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MONEY, REAL INTEREST RATES, AND OUTPUT: A REINTERPRETATION OF POSTWAR U.S. DATA[1]

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This paper reexamines U.S. postwar data to investigate if the observed comovements between money, interest rates, inflation, and output are compatible with the money to real interest to output links suggested by existing monetary theories of the business cycle, which include both Keynesian and equilibrium models. We find these theories are incompatible with the data, and in light of these results, we propose an alternative structural model which can account for the major dynamic interactions among the variables. This model has two central features: (i) output is unaffected by the money supply, and (ii) the money supply process is influenced by policies designed to achieve short-run price stability.

1. INTRODUCTION

Does money matter? This paper reexamines the time-series evidence that changes in the money supply have been an important factor in generating postwar U.S. business cycles. Specifically, we investigate whether the observed comovements between money, real interest rates, prices, and output are compatible with existing monetary theories of income determination, which include both traditional Keynesian analysis as well as the newer informationally based equilibrium theories. Our main empirical findings cast strong doubt on the importance of these theories for understanding recent U.S. experience. Rather, we find that most of the dynamic interactions among the key variables can best be explained as arising from an economic structure in which monetary phenomena do not affect real variables. Thus, we conclude that monetary instability has not played an important role in generating fluctuations.

The major result of the paper is to show that certain Granger causal orderings fit the data well and that these empirical findings have implications for the validity of various monetary theories of output. This type of time-series methodology was pioneered by Sims [**13**], who showed that in postwar U.S. data, causality is unidirectional from money to income. Although this result is compatible with a variety of theories, it was generally accepted as evidence that "money matters" for real output. However, this interpretation has been recently challenged by Sims' [**15, 16**] subsequent finding that money is no longer Granger-causally prior for output when nominal interest rates are added to a vector autoregression containing money, output, and prices. Sims found that an upward innovation in nominal interest rates leads to a decline in both money and output, and he concluded [**15**, p. 253] that "some of the observed comovements of industrial production and money stock are attributed to common responses to surprise changes in the interest rate." This relationship appears in pre- and postwar U.S. data and in postwar French, British, and German data.

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From the standpoint of most monetary theories of output, these empirical results are anomalous since the nominal interest rate is a poor proxy for the theoretically meaningful ex ante real interest rate. Fama [**6**] has shown that a substantial part of the movement in short-term interest rates, at least over postwar U.S. experience, can be attributed to changes in expected inflation. These results, as Shiller [**12**, p. 148] notes, "must give pause to those who believe that inflationary expectations are highly sluggish or follow a trend and that medium-run movements in short-term interest rates are movements in ex ante real rates."

The main novelty of this paper is to reexamine the time-series evidence, emphasizing the distinction between movements in expected (ex ante) real interest rates and movements in expected inflation rates. To find empirical counterparts to these unobservables, we assume agents' expectations of future inflation are rational and thus identify the projection of future inflation on current observables with agents' expectations. A key result of this procedure is that we cannot reject the joint hypothesis that agents' expectations are rational and that ex ante real rates are exogenous, or Granger-causally prior, relative to a universe containing money, prices, nominal rates, and output. Since both Keynesian models and the newer equilibrium theories share the feature that money affects current real activity by altering agents' perceptions of the intertemporal terms of trade, the finding of real rate exogeneity would appear inconsistent with the nexus of money, real interest rates, and output suggested by these models.

In light of this finding, we construct a prototypic alternative model which is consistent with the data. This model builds on an insight suggested by Fama [**6**] that the incremental predictive content of nominal variables for future real variables arises solely because economic agents have some information about future real activity—beyond that contained in current and lagged real variables—which shows up first in the equilibrium price of financial assets, particularly nominal interest rates. This occurs because expectations of changes in future output induce changes in expected future prices through a neoclassical money demand function and hence affect current inflation rates and current nominal rates. In this context, the comovements between money and future real activity are consistent with a Fed reaction function which attempts to offset, at least partially, the movements in expected inflation rates arising from anticipated output shocks. We emphasize that this model is far more "classical" than even the "new classical macroeconomics" models of Lucas [**10**] or Barro [**2**] because output is assumed to be independent of current, past, and expected future money, whether anticipated or not. When our model is tested, we find it to be surprisingly consistent with the data.

The paper is organized as follows: In Section 2 we replicate the basic results of Sims' four-variable vector autoregressions. In Section 3 we formulate and test exogeneity of the ex ante real rate and discuss why we believe this test applies to the empirical validity of both Keynesian IS-LM analysis and the informationally based equilibrium theories. In Section 4 we formulate our alternative model, which we believe can explain the comovements between real and financial variables, and present a test of this model. In Section 5 we apply our testing procedure to a number of other hypotheses concerning the causal structure of real and nominal variables. Although these tests do not bear on the validity of any completely articulated theory, we present these results both to demonstrate that our test procedure has power to discriminate among alternatives and to provide a convenient data summary technique of some independent interest. Section 6 provides a summary.

2. REVIEW OF EARLIER WORK

Using a multivariate, linear time-series model, Sims [**15**] showed that nominal interest rate innovations explain a substantial fraction of the variance of industrial production. Furthermore, the inclusion of interest rates substantially decreases the variance of industrial production attributed to innovations in the money supply. When interest rates are omitted from the system, monetary innovations explain 37 per cent of the forecast error variance of industrial production at the 48-month horizon; when interest rates are added, the proportion falls to 4 per cent. We duplicate this result in both monthly and quarterly U.S. postwar data, which added several recent years of volatile nominal rate movements to Sims' data set. For the sake of brevity we report here—and throughout this paper—only on the results obtained with the quarterly data set.[3]

A Granger causality test rejects exogeneity of output with respect to money at the one per cent marginal significance level in both a three-variable (industrial production, money, inflation) and a four-variable (plus nominal interest rate) vector autoregression. The regressions include four lags of each variable and a constant; observations are for the period 1949:2 to 1983:2. These Granger test results tell us only that information in the lags of money helps to reduce the one-step-ahead forecast errors of output.

We find the results of a decomposition of variance for these systems, shown in Table I, more revealing. This measure is based on a decomposition of the variance of forecast errors at various time horizons into a sum of components associated with each of a set of orthogonal innovations. A more complete description of this decomposition is given in Sims [**18**]. As can be seen in the table, the dominance of interest rate innovations over money innovations becomes stronger as the time horizon for predicting output lengthens. This accords with Sims' finding that the response of output to interest rate innovations is essentially

[3] Our data include observations from 1948:1 through 1983:2 on measures of the money stock (M1), nominal interest rates (the market average yield on 90-day treasury securities), a price series (the consumer price index less shelter), and output (the industrial production index). An attempt was made to measure all series as closely as possible to a point in time near the middle of the third month of the quarter. For money and interest rates we took weekly averages of the second week of the month (the third week was used if there were five weeks in the month). For prices the monthly figure represents a sample taken approximately during the middle week, whereas for output the best measure available is of the flow throughout the month. The seasonally adjusted versions of the money, price, and output series were used. Logs of the level of money and industrial production were used. Inflation was measured as 400.0 times the change in the log of the price level. The nominal interest rate was measured as 100.0 times the log of one plus the per cent yield divided by 100.

