct mail, peel-off, on-pack, and in-pack methods. Consumers may redeem these coupons at a retail store and enjoy a price discount. In general, the retailer will be compensated by the manufacturer at coupon value, plus a handling fee. In this section we focus on the case when only the manufacturer issues coupons. Here, the manufacturer offers coupons of value m per unit of product to the low-demanders. The effective demand function of low-demanders then becomes $D_L = L - p + m$ and that of high-demanders is unchanged, i.e., $D_H = H - p$. As a result, the manufacturer's and retailer's profit maximization problems are, respectively,

$$\underset{w,m}{Max}\ \Omega = wD_H + (w - m)D_L, \tag{9}$$

$$\underset{p}{Max}\ \pi = (p - w)(D_H + D_L). \tag{10}$$

The game structure is similar to that in the previous subsection. In the first stage, the manufacturer chooses the wholesale price (w) and the coupon value (m). Given the wholesale price, the retailer chooses the retail price (p) in the second stage. We again use backward induction to solve this game. The SPNE manufacturer coupon value, wholesale price, retail price, demand quantities, consumer surplus, and profits are all as follows (the superscript u represents the case when only the upstream manufacturer issues the coupon):

$$m^u = \tfrac{1}{2}(H - L), \quad w^u = \tfrac{1}{2} H, \quad w^u - m^u = \tfrac{1}{2}L,$$

$$p^u = \tfrac{1}{8}(5H + L), \quad p^u - m^u = \tfrac{1}{8}(H + 5L),$$

$$D_H^u = \tfrac{1}{8}(3H - L), \quad D_L^u = \tfrac{1}{8}(3L - H),$$

$$D_H^u + D_L^u = \tfrac{1}{4}(H + L),$$

$$CS_H^u = \tfrac{1}{128}(3H - L)^2, \quad CS_L^u = \tfrac{1}{128}(3L - H)^2,$$

$$\Omega^u = \tfrac{1}{16}(3H^2 - 2HL + 3L^2), \quad \pi^u = \tfrac{1}{32}(H + L)^2,$$

$$SS^u = \tfrac{1}{64}[19(H - L)^2 + 28HL]. \tag{11}$$

Comparing the equilibrium values in Equations (5), (8), and (11), we conclude that the SPNE coupon values, effective prices, consumer surpluses, and total demand quantities when either the manufacturer or the retailer issues coupons are all the same and are equal to those under third-degree price discrimination without coupons. With only the manufacturer coupon, the wholesale price will increase ($w^u > w^d = \bar{w}$). However, since the increase in w is equal to the increase in p, the retailer's profit is unchanged ($\pi^u = \bar{\pi}$). In contrast, the manufacturer's profit definitely goes up ($\Omega^u > \bar{\Omega}$).

At the SPNE, the demand elasticity of high-demanders $\varepsilon_H^u = (5H + L)/(3H - L)$ is strictly less than that of low-demanders $\varepsilon_L^u = (H + 5L)/(3L - H)$. The effective wholesale price to sell to high-demanders $w^u = \frac{1}{2}H$ is strictly higher than that of low-demanders $w^u - m^u = \frac{1}{2}L$. This is exactly the outcome in which the retailer price-discriminates against the two groups of consumers. It also indicates that third-degree price discrimination equilibrium can be achieved by the manufacturer's coupon. Through the use of coupons, the manufacturer can set a high price on high-demanders with a low elasticity (i.e., the high-demanders) and a low price on demanders with a high elasticity (i.e., the low-demanders). Compared to the outcome without coupons, the manufacturer's coupon expands the difference in demand elasticities; that is, $\varepsilon_H^u > \bar{\varepsilon}_H$ and $\varepsilon_L^u < \bar{\varepsilon}_L$. The social surplus thus declines due to third-degree price discrimination made by the coupon ($SS^u - \overline{SS} = -\frac{1}{16}(H - L)^2 < 0$).

In order to verify that by issuing coupons, the manufacturer can move the equilibrium to what would be third-degree price discrimination equilibrium without coupons, we define $w_H = w$ and $w_L = w - m$, and substitute them into Equations (9) and (10) to yield the two firms' profit maximization problems as:

$$\underset{w_H, w_L}{Max}\ \Omega = w_H D_H + w_L D_L, \tag{9'}$$

$$\underset{p}{Max}\ \pi = (p - w_H)(D_H + D_L). \tag{10'}$$

By comparing Equations (9) and (10) to (9′) and (10′), we find that the coupon which allows the low-demanders to purchase the product at a discounted price can help the manufacturer facilitate price discrimination against the high-demanders. The manufacturer nominally imposes one wholesale price on its sales to the retailer, but it can (by issuing coupons directly to the low-demanders) implicitly charge two wholesale prices (w_H and w_L), according to the type of consumers to whom the retailer sells!