TABLE I

DECOMPOSITION OF VARIANCE OF INDUSTRIAL PRODUCTION IN THREE- AND FOUR-VARIABLE SYSTEMS[a]

	3-Variable System			4-Variable System			
Forecast Horizon (Quarters)	Output	Inflation	Money	Output	Inflation	Money	Nominal Rate
1	100.0	0.0	0.0	100.0	0.0	0.0	0.0
4	79.5	4.1	16.5	72.3	3.6	11.3	12.8
8	55.6	18.5	25.9	39.4	16.4	12.9	31.3
16	40.1	34.1	25.8	22.0	32.0	8.4	37.6
24	35.2	38.6	26.2	16.9	37.0	7.2	39.0

[a] Entries give the percentage of forecast error variance accounted for by orthogonalized innovations in the listed variables. The order of orthogonalization is as listed.

flat for about six months, followed by a smooth decline reaching a minimum about 18 months later.[4]

As a further check of the robustness of this link between the nominal interest rate and output,[5] we split the four-variable, four-lags system in half and reestimate the system separately for the two nonoverlapping subperiods—1949:2 to 1966:1 and 1966:2 to 1983:2. Although a test of equality of the estimated coefficients across the two periods is strongly rejected, we find that the qualitative properties of the output response to interest rate innovations is remarkably similar in the two periods. In Figure 1, the moving average response of each of the four variables to an innovation in nominal interest rates orthogonal to the other variables is presented for each period. In both periods, output declines in response to interest rate innovations. This response is much quicker in the more recent period: there is no discernible lag and the response is strongest at the five-quarter horizon. In the earlier period, a two-quarter lag is evident and the maximum impact is at the six-quarter horizon. In both periods, interest rate innovations are followed by a decrease in nominal balances.

3. IS THE REAL RATE EXOGENOUS?

We begin our investigation of the relationship between real and nominal variables by testing a restriction which we feel is incompatible with theories that

[4] The decomposition results remained essentially unchanged when trend or trend and trend-squared were added to the system. For example, when trend and trend-squared are included in the regressions, the explanatory contribution of money to industrial production at the 24-quarter horizon drops from 33.0 per cent to 19.4 per cent when Treasury bills are added. The bill rate itself accounts for 29.7 per cent of the forecast error variance at this horizon with trend and trend-squared included. Similar patterns emerged when the post-October 1979 period of the Federal Reserve's new operating procedures was dropped and when monthly data were used.

[5] We also estimated a number of larger systems including (not all at one time) inventories, retail sales, real wages, wage settlements, the monetary base, a stock price index, the unemployment rate, 10-year bond yields, and a trade-weighted index of the value of the dollar. The qualitative behavior of the output response to interest rate innovations described above appeared in every system estimated.

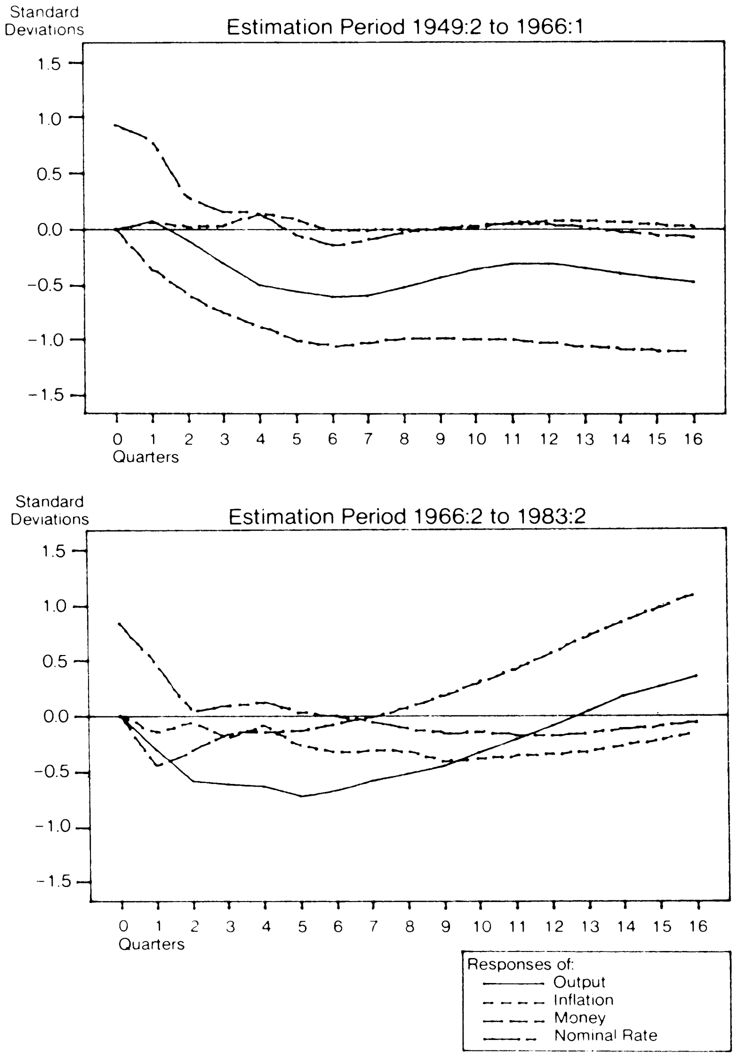

FIGURE 1—Responses to a Nominal Interest Rate Innovation

emphasize a role for the real rate of interest in transmitting monetary disturbances to the real economy. In particular, we test the restriction that past money, prices, and income have no additional predictive content for current real rates, given past real rates. That is, we test the hypothesis that the real rate is exogenous, or Granger-causally prior, in the context of this four-variable system.[6]

[6] Shiller [**12**] tested and rejected the hypothesis that ex post (realized) real rates were exogenous. As he notes (p. 153), this test does not bear directly on the proposition tested here except under some additional and rather unattractive assumptions.

Interpretation of causal orderings as indicative of behavioral or structural relationships is a complicated and subtle issue (see Sims [**13, 14**]). In general, when there are as many independent shocks to the system as there are variables, we would expect that each variable would have some incremental predictive power for each other variable, and thus no causal ordering would arise. Thus, failure to find a causal ordering would be compatible with many competing hypotheses, and as a result, we could not distinguish among the hypotheses. When we do find a causal ordering, however, then we can place restrictions on either the dimensionality of the exogenous stochastic terms or the behavioral relationships which describe the economy.

The compatibility of this causal ordering with the IS-LM model, the Lucas-Barro models, and the Grossman–Weiss model will each be considered in turn. We would expect that IS-LM models, in general, would not be consistent with exogeneity of the real rate. Thus, we believe the failure to reject would raise questions about the validity of such models. We believe the test also bears on the empirical validity of the informationally constrained equilibrium models, even though our measure of the expected real rate ignores the limitations on current period information, which are essential ingredients of these models. While in both cases we can imagine versions of the model which would fool us into acceptance of the hypothesis that the real rate is exogenous, we find these special cases implausible.

The IS-LM Model

A central feature of Keynesian IS-LM analysis is the idea that changes in the demand or supply of nominal balances can change the real interest rate. Keynesian theory achieves this connection by invoking sluggish nominal price adjustments in nonfinancial markets, particularly the labor market.

Consider the following IS-LM model:

$$\text{(1)}\qquad \begin{aligned} &\text{IS} \quad Y_t = -\beta_1 r_t + \varepsilon_t, \quad \beta_1 > 0, \\ &\text{LM} \quad \frac{M_t}{P_t} = \alpha_1 Y_t - \alpha_2 (r_t + \hat{\Pi}_t^{t+1}) + \phi_t, \quad \alpha_1 > 0, \alpha_2 > 0, \end{aligned}$$

where $\hat{\Pi}_t^{t+1}$ is expected inflation,

$$\text{(2)}\qquad \hat{\Pi}_t^{t+1} \equiv E[\Pi_{t+1} | Y_{t-s}, M_{t-s}, R_{t-s}, \Pi_{t-s}, s \geq 0],$$

where r_t is the real interest rate,

$$\text{(3)}\qquad r_t \equiv R_t - \hat{\Pi}_t^{t+1},$$

where ε_t represents all exogeneous spending (including government spending and variations in desired investment unrelated to interest rate movements) and where ϕ_t represents random influences on real money demand (the state of "liquidity preference"). The reduced form equations for the endogenous variables

r_t and Y_t are given by

$$(4)\qquad \begin{aligned} r_t &= \gamma_1 \varepsilon_t + \gamma_2(m_t - \phi_t) + \gamma_3 \hat{\Pi}_t^{t+1}, \\ Y_t &= \gamma_4 \varepsilon_t + \gamma_5(m_t - \phi_t) + \gamma_6 \hat{\Pi}_t^{t+1}, \end{aligned}$$

where

$$\gamma_1 = \frac{\alpha_1}{\alpha_2 + \alpha_1 \beta_1}, \quad \gamma_2 = \frac{-1}{\alpha_2 + \alpha_1 \beta_1}, \quad \gamma_3 = \frac{-1}{\alpha_2 + \alpha_1 \beta_1},$$

$$\gamma_4 = \frac{\alpha_2}{\alpha_2 + \alpha_1 \beta_1}, \quad \gamma_5 = \frac{\beta_1}{\alpha_2 + \alpha_1 \beta_1}, \quad \gamma_6 = \frac{\beta_1 \alpha_2}{\alpha_2 + \beta_1 \beta_1},$$

and

$$m_t = \frac{M_t}{P_t}.$$

An implication of this theory is that, unless the interest elasticity of investment demand β_1 is infinite, monetary policy can affect output only to the extent it affects the ex ante real rate.

We then ask under what auxiliary hypothesis can this model be compatible with the finding that

$$(5)\qquad E(r_{t+1} | r_{t-s}, s \geqslant 0) = E(r_{t+1} | r_{t-s}, M_{t-s}, \Pi_{t-s}, Y_{t-s}, s \geqslant 0).$$

One possibility is that, over the observed sample, it was the deliberate objective of Fed policy to set expected real rates in such a way that the two hypotheses are observationally equivalent. This might arise, for example, if the policy objective were to minimize the variance of output $E(Y_t - \bar{Y})^2$ by setting $r_t = -(1/\beta_1)(\bar{Y} - \varepsilon_t)$. If ε_t followed a univariate autoregressive process, then so would r_t. Although we cannot reject this possibility a priori, it is unlikely that desired interest rate targets could be expressed in terms of any single factor, let alone the past history of interest rates. It certainly appears as if policy has aimed for both price and output stability. Since prices and output exhibit some independent variation, it is implausible to take the finding that the real rate is exogenous as indicative of a particular policy reaction function.

Another possibility which could explain the lack of any influence from past money, prices, and output on current ex ante real rates is that the IS curve is horizontal. This would be true if the interest sensitivity of demand β_1 were infinite, so that variations in money supply or demand affected only output without a measurable impact on interest rates. This possibility is both highly implausible and easily rejected by subsequent findings.

Still a third possibility, less easily dismissed, is that over the sample period, most variations in money supply m_t were passive responses to money demand shocks ϕ_t. Under this hypothesis, there would be no added explanatory power from past money to future real rates. This hypothesis requires either no deliberate attempt on behalf of the Fed for controlling real rates, except insofar as interest rate targets depend only on lagged values, or that policy-induced interest rate variations have been sufficiently small compared with exogenous money demand shifts so that our procedure cannot distinguish this variation from a variation due to sample errors.

These possibilities, while being neither mutually exclusive nor exhaustive, seem sufficiently implausible to us that the data's failure to reject the hypothesis of real rate exogeneity casts strong doubt on the Keynesian notion that monetary policy has affected output through changes in the real rate of interest.

The Lucas-Barro Models

The model presented in Lucas [**10**] and modified by Barro [**2, 3**] emphasizes the effects of unperceived monetary injections on the labor supply by altering perceptions of real rates of return. By positing barriers on current period information flows, these models draw a sharp distinction between expectations based on complete current period information and the expectations held by a representative trader. Nevertheless, the hypothesis that the real rate (based on complete current information) is exogenous would seem incompatible with most intertemporal versions of these models.

Lucas' original model assumed all random disturbances to be serially uncorrelated and all information lags to be, at most, a single period. These features, while inessential, imply that both concepts of the real rate would be serially independent. Thus, in this limited sense, the models are compatible with the finding that the real rate is exogenous. However, if these models are appended to be consistent with the fact that there are substantial serial correlations in most macroeconomic time series, then it is difficult to reconcile the models.

To see this, imagine that the ex ante real rate, conditioned on aggregate information, is given by

$$r_t = \sum_{j=1}^{n_1} \lambda_j r_{t-j} + \sum_{j=0}^{n_2} \alpha_j \eta_{t-j} + \phi \tilde{m}_t + \varepsilon_t \tag{6}$$

where $\tilde{m}_t = m_t - E[m_t|\text{information as of } t-1]$ is the unexpected component of money and η_t is a stochastic vector of real factors which affect real rates (e.g., productivity, thrift, government expenditures). Barro [**4**] argues that the sign of ϕ in equation (6) should be negative. Exogeneity of the real rate, in the context of a system which includes a measure of real production, requires either that the measure is uncorrelated with components of η_t or that the α_j are all zero. Theories which emphasize a confusion between unperceived monetary injections and persistent real factors affecting the ex ante real rate would generally predict a systematic response of the real rate to changes in real production. A failure to reject exogeneity of the real rate thus raises questions about the empirical importance of this channel for monetary disturbances to have real effects.

The Grossman-Weiss Model

The Grossman-Weiss [9] model also assumes incomplete information so that the expected real rate based on complete current period information differs from the expectations held by a representative trader. This model determines ex ante real rates by the possibility of intertemporal substitution of consumption. A necessary condition for equilibrium in the bond market is that each agent chooses consumption to satisfy the first order condition $u'(c_t) = \beta E[r_t u'(c_{t+1})]$ where the expectation is taken over the agent's information available in period t. The model determines the ex ante real rate based on complete current period information $\hat{r}_t = (1-\alpha)[c_{t+1} - c_t]$ where c_t is (log) per capita consumption and α is the coefficient of relative risk aversion. (Note that for this model, consumption is perfectly predictable one period ahead on the basis of complete current period information.)

As in the Lucas-Barro models, the compatibility of this theory with an exogenous ex ante real rate depends crucially on the nature of the exogenous stochastic disturbances. Since ex ante real rates are a linear function of the first difference of (log) per capita consumption, ex ante real rates will be exogenous if and only if per capita consumption is exogenous relative to the same universe. In the original version of the model, it was assumed that all disturbances were serially independent, resulting in serially independent, and hence exogenous, consumption. If, however, the model is modified to be consistent with serially correlated consumption by imposing serially correlated productivity shocks, then consumption and real rates will not be exogenous. As in the models which emphasize unperceived money, when there are both persistent real and transitory monetary factors which determine ex ante real rates, we would not expect the real rate to be exogenous.

What these three theories we've examined have in common is that the real interest rate plays a crucial role in the generation of business cycles and that (except under special circumstances) its behavior is a function of lagged real and monetary disturbances. Any model with these two characteristics would appear to be challenged by the finding that, in a system with real and monetary variables, the real interest rate is exogenous.

Some people have argued that the finding of exogeneity is sensitive to the universe of variables examined, which by necessity is limited. Although it is plausible that a finding that one variable has incremental predictive power for future values of another variable could be overturned (as we saw in Section 2), a finding of exogeneity could be reversed only under very special circumstances. For example, suppose the true reduced form for ex ante real rates is given by

$$r_t = \sum_{j=0}^{\infty} v_j m_{t-j} + \sum_{j=0}^{\infty} w_j z_{t-j} + \varepsilon_t \tag{7}$$

where z_t is a vector stochastic process of omitted variables and w_j is a vector conformable to z_t. Suppose

$$E[z_{t-K} \mid m_t, m_{t-1}, \ldots] = \sum_{j=0}^{\infty} \alpha_{Kj} m_{t-j}. \tag{8}$$

Then, in population, the regression coefficients of r_t on lagged m's are given by

$$h_j = v_j + \sum_{K=0}^{\infty} w_K \alpha_{Kj} \qquad (j = 0, \ldots). \tag{9}$$

While it is certainly possible that h_j's will be zero, even though the v_j's are nonzero, this is highly unlikely as it requires an extreme coincidence between the v's, w's, and α's.