From Equation (9′), it is clear that the SPNE outcomes, when only the manufacturer issues coupons, are exactly the same as those when the manufacturer practices third-degree price discrimination on the retailer. It is worth noting that there is only one retailer in the model. According to the conventional wisdom, it is impossible for an upstream manufacturer who faces only one retailer to practice third-degree price discrimination; however, here we have proved that an upstream manufacturer can in fact engage in third-degree price discrimination on a unique retailer through manufacturer coupons. The manufacturer coupon enables the manufacturer to charge the retailer two effective wholesale prices, $w_H = w$ and $w_L = w - m$, based on different consumer types, and to transfer the manufacturer coupon value to the retailer's cost.[3]

Note that the manufacturer coupon can also increase the channel profit and help alleviate the channel profit loss caused by double marginalization, through moving the equilibrium to third-degree price discrimination. This can be easily proven by $\Omega^u + \pi^u - (\bar{\Omega} + \bar{\pi}) = \frac{1}{32}[8(H-L)^2 + 6HL] > 0$. That is, the channel profit under the manufacturer coupon is strictly higher than that without coupon. According to the above analyses, we obtain the following propositions:

Proposition 1:
Under the linear demand function and successive monopoly market structure, the coupon issuance by either the manufacturer or the retailer does not change product sales, but it does reduce consumer and social surpluses.

Proposition 2:
Under the linear demand function and successive monopoly market structure, no matter which one of those two firms issues the coupon, the resulting effective retail prices and consumer surplus are the same. Moreover, the coupon issuer makes more profit while the other firm's profit remains the same as without coupons.

Proposition 3:
Coupons issued by either the manufacturer or the retailer can move the successive monopoly

equilibrium to what would be a third-degree price discrimination equilibrium without coupons.

Market Equilibrium with Two Coupon Issuers

In addition to the cases in which either the manufacturer or the retailer is allowed to issue coupons, it is common for both upstream and downstream firms to issue coupons. Consumers are usually allowed to present both manufacturer and retailer coupons at time of purchase. In this section we will study the case in which both firms are allowed to issue coupons to consumers. The values of manufacturer and retailer coupons are denoted, respectively, by m and r. The demand function of low-demanders is then $D_L = L - p + m + r$ and that of high-demanders is $D_H = H - p$. Therefore, the profit maximization problems of the manufacturer and the retailer become, respectively:

$$\underset{w,m}{Max}\ \Omega = wD_H + (w-m)D_L,\ s.t.\ m \geq 0, \tag{12}$$

$$\underset{p,r}{Max}\ \pi = (p-w)D_H + (p-w-r)D_L,\ s.t.\ r \geq 0. \tag{13}$$

There are two stages in this pricing game. In the first stage, the manufacturer chooses the wholesale price (w) and the manufacturer coupon value (m). Given the wholesale price and the manufacturer coupon value, the retailer chooses the retail price (p) and the retailer coupon value (r) in the second stage. By backward induction, we obtain the SPNE outcomes as shown below (the superscript *ud* represents the case when both firms issue coupons):

$$m^{ud} = \tfrac{1}{2}(H-L), \quad r^{ud} = \tfrac{1}{4}(H-L),$$

$$w^{ud} = \tfrac{1}{2}H, \quad w^{ud} - m^{ud} = \tfrac{1}{2}L,$$

$$p^{ud} = \tfrac{3}{4}H, \quad p^{ud} - m^{ud} - r^{ud} = \tfrac{3}{4}L,$$

$$D_H^{ud} = \tfrac{1}{4}H, \quad D_L^{ud} = \tfrac{1}{4}L, \quad D_H^{ud} + D_L^{ud} = \tfrac{1}{4}(H+L),$$

$$CS_H^{ud} = \tfrac{1}{32}H^2, \quad CS_L^{ud} = \tfrac{1}{32}L^2,$$

$$\Omega^{ud} = \tfrac{1}{8}(H^2+L^2), \quad \pi^{ud} = \tfrac{1}{16}(H^2+L^2),$$

$$SS^{ud} = \tfrac{7}{32}(H^2+L^2). \tag{14}$$

From the outcomes, we know that the manufacturer coupon value is strictly higher than the retailer coupon value. This is because under the linear demand function, the demand elasticity faced by the manufacturer are strictly higher than those faced by the retailer. As a result, the manufacturer's optimal discriminatory pricing decision is to make a higher coupon value (price difference) than the retailer.