Another possible objection to our test of exogeneity is that it neglects possible effects of changes in the conduct of monetary policy during our sample. The pre-accord period (prior to 1951:2) and the recent explicit "monetarist" experiment (1979:4 to 1982:1) stand out as two episodes when we might expect different interactions among the key variables. Our reaction to this type of objection is mixed. While the hypotheses of structural stability during the periods 1950:2 through 1951:2 and 1979:4 through 1982:1 relative to the rest of the sample are rejected,[7] it is difficult to see a priori how this should affect our interpretation of real rate exogeneity over the entire period. In any event, a finding of real rate exogeneity is noteworthy only if it holds over various subperiods; so as a kind of sensitivity check, we test on the full sample as well as on two partial data sets, first with the 1950:2 to 1951:2 period removed and, second, with both it and the 1979:4 to 1982:1 period removed. It turns out that these sample periods produce consistent results, so we concentrate our attention on the full period.

Our tests are based on the standard likelihood ratio statistic. In interpreting our results we use both the Akaike [**1**] criterion and the marginal significance levels giving the probability, under the null hypothesis, of observing test statistics of the given magnitude. In the context of hypothesis testing, the Akaike criterion suggests rejection of the null hypothesis if the log likelihood ratio is greater than the number of restrictions k. The marginal significance levels are based on asymptotic distribution of twice the log likelihood ratio. (The distribution is chi-squared with k degrees of freedom.) We find the classical hypothesis-testing framework, with a fixed unrestricted vector autoregression as the alternative, a useful device through which we can investigate specific questions by looking at the degree to which various hypotheses are consistent with the data. In this context, we interpret the Akaike criterion and the calculation of a significance level of a likelihood ratio statistic as alternative ways to correct the relative fits of different restrictions for differences in degrees of freedom.

Because the ex ante real rate is unobservable, testing this hypothesis requires an auxiliary hypothesis of how agents forecast future prices. We assume that agents' expectations are rational, which in the context of our information set and in the absence of any further restrictions, identifies price expectations with the projection of future prices on current and lagged endogenous variables. Thus, we define

$$\hat{\Pi}_t^{t+1} \equiv E[\Pi_{t+1} | Y_{t-s}, M_{t-s}, R_{t-s}, \Pi_{t-s}, s = 0, 1, 2, 3] \tag{10}$$

[7] These tests for structural stability follow the methodology described by Sims [**17**].

and

$$r_t \equiv R_t - \hat{\Pi}_t^{t+1}.$$

As is often the case, the imposition of the rational expectations hypothesis leads to complicated, nonlinear, cross-equation restrictions. While the imposition of these restrictions is costly in terms of computations, we find that it generates test statistics which have greater power to differentiate among hypotheses than other approaches such as Fama [**5**], Fama and Gibbons [7], Nelson and Schwert [**11**], and Garbade and Wachtel [**8**]. For evidence of this, see the results in Section 5.

The hypothesis that the ex ante real rate of interest r_t is a function of only its own lagged values, a constant term, and an uncorrelated random error can be written as follows:

$$r_t = \sum_{j=1}^{m} b_j r_{t-j} + c^r + u_t. \tag{11}$$

Substitution of (10) into (11) leads to the following expression for the nominal interest rate:

$$R_t = \hat{\Pi}_t^{t+1} + \sum_{j=1}^{m} b_j R_{t-j} - \sum_{j=1}^{m} b_j \hat{\Pi}_{t-j}^{t-j+1} + c^r + u_t. \tag{12}$$

This equation imposes testable restrictions across the autoregressive representation for R_t, Π_t, and the other variables, Z_t, in the information set that individuals use in projecting future values of Π.

Suppose that for the K-vector X_t, a finite order autoregressive representation exists:

$$\begin{aligned} X'_t &= [R_t \Pi_t Z'_t], \\ X_t &= \sum_{l=1}^{L} A_l X_{t-l} + C + \eta_t. \end{aligned} \tag{13}$$

The ith equation of this representation has the scalar form

$$X_t^i = \sum_{j=1}^{K} \sum_{l=1}^{L} a_l^{ij} X_{t-l}^j + C^i + \eta_t^i \tag{14}$$

where a_l^{ij} is the coefficient on the lth lag of the jth component of X. Thus, for example, the projection of inflation during period t on observables at time $t-1$ is given by

$$\hat{\Pi}_{t-1}^t = \sum_{l=1}^{L} a_l^{21} R_{t-l} + \sum_{l=1}^{L} a_l^{22} \Pi_{t-l} + \sum_{j=3}^{K} \sum_{l=1}^{L} a_l^{2j} Z_{t-l}^{j-2} + C^2. \tag{15}$$

The restrictions on a vector autoregression implied by (11) are generated by using (15) to replace all expected inflation terms in (12) with projections on observables, collecting terms with R_t on the left-hand side, and then by projecting both sides on information available at time $t-1$. The resulting equation is a projection of R_t on information available at time $t-1$ that equates each of the coefficients in the R_t equation, a_l^{1j}, with a function of the b_l's and the a_l^{ij}'s for $i=2,\ldots,K$. For example, for $l\leq m$,

$$a_l^{11}=\frac{1}{(1-a_1^{21})}\left[b_l+a_{l+1}^{21}+\sum_{j=2}^{K}a_1^{2j}a_l^{j1}-\sum_{j=1}^{l}b_ja_{l-j+1}^{21}\right]. \tag{16}$$

Because there are L lags in each of the projections of the observed variables, lags of the real rate become functions of observations more than L periods earlier than the current period. Thus, the reduced form projection for R must include $m-1$ more lags than each of the other equations. This requires us to impose (11) as a restriction on a vector autoregressive system with $L+m-1$ lags on all variables in the R projection and L lags on all variables in the other projections.

Equations similar to (16) express each of the coefficients in the R projection as a function of the other coefficients. Given the introduction of the $m+1$ new free parameters, $b_1,\ldots,b_m$ and c^r, these equations impose $K^*(L+m-1)-m$ nonlinear restrictions on the parameters of the vector autoregression.

The results of our test of exogeneity of the real rate, given in Table II, are clear. By any conventional significance level or the Akaike criterion, and for each

TABLE II

RESULTS OF TESTING EXOGENEITY OF THE REAL RATE

Full Period Results		
49:2-83:2		
Restricted Equation (standard errors)	$r_t=.760\ r_{t-1}+\ .156+u_t$ $(.051)\quad(.108)$	
Log Determinants	Restricted Unrestricted	-16.4987 -16.5761
Likelihood Ratio Test Two times adjusted[a] log likelihood ratio $=9.29\sim\chi^2(15)$ Marginal significance level $=.86$		
Akaike Criterion Number of restrictions—log likelihood ratio $=9.70$ >0 implies failure to reject the null hypothesis		
Partial Data Set I		
49:2-83:2 with 50:2-51:2 removed		
Likelihood Ratio Test marginal significance level $=.12$ Akaike Criterion $=2.6$		
Partial Data Set II		
49:2-83:2 with 50:2-51:2 and 79:4-82:1 removed		
Likelihood Ratio Test marginal significance level $=.28$ Akaike Criterion $=4.75$		

[a] Sims' [17] adjustment for degrees of freedom is incorporated.

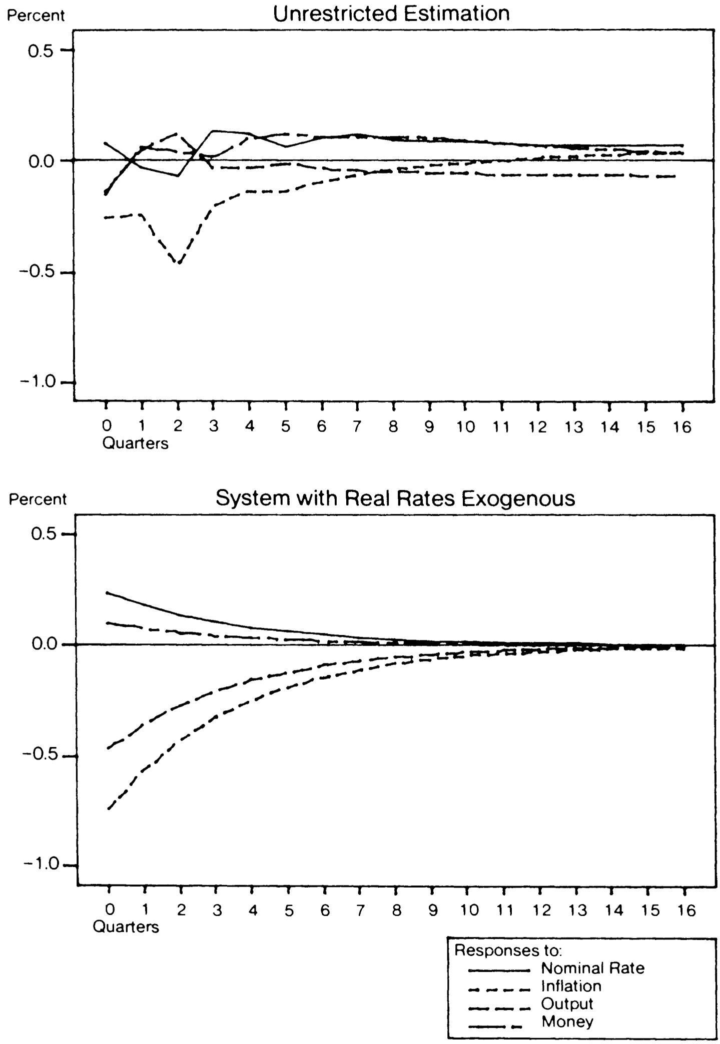

FIGURE 2—Responses of the Real Interest Rate

of our three samples, we cannot reject the null hypothesis. Under the null, twice the log likelihood ratio for this set of restrictions with $m = 1$ is distributed chi-squared with 15 degrees of freedom. For the full data set, the statistic is 9.3, which gives a .86 marginal significance level. In Figure 2 the responses of the real rate to orthogonalized innovations in the observable variables are shown for both the unrestricted and the restricted systems. The first order Markov nature of the exogenous real rate process is apparent. The responses also show that even if the real rate is exogenous, it can respond to the contemporaneous components of the innovation to other variables. Notice that there are contemporaneous components even for innovations orthogonal to nominal rate and inflation innovations, because there is still a correlation with the expected inflation, and thus the real rate, innovations.

4. WHAT "CAUSES" OUTPUT

Most macroeconomic theories suggest that real rates, not nominal rates, should play an important role in the determination of future output. Since much of the variation in nominal interest rates reflects changes in anticipated inflation, the consistent response of output to nominal rate innovations, documented in Section 2, is surprising. Conventional theories would lead us to expect the response of output to a real rate innovation, where the expected inflation "noise" has been removed, to be much stronger. However, in this section we show that the information content of nominal rates is due primarily to their reflection of changes in anticipated inflation rather than changes in the real rate. We suspect that this statistical link arises because agents in the economy have some information about the level of future output—information which is not directly observable to the econometric investigator and which is first reflected in nominal quantities. Then we develop and test a model in which this is the case in order to demonstrate that such a structure is consistent with the data.

As in Section 3, our proxy for the unobservable ex ante real rate is generated by attributing to agents a knowledge of the hypothesized time-invariant autoregressive structure of the economy and by identifying agents' inflationary expectations with the projection of the annualized growth rate of the price level from t to $t+1$ on information available at t. In order to decompose the output response to nominal rate innovations into that response due to the real rate component, as opposed to that due to the expected inflation component, we start by defining the expected inflation innovation to be the unpredictable change in expected inflation, that is,

$$\tilde{\hat{\Pi}}_t^{t+1} \equiv \hat{\Pi}_t^{t+1} - E[\hat{\Pi}_t^{t+1} | Y_{t-s}, M_{t-s}, R_{t-s}, \Pi_{t-s}, \quad s = 1, 2, 3, 4]. \tag{17}$$

It is easy to see that the time t innovation in expected inflation is a linear combination of that period's innovations in the observed variables. Furthermore, it is clear that with the innovation to real rates $\tilde{r}_t$ similarly defined to be the unpredictable change in ex ante real rates, we have a natural decomposition of nominal interest rate innovations,

$$\tilde{R}_t = \tilde{r}_t + \tilde{\hat{\Pi}}_t^{t+1}. \tag{18}$$

We find that nominal interest rate innovations in our quarterly data reflect approximately equal contributions from real rate and expected inflation innovations. This result can be derived from Table III, which gives the covariance matrix of innovations in our unrestricted vector autoregression. The matrix is expanded to show real rate and expected inflation innovation covariance. Based on these

TABLE III

COVARIANCE MATRIX OF INNOVATIONS[a]

(Boldface entries below the diagonal are correlations.)

	$\tilde{Y}$	$\tilde{\Pi}$	$\tilde{M}$	$\tilde{R}$	$\tilde{\hat{\Pi}}$	$\tilde{r}$
$\tilde{Y}$	.000472	.00469	.0000274	.00756	.0108	−.00326
$\tilde{\Pi}$	**.090**	5.776	.00173	.507	2.096	−1.589
$\tilde{M}$	**.228**	**.130**	.0000307	.000199	.000550	−.000351
$\tilde{R}$	**.353**	**.214**	**.036**	.972	.431	.541
$\tilde{\hat{\Pi}}$	**.508**	**.889**	**.101**	**.446**	.963	−.532
$\tilde{r}$	**−.145**	**−.638**	**−.061**	**.530**	**−.523**	1.073

[a] The variables are defined as follows: $\tilde{Y}$ = output innovation; $\tilde{\Pi}$ = inflation innovation; $\tilde{M}$ = money innovation; $\tilde{R}$ = nominal interest rate innovation; $\tilde{\hat{\Pi}}$ = expected inflation innovation; and $\tilde{r}$ = real interest rate innovation.

covariances it can be seen that a 1 per cent innovation in nominal rates is most likely to reflect an increase of .44 per cent in expected inflation and of .56 per cent in expected real rates. This result is in contrast to, but not inconsistent with, the results of Fama [**5**] and Shiller [**12**], which show that most variations in the *level* of nominal rates can be attributed to changes in the *level* of expected inflation.

Another aspect of Table III is the strong negative correlation between expected real rates and expected inflation. Since both inflation and expected real rates have some persistent component, this can explain the well-documented negative correlation between the level of past and current inflation and the level of real rates, even in the absence of any structural link between past inflation and future real rates.

Because of the high negative correlation between real rate innovations and expected inflation innovations, the qualitative properties of the impulse response functions and the decomposition of variance with these innovations will depend on the particular ordering chosen. This sensitivity is confirmed in Table IV, which reports the variance decomposition of output in three alternative representations—all of which lead to equivalent predictions of future values.

The linearity of the vector autoregressive system and identity (18) implies that, given the innovation to any one of the three variables—nominal rate, real rate, or expected inflation rate—the orthogonalized innovations to either of the other two are equivalent. Or, to put this result another way, given any one of these variables, the incremental predictive content for output is identical whichever of the other two variables is included. Thus, for example, when nominal rates come first, the subsequent innovation can be viewed equivalently as the orthogonalized expected inflation or real rate innovation. This identity makes it difficult to interpret the residual orthogonalized innovation. Instead, in order to summarize the qualitative importance of each component, we focus on the relative contributions to output variance of the first innovation when it is, in turn, a nominal rate, a real rate, or an expected inflation innovation.

When nominal interest rate innovations are ordered ahead of either of the other components, the nominal rate innovations explain 51.6 per cent of the

TABLE IV

DECOMPOSITION OF VARIANCE OF INDUSTRIAL PRODUCTION AT VARIOUS FORECAST HORIZONS WITH VARIOUS ORDERINGS OF EX ANTE INFLATION AND REAL RATES

		First Ordering		
Quarter	Industrial Production	Money	Nominal Rate	Expected Inflation or Real Rate
4	72.3	9.8	15.6	2.3
8	39.4	10.0	40.4	10.1
12	27.6	7.2	48.6	16.7
16	22.0	5.8	51.6	20.6

		Second Ordering		
Quarter	Industrial Production	Money	Real Rate	Expected Inflation or Nominal Rate
4	72.3	9.8	1.9	16.1
8	39.4	10.0	2.5	48.0
12	27.6	7.2	1.9	63.3
16	22.0	5.8	1.5	70.6

		Third Ordering		
Quarter	Industrial Production	Money	Expected Inflation	Nominal Rate or Real Rate
4	72.3	9.8	7.4	10.5
8	39.4	10.0	25.9	24.7
12	27.6	7.2	37.5	27.7
16	22.0	5.8	43.9	28.3

variance of forecast errors of output at a four-year horizon. As noted above, a nominal rate innovation is most likely to reflect approximately equal contributions from real rates and expected inflation. If the effect of nominal rate innovations is due to changes in the real rate, then we would expect the impact on output to be even larger when we isolate the real rate component. It turns out that this is not the case; in fact, the result is just the reverse. If we reorder the innovations so that the real rate innovation comes first, the per cent of output forecast variance explained at the same four-year horizon drops to 1.5 per cent.