In order to mathematically show these price discrimination effects when both firms issue coupons, we define $w_H = w$, $w_L = w - m$, $p_H = p$, and $p_L = p - r$. The two firms' profit maximization problems in Equations (12) and (13) can then be rewritten as:

$$\underset{w_H,w_L}{Max}\ \Omega = w_H D_H + w_L D_L, \tag{12'}$$

$$\underset{p_H,p_L}{Max}\ \pi = (p_H - w_H)D_H + (p_L - w_H)D_L. \tag{13'}$$

From Equations (12′) and (13′), we find that the SPNE when both firms issue coupons is equivalent to the SPNE when the manufacturer engages in third-degree price discrimination on the retailer, and the retailer engages in third-degree price discrimination on consumers. After the manufacturer issues the coupon, the wholesale prices for the product to be sold by the retailer to the high- and low-demanders become, respectively, $w_H = w$ and $w_L = w - m$. That is, the manufacturer coupon enables the upstream manufacturer to artificially segment the downstream retailer's sales into two parts: those to high-demanders and those to low-demanders. Moreover, Equation (13′) shows that the retailer coupon enables the downstream retailer to engage in third-degree price discrimination on consumers. That is, the retail prices for high- and low-demanders become, respectively, $p_H = p$ and $p_L = p - r$.

The upstream manufacturer will engage in third-degree price discrimination on the downstream retailer through the manufacturer coupon, and the downstream retailer will engage in third-degree price discrimination on consumers through the retailer coupon. As both the retailer and consumers are charged discriminatory prices, we define this phenomenon as 'the successive third-degree price discrimination', which differs from the 'third-degree price discrimination' by a monopolist in the traditional literature.

We further find that the channel profit with a single coupon issuer is strictly higher than that with two coupon issuers; that is, $(\Omega^{ud} + \pi^{ud}) - (\Omega^{d} + \pi^{d}) = (\Omega^{ud} + \pi^{ud}) - (\Omega^{u} + \pi^{u}) = -\frac{1}{32}(H-L)^2 < 0$. At the same time, the consumer surplus with a single coupon issuer is

strictly higher than that with two coupon issuers; that is:

$$(CS_H^{ud} + CS_L^{ud}) - (CS_H^{d} + CS_L^{d}) = (CS_H^{ud} + CS_L^{ud}) - (CS_H^{u} + CS_L^{u}) = -\tfrac{3}{64}(H-L)^2 < 0.$$

Moreover, the social surplus with a single coupon issuer is strictly higher than that with two coupon issuers; that is:

$$SS^{ud} - SS^{d} = SS^{ud} - SS^{u} = -\tfrac{5}{64}(H-L)^2 < 0.$$

These rankings imply that the social surplus under 'the successive third-degree price discrimination' is strictly less than that under third-degree price discrimination. According to Subsections 2.2 to 2.5, we have the following proposition:

Proposition 4:
Under non-cooperative coupon strategies,

(1) the SPNE outcome is that both firms issue coupons;
(2) the manufacturer coupon value is strictly larger than the retailer coupon value;
(3) the consumer and social surpluses with a single coupon issuer are strictly higher than those with two coupon issuers;
(4) the total demand quantities, with a single coupon issuer, and with two coupon issuers or without coupons are the same.

COOPERATIVE COUPON STRATEGIES

Both the manufacturer and retailer can also coordinate their pricing strategies to maximize the channel profit, with the division of the channel profit depending on their relative bargaining power. A necessary condition for cooperation requires that the payoffs at cooperation be no less than those at non-cooperation. The game structure is the same as that depicted in Figure 1 except that the goal of both firms is now to maximize their joint profits.

Under cooperation, the manufacturer and retailer coordinate their coupon values. The demand function of low-demanders becomes $D_L = L - p + m + r$. The demand function of high-demanders is still $D_H = H - p$. The channel profit maximization problem is then:

$$\underset{p,m,r}{Max}\ \Omega + \pi = pD_H + (p - m - r)D_L. \tag{15}$$

After rearrangement, Equation (15) can be rewritten as:

$$\underset{p,h}{Max}\ \Omega + \pi = pD_H + (p - h)D_L, \tag{15'}$$

where $h \equiv m + r$. Solving the maximization problem in Equation (15′), we find the following SPNE outcomes:

$$\begin{aligned}
&h^* = \tfrac{1}{2}(H - L),\\
&p^* = \tfrac{1}{2}H, \quad p^* - h^* = \tfrac{1}{2}L,\\
&D_H^* = \tfrac{1}{2}H, \quad D_L^* = \tfrac{1}{2}L,\\
&CS_H^* = \tfrac{1}{8}H^2, \quad CS_L^* = \tfrac{1}{8}L^2,\\
&\Omega^* + \pi^* = \tfrac{1}{4}(H^2 + L^2), \quad SS^* = \tfrac{3}{8}(H^2 + L^2).
\end{aligned} \tag{16}$$

According to Equation (16), we obtain the following proposition:

Proposition 5:
Under the linear demand function and cooperative coupon strategies, the sum of coupon values is equal to that with only one coupon issuer under non-cooperation.

Comparing Propositions 3 and 5, we find that irrespective of whether the two firms adopt non-cooperative or cooperative coupon strategies, the total coupon values are the same. Recall that when there is only a single coupon issuer, coupons can result in third-degree price discrimination effect. If the two firms issue coupons cooperatively, the coupons would then enable them to jointly achieve third-degree price discrimination. Table 1 lists the SPNE outcomes in various behavioral regimes and from those outcomes we derive Lemma 1.

Lemma 1:
The rankings in channel profits, consumer surpluses, and social surpluses of the behavioral intervals are as follows:

(1) $\Omega^* + \pi^* > \Omega^d + \pi^d = \Omega^u + \pi^u > \Omega^{ud} + \pi^{ud} > \bar{\Omega} + \bar{\pi}$
(2) $CS_H^* + CS_L^* > \overline{CS}_H + \overline{CS}_L > CS_H^d + CS_L^d = CS_H^u + CS_L^u > CS_H^{ud} + CS_L^{ud}$
(3) $SS^* > \overline{SS} > SS^d = SS^u > SS^{ud}$.

Cooperative coupon strategies yield a higher consumer surplus, channel profit, and social surplus than all non-cooperative strategies. This

Table 1. Producer, Consumer and Social Surpluses under Different Coupon Strategies

Behavioral regime	Manufacturer profit Ω	Retailer profit π	Channel profit $\Omega+\pi$	Consumer surplus $CS_H + CS_L$	Social surplus SS	Economic effects
Non- cooperative strategies without coupons	$\frac{1}{16}(H+L)^2$	$\frac{1}{32}(H+L)^2$	$\frac{3}{32}(H+L)^2$ lowest	$\frac{1}{128}[(5H-3L)^2+(5L-3H)^2]$ medium	$\frac{1}{64}[23(H-L)^2+28HL]$ medium	1. Double marginalization
Non- cooperative strategies with retailer coupon	$\frac{1}{16}(H+L)^2$	$\frac{1}{32}(5H^2-6HL+5L^2)$	$\frac{1}{16}(7H^2-2HL+7L^2)$ medium	$\frac{1}{64}(5H^2-6HL+5L^2)$ low	$\frac{1}{64}[19(H-L)^2+28HL]$ low	1. Double marginalization 2. Third-degree price discrimination
Non- cooperative strategies with manufacturer coupon	$\frac{1}{16}(3H^2-2HL+3L^2)$	$\frac{1}{32}(H+L)^2$	$\frac{1}{16}(7H^2-2HL+7L^2)$ medium	$\frac{1}{64}(5H^2-6HL+5L^2)$ low	$\frac{1}{64}[19(H-L)^2+28HL]$ low	1. Double marginalization 2. Third-degree price discrimination
Non- cooperative strategies with two coupon issuers	$\frac{1}{8}(H^2+L^2)$	$\frac{1}{16}(H^2+L^2)$	$\frac{3}{16}(H^2+L^2)$ low	$\frac{1}{32}(H^2+L^2)$ lowest	$\frac{7}{32}(H^2+L^2)$ lowest	1. Double marginalization 2. Successive third-degree price discrimination
Cooperative strategies with coupon	—	—	$\frac{1}{4}(H^2+L^2)$ highest	$\frac{1}{8}(H^2+L^2)$ highest	$\frac{3}{8}(H^2+L^2)$ highest	1. Third-degree price discrimination by a vertically- integrated firm

Note: 'Lowest', 'low', 'median', 'high' and 'highest' are used for rankings among the five coupon strategies.

is because the two firms can eliminate double marginalization and achieve the vertical integration effect (Spengler, 1950) through coordination in coupon strategies. Therefore, vertical firms cooperating on coupons may achieve third-degree price discrimination while eliminating double marginalization. Since there is no more double marginalization at vertical cooperation, the consumer surplus, channel profit, and social welfare all increase.