This striking change in the variance decomposition means that while an unexpected increase in nominal interest rates, given current values for output and money, implies a major revision in the forecast of output, an unexpected increase in real rates signals essentially no change in the prospect for output. This pattern is consistent with two possibilities: One possibility is that nominal interest rates as such contain the information concerning output and that distinguishing real rates from expected inflation only masks this signal. The second is that expected inflation innovations are the crucial component of nominal rate changes. These two possibilities can be distinguished by considering a third ordering with expected inflation innovations ordered first. If the first possibility is true, we would again see no explanatory power in these innovations. If the second is true, the explanatory power would be expected to increase above that of nominal rates.

The results from this third ordering are most consistent with the second possible explanation. Expected inflation innovations explain 43.9 per cent of the output forecast variance at the four-year horizon in this ordering. While this amount is slightly less than the amount for nominal rate innovations, it is much greater than the amount for real rate innovations. The slight decrease in explanatory power of expected inflation innovations relative to nominal rate innovations may be due either to error in our measurement of expected inflation innovations or to a much smaller (but nonzero) effect on output from the real rate component of nominal rate innovations. Given the negative correlation between real rate and expected inflation innovations, separate contributions from each component, which reinforce each other in the nominal rate innovation, will tend to offset each other in either the real rate or expected inflation innovations, each of which when ordered first contains a small negative component of the other.

The relationship between expected inflation innovations and subsequent movements in output, which we have documented here, could reflect new information about the future course of output showing up first in the nominal interest rate. To see how this could arise, consider the following structural model in which output is independent of the money supply process:

$$
\begin{aligned}
(19)\qquad & Y_{t+1} = Y_t + \alpha r_t + Z_t + u_{t+1}, \\
& M_t - P_t = \beta Y_t - \gamma R_t, \\
& R_t \equiv \hat{\Pi}_t^{t+1} + r_t, \\
& \hat{\Pi}_t^{t+1} \equiv \hat{P}_t^{t+1} - P_t, \\
& r_t = \lambda r_{t-1} + v_t.
\end{aligned}
$$

This model is meant to illustrate a particular causal structure. Its crucial feature is that some information in Z_t is known to agents in the economy and is useful for predicting future output, but is not directly observable to the econometric investigator. Therefore, in developing a test of the model, we permit the right-hand variables to have longer lag lengths.

Suppose the model is closed by specifying a money supply process

$$(20)\qquad M_t \equiv 0$$

and the exogenous disturbances Z_t, u_t, and v_t are serially independent. Using the method of undetermined coefficients, it is straightforward to show that the reduced form equations for expected inflation and nominal rates are given by

$$(21)\qquad \hat{\Pi}_t^{t+1} = \left(\frac{-\beta}{1+\gamma}\right) Z_t - \left(\frac{\gamma(1-\lambda)+\beta\alpha}{1+\gamma(1-\lambda)}\right) r_t$$

and

$$R_t = \left(\frac{-\beta}{1+\gamma}\right) Z_t + \left(\frac{1-\beta\alpha}{1+\gamma(1-\lambda)}\right) r_t,$$

and the solution for the innovations in these variables are

$$\tilde{\hat{\Pi}}_t = \left(\frac{-\beta}{1+\gamma}\right) Z_t - \left(\frac{\gamma(1-\lambda)+\beta\alpha}{1+\gamma(1-\lambda)}\right) v_t \tag{22}$$

and

$$\tilde{R}_t = \left(\frac{-\beta}{1+\gamma}\right) Z_t + \left(\frac{1-\beta\alpha}{1+\gamma(1-\lambda)}\right) v_t.$$

This model shows most simply that nominal interest rate innovations or expected inflation innovations will be correlated with Z innovations and thereby will be useful for predicting output when Z_t is not observed directly. This occurs despite the lack of structural feedback from past, current, or future money and prices to output.

Of course, this model could not account for the predictive content of money in a bivariate system. However, it would not be difficult to change the specification of the money supply process to be consistent with this finding, as well as with other characteristic features of the data. Consider the money supply process

$$\Delta M_t = \Delta M_{t-1} + \delta \tilde{\hat{\Pi}}_t^{t+1} + \tau u_t + w_t. \tag{23}$$

We would expect δ to be negative because the monetary authority reacts to an increase in inflationary expectations by contracting. We would expect τ to be positive since the money supply reacts positively to an unexpected increase in output. With this specification, the reduced form equations for changes in money supply and expected inflation are given by

$$\begin{aligned}\Delta M_t = \Delta M_{t-1} &- \left(\frac{\delta\beta}{(1-\delta)(1+\gamma)}\right) Z_t - \left(\frac{\delta}{1-\delta}\right)\left(\frac{\gamma(1-\lambda)+\beta\alpha}{(1+\gamma(1-\lambda)}\right) v_t \\ &+ \left(\frac{\tau}{1-\delta}\right) u_t + \left(\frac{1}{1-\delta}\right) w_t\end{aligned} \tag{24}$$

and

$$\hat{\Pi}_t^{t+1} = \Delta M_t - \left(\frac{\beta}{1+\gamma}\right) Z_t - \left(\frac{\gamma(1-\lambda)+\beta\alpha}{1+\gamma(1-\lambda)}\right) r_t,$$

and the equations for the innovations are

$$\begin{aligned}\tilde{M}_t = &- \left(\frac{\delta\beta}{(1-\delta)(1+\gamma)}\right) Z_t - \left(\frac{\delta}{1-\delta}\right)\left(\frac{\gamma(1-\lambda)+\beta\alpha}{1+\delta(1-\lambda)}\right) v_t \\ &+ \left(\frac{\tau}{1-\delta}\right) u_t + \left(\frac{1}{1-\delta}\right) w_t\end{aligned} \tag{25}$$

and

$$\tilde{\hat{\Pi}}_t^{t+1} = -\left(\frac{\beta}{1+\gamma}\right) Z_t - \left(\frac{\gamma(1-\lambda)+\beta\alpha}{1+\delta(1-\lambda)}\right) v_t + \tilde{M}_t.$$

This modification to the money supply process shows how monetary innovations could be positively associated with Z innovations. Thus the monetary innovations could be useful for predicting real output in a bivariate system and yet contribute no additional explanatory power in a larger system which contains either nominal interest rates or the level of expected inflation. A Phillips curve relationship—a positive association between inflation and lagged output growth—could arise if τ is positive, meaning that money growth rises with unexpected output growth.

This model suggests an empirical test of the hypothesis that the expected inflation-output link is spurious because inflation is proxying for other information relevant to predicting future output. To test the hypothesis, we define a new variable $\Pi_t^* = \tilde{\hat{\Pi}}_t^{t+1} - E[\tilde{\hat{\Pi}}_t^{t+1} | \tilde{r}_t, \tilde{Y}_t]$, which is that component of the expected inflation innovation orthogonal to the contemporaneous innovations in the real variables. From equation (25) above, it can be seen that Π_t^* is a linear combination of the Z_t and w_t disturbances. If inflation is simply proxying for the Z disturbance, then in light of our previous finding that the ex ante real rate is exogenous, we would expect that the real variables (output and ex ante real rates) together with Π^* will be block exogenous. In other words, we would expect no additional explanatory power for future real variables from current and past money, prices, or nominal interest rates—given current and past real variables and Π^*. Formally, we may state our hypothesis for the output equation as

$$\begin{aligned}(26) \qquad & E[Y_{t+1} | Y_{t-k}, r_{t-k}, \Pi^*_{t-k}, k \geq 0] \\ & = E[Y_{t+1} | Y_{t-k}, R_{t-k}, M_{t-k}, \Pi_{t-k}, k \geq 0]\end{aligned}$$

where

$$r_t \equiv R_t - E[\Pi_{t+1} | Y_{t-k}, R_{t-k}, M_{t-k}, \Pi_{t-k}, k \geq 0].$$

To implement a test of this hypothesis, we will assume that agents' expectations of inflation at time t, $\hat{\Pi}_t^{t+1}$, are equivalent to a projection of inflation from t to $t+1$ on observable data at t:

$$\begin{aligned}(27) \qquad \hat{\Pi}_t^{t+1} &= E[\Pi_{t+1} | Y_{t-j}, R_{t-j}, \Pi_{t-j}, M_{t-j}, j = 0, \ldots, \infty] \\ &= \sum_{j=1}^{\infty} a_{0j} R_{t-j} + a_{1j} Y_{t-j} + a_{2j} M_{t-j} + a_{3j} \Pi_{t-j}.\end{aligned}$$

Innovations in expected inflation are defined by

$$\begin{aligned}(28) \qquad \tilde{\hat{\Pi}}_t^{t+1} = {} & E[\Pi_{t+1} | Y_{t-j}, R_{t-j}, \Pi_{t-j}, M_{t-j}, j = 0, \ldots, \infty] \\ & - E[\Pi_{t+1} | Y_{t-j}, R_{t-j}, \Pi_{t-j}, M_{t-j}, j = 1, \ldots, \infty].\end{aligned}$$

Innovations in the observables, such as nominal interest rates, are defined by

$$(29) \qquad \tilde{R}_t = R_t - E[R_t | Y_{t-j}, R_{t-j}, \Pi_{t-j}, M_{t-j}, j = 1, \ldots, \infty].$$

Upon substituting the expectations implied by the autoregressive representation defined above, it is easily seen that

$$\tilde{\hat{\Pi}}_t^{t+1} = a_{01}\tilde{R}_t + a_{11}\tilde{Y}_t + a_{21}\tilde{M}_t + a_{31}\tilde{\Pi}_t \tag{30}$$

and hence

$$\begin{aligned} \Pi_t^* &= a_{01}\{\tilde{R}_t - E[\tilde{R}_t|\tilde{r}_t, \tilde{Y}_t]\} + a_{11}\{\tilde{Y}_t - E[\tilde{Y}_t|\tilde{r}_t, \tilde{Y}_t]\} \\ &\quad + a_{21}\{\tilde{M}_t - E[\tilde{M}_t|\tilde{r}_t, \tilde{Y}_t]\} + a_{31}\{\tilde{\Pi}_t - E[\tilde{\Pi}_t|\tilde{r}_t, \tilde{Y}_t]\} \\ &= a_{01}\tilde{R}_t + a_{21}\tilde{M}_t + a_{31}\tilde{\Pi}_t + k_1\tilde{r}_t + k_2\tilde{Y}_t \end{aligned} \tag{31}$$

where the k coefficients are "mongrel" coefficients involving both the a coefficients and the covariance matrix of the innovations.

Thus, letting $Q_t \equiv a_{01}\tilde{R}_t + a_{21}\tilde{M}_t + a_{31}\tilde{\Pi}_t$, we may rewrite our hypothesis as

$$\begin{aligned} &E[Y_{t+1}|\text{all available information at } t] \\ &\quad = E[Y_{t+1}|Y_{t-j}, r_{t-j}, \Pi_{t-j}^*, j = 0, \ldots, \infty] \\ &\quad = E[Y_{t+1}|Y_{t-j}, r_{t-j}, Q_{t-j}, \tilde{r}_{t-j}, \tilde{Y}_{t-j}, j = 0, \ldots, \infty] \\ &\quad = E[Y_{t+1}|Y_{t-j}, r_{t-j}, Q_{t-j}, j = 0, \ldots, \infty] \end{aligned} \tag{32}$$

where the last equality follows from the fact that, under our hypothesis, the innovations in the real variables are spanned by the same space as the level of the real variables and our Q variable.

In implementing empirical tests it is common practice to truncate lag lengths, even though it is recognized that such restrictions are only approximately true. In our case, however, it should be noted that the approximation may be of somewhat greater concern because under our null hypothesis, unless $a_{11} = 0$ or lags of Π^* do not appear in the output equation, the autoregressive representation for output will be infinite-dimensional.

Nonetheless, we will follow the usual practice and assume that a finite autoregressive representation of Y exists in terms of past Y, r, and Q. Specifically, we assume that

$$\begin{aligned} &E[Y_{t+1}|Y_{t-j}, r_{t-j}, Q_{t-j}, j = 0, 1, 2, 3] \\ &\quad = E[Y_{t+1}|Y_{t-j}, r_{t-j}, Q_{t-j}, j = 0, 1, \ldots, \infty] \end{aligned} \tag{33}$$

and that all other observable variables (R, Π, M) also have a finite autoregressive representation with four lags of all past variables sufficient to capture all lagged effects.

With these auxiliary assumptions, we may test this hypothesis as a restriction on an unrestricted vector autoregression. Our hypothesis is that four lags of r, Y, and Q are sufficient to capture all past effects. Since Q is a linear combination of the innovations to the observables, Q_{t-4} is a linear combination of the observables from $t-4$ to $t-8$. Thus, our hypothesis is a restriction on an autoregression with eight lags of each of the observables in the output equation and with four lags of each of the observables in each of the other three equations.

TABLE V

RESULTS OF TESTING THAT OUTPUT IS A FUNCTION OF LAGGED OUTPUT, REAL RATES, AND EXPECTED INFLATION INNOVATIONS

Full Period Results

50:2-83:2

Restricted Equation
(standard errors)

$$\begin{aligned} Y_t = {} & \underset{(.071)}{1.313}\, Y_{t-1} + \underset{(.166)}{.028}\, Y_{t-2} + \underset{(.135)}{.532}\, Y_{t-3} - \underset{(.083)}{.073}\, Y_{t-4} \\ & - \underset{(.0019)}{.0100}\, Q_{t-1} - \underset{(.0025)}{.0159}\, Q_{t-2} - \underset{(.0025)}{.0089}\, Q_{t-3} \\ & + \underset{(.0018)}{.0001}\, Q_{t-4} - \underset{(.0012)}{.0079}\, r_{t-1} - \underset{(.0015)}{.0031}\, r_{t-2} \\ & + \underset{(.0015)}{.0019}\, r_{t-3} + \underset{(.0011)}{.0045}\, r_{t-4} + \underset{(.0160)}{.0202} + u_t \end{aligned}$$

Log Determinants

Restricted	−16.5376
Unrestricted	−16.7841

Likelihood Ratio Test

Two times adjusted[a] log likelihood ratio = 28.59 ~ $\chi^2(20)$

Marginal significance level = .10

Akaike Criterion

Number of restrictions—log likelihood ratio = 3.61

>0 implies failure to reject the null hypothesis

Partial Data Set I

51:3-83:2

Likelihood Ratio Test marginal significance level = .12

Akaike Criterion = 4.14

Partial Data Set II

51:3-83:2 with 79:4-82:1 removed

Likelihood Ratio Test marginal significance level = .12

Akaike Criterion = 3.78

[a] Sims' [17] adjustment for degrees of freedom is incorporated.

As with our test of exogeneity of ex ante real rates, this test requires the imposition of complicated, nonlinear, cross-equation restrictions. The results, given in Table V, again show no evidence for rejection of the null hypothesis for the whole sample or either of the two subperiods examined.