WELFARE ANALYSIS

When the manufacturer and retailer adopt non-cooperative strategies, the SPNE outcome is that both issue coupons; in other words, at non-cooperation the retailer's dominant strategy in the second stage is to issue coupons, regardless of whether the manufacturer issues the coupon in the first stage. Given that the retailer always issues a coupon, the manufacturer will also issue the coupon in the first stage. As a result, the SPNE outcome at non-cooperation is that both firms issue coupons.

Under cooperation the two firms will coordinate the total coupon values and it does not matter which coupon has a higher value. Although competition in a horizontal market usually promotes consumer and social surpluses, it does not hold in a vertical market. Cooperative coupon strategies here bring in more consumer and social surplus than non-cooperative coupon strategies, because cooperative coupon strategies achieve *de facto* vertical integration, which eliminates double marginalization. Moreover, non- cooperative coupon strategies generate the successive third-degree price discrimination in addition to double marginalization, resulting in the lowest consumer and social surpluses. Based on the above discussion, we have the following proposition:

Proposition 6:
Assuming that both manufacturer and retailer can issue coupons:

(1) If they adopt non-cooperative strategies, then the SPNE outcome is that both firms issue coupons.
(2) The case when only one of them issues the coupon is an SPNE outcome only if they adopt cooperative strategies.

(3) No matter whether they adopt non-cooperative or cooperative strategies, coupons will always bring the issuer with a profit higher than that when it does not issue coupons.
(4) Cooperative coupon strategies result in a higher level of channel profit, consumer surplus and social surplus than non-cooperative coupon strategies.

CONCLUDING REMARKS

At first sight, coupons are simply a price-discount favor to consumers; however, coupons can serve as a means for the firm to achieve third-degree price discrimination. That is, with coupons, consumers with a lower elasticity are charged a higher price by firms, and consumers with higher elasticity are charged a lower price. This practice tends to increase the coupon issuer's profit at the expense of consumers.

Most existing literature focuses on coupon strategies of downstream retailers and neglects those of upstream manufacturers, not to mention the case in which both firms issue coupons. This paper allows both the upstream manufacturer and the downstream retailer to issue coupons either unilaterally, or cooperatively, and has found that coupons issued by either the upstream or the downstream firm can move the equilibrium to what would be third-degree price discrimination equilibrium without coupons. When only one firm issues the coupon, both manufacturer and retailer coupons can help alleviate the channel profit loss due to double marginalization.

It is also found that cooperative coupon strategies bring in more consumer and social surplus than non-cooperative coupon strategies, because cooperative coupon strategies achieve *de facto* vertical integration, which eliminates double marginalization. It is therefore suggested that cooperative coupon strategies should be encouraged.

Most existing literature on coupon strategies assumes inelastic demand. This paper represents the first effort to use a continuous demand function which enables us to mathematically prove that coupons can be used as an instrument to achieve third-degree price discrimination in terms of demand elasticities. Moreover, the continuous demand function enables us to solve for equilibrium coupon values from the first-order conditions of the manufacturer's and the retailer's profit maximization problems.

In a model with one upstream manufacturer selling its entire product to one downstream retailer, it is common to think that the upstream manufacturer who faces only one derived demand is not able to engage in third-degree price discrimination. However, this paper has shown that the upstream manufacturer can, in fact, engage in third-degree price discrimination through issuing coupons. The manufacturer's coupon enables the upstream manufacturer to segment the derived demand from the downstream retailer into two sets: sales to high-demand consumers and sales to low-demand consumers. The manufacturer's coupon then enables the manufacturer to charge the retailer two wholesale prices.

We also find a phenomenon that has never been discussed in the existing literature: 'the successive third-degree price discrimination', which differs from the conventional third-degree price discrimination in the literature. This occurs when the upstream manufacturer engages in third-degree price discrimination on the downstream retailer through the manufacturer's coupon, while the downstream retailer also engages in third-degree price discrimination on consumers through the retailer's coupon.

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NOTES

1. This assumption will not change the qualitative results of this paper.
2. Please refer to Gibbons (1992) and Fudenberg and Tirole (1993) for game tree and SPNE solution concepts.
3. This result is consistent with the empirical finding in Hess and Gerstner (1993): With the collected receipts from shoppers leaving five major North Carolina supermarkets, they showed that double couponing can be served as an instrument for price discrimination.

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