To further illustrate the fit of our restriction, we show the response of industrial production to various innovations, both with and without the imposition of our hypothesis, in Figure 3. The two graphs in this figure show that the impact of an orthogonalized expected inflation innovation on output is an immediate and persistent negative response. In both systems, orthogonalized inflation innovations explain much more of the forecast variance of output than real rate

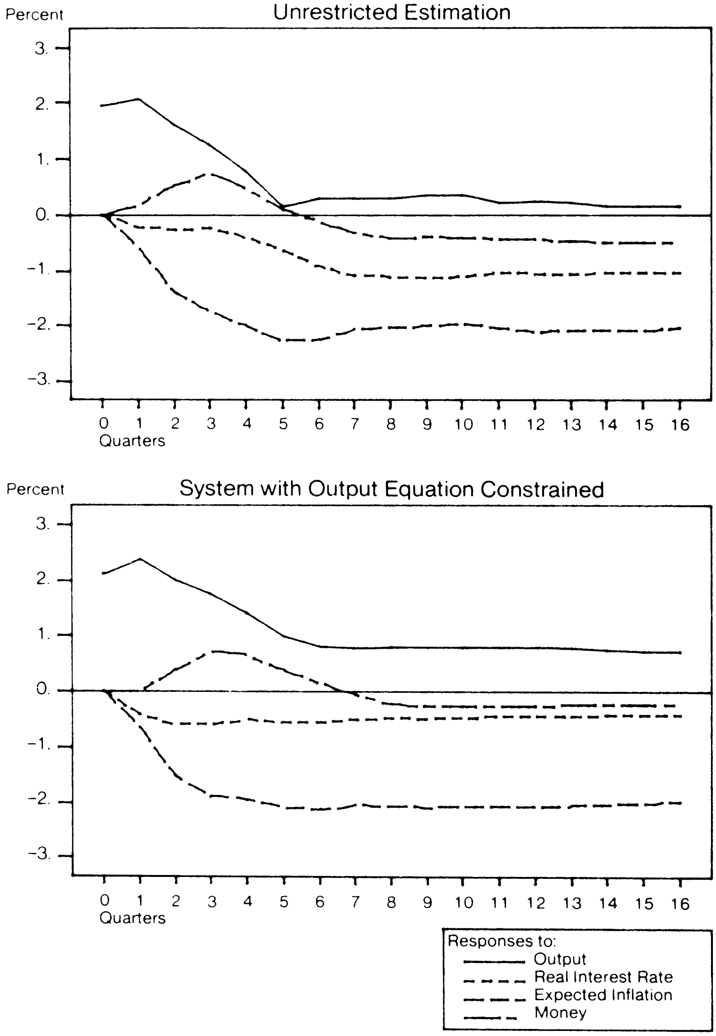

FIGURE 3—Responses of Output

innovations. The decomposition of variance of the restricted system is shown in Table VI. Given our previous finding of real rate exogeneity, it is not surprising that orthogonalized money innovations, which in this system can affect output only through their impact on the real rate, explain only 2.0 per cent of the forecast variance of output at the sixteen-quarter horizon. This lack of explanatory power is not due to the imposition of our restrictions, however; even in the unrestricted system, money innovations at this horizon explain only 3.4 per cent of the forecast variance of output.

TABLE VI

DECOMPOSITION OF VARIANCE OF OUTPUT IN SYSTEM WITH THE RESTRICTIONS OF SECTION 4 IMPOSED

Forecast Horizon (Quarters)	Per Cent of Forecast Variance Explained by Innovations to: Output	Real Rates	Expected Inflation	Money
1	100.0	0.0	0.0	0.0
4	68.2	3.6	25.5	2.7
8	44.4	4.3	48.7	2.7
16	29.4	4.1	64.4	2.0

5. OTHER TESTS

In Sections 3 and 4 we have presented two tests of a hypothesis using as the alternative an unrestricted vector autoregression. In neither case was the hypothesis rejected. Since the lack of rejection of a hypothesis is only of interest to the extent that a test procedure has power to identify false restrictions, it would appear to be useful to show that the procedure we use does indeed discriminate between those restrictions which are consistent with the data and those which are not.

In Table VII we present a number of tests of what causes real rates and output. These tests impose roughly the same number of restrictions as do our previous hypotheses (which we repeat here for convenience as tests 1 and 2). We have adopted a convenient shorthand in Table VII for describing our restrictions. For example, the null hypothesis (that Y is explained by only its own lags, a constant, and lags of innovations in money) is written as "Y explained by $\tilde{M}$." "Y, R block exogenous" refers to the restrictions that only a constant and lags of Y and R appear in the Y and R equations. The hypothesis "r a random walk" tests the restriction that in (11) $b_1 = 1$ and $m = 1$, while "r constant" tests the restriction that $b_1 = 0$ and $m = 1$. These tests provide a diagnostic device for our testing procedure, demonstrating that in many cases it does reject restrictions similar to those we focus on. Moreover, although the hypotheses in Table VII are not generally motivated by particular economic theories, the test results can also be viewed as a convenient device for data summary. As a metric for ranking the relative fit of the various restrictions, we again present the marginal significance level of the log likelihood ratio statistic as well as Akaike's criterion (the number of degrees of freedom less the log likelihood ratio). In tests 3 and 4, for example, we see that Fama's [**5**] hypothesis—that the real rate is constant—is soundly rejected on all samples, whereas the more recent hypothesis of Fama and Gibbons [7]—that the real rate is a random walk—is rejected only when the period of the Federal Reserve's new operating procedures is dropped. The hypotheses in tests 3 and 4 are an additional restriction on the first order Markov restriction of the

TABLE VII

HYPOTHESIS TEST RESULTS
Full Data Set: 49:2-83:2

Null Hypothesis	Alternative	Log Determinant	Degrees of Freedom	χ^2 Statistic	Marginal Significance Level	Akaike's Criterion
1. r exogenous	A	−16.4987	15	9.289	.8619	9.70
2. Y explained by r, Q	B	−16.5376	20	28.592	.0961	3.61
3. r constant	A	−16.2776	16	35.819	.0030	−4.45
4. r a random walk	A	−16.3870	16	22.695	.1221	3.05
5. Y exogenous	A	−16.2755	12	36.069	.0003	−8.59
6. Y, R block exogenous	A	−16.3212	16	30.590	.0152	−1.46
7. Y explained by R	A	−16.4057	8	20.455	.0087	−3.67
8. Y explained by M	A	−16.3861	8	22.806	.0036	−5.02
9. Y explained by Π	A	−16.3334	8	29.120	.0003	−8.63
10. Y exogenous	B	−16.2532	28	61.586	.0003	−7.30
11. Y, R block exogenous	B	−16.2864	32	57.732	.0035	−1.10
12. Y explained by R	B	−16.3899	24	45.729	.0048	−2.21
13. Y explained by M	B	−16.3675	24	48.320	.0023	−3.70
14. Y explained by Π	B	−16.3128	24	54.671	.0003	−7.34
15. Y explained by $\tilde{R}$	B	−16.4036	24	44.136	.0074	−1.30
16. Y explained by $\tilde{M}$	B	−16.3658	24	48.519	.0022	−3.82
17. Y explained by $\tilde{\Pi}$	B	−16.2770	24	58.818	.0001	−9.72
18. Y explained by r	B	−16.3494	24	50.420	.0013	−4.91
19. Y explained by $\hat{\Pi}$	B	−16.4243	24	41.736	.0138	.07
20. Y explained by Q	B	−16.3921	24	45.471	.0051	−2.07
21. Y explained by r, $\hat{\Pi}$	B	−16.4551	20	38.167	.0084	−1.88

Alternative Vector Autoregressions

Alternative	Lag in Equation R	Y	M	Π	Log Determinant	Period	Correction Factor	Effective Number of Observations[a]
A	4	4	4	4	−16.57611	49:2-83:2	17	120
B	4	8	4	4	−16.78408	50:2-83:2	17	116

[a] Sims' [17] adjustment for degrees of freedom is incorporated.

real rate estimate in test 1. Relative to this alternative, both restrictions—that the Markov parameter equals zero and one, respectively—are soundly rejected on all samples.

Of the alternative hypotheses that we tested for what determines output, most are clearly rejected in all cases. The only restrictions to output which fit nearly as well as the hypothesis in test 2 are those on the partial samples in tests 20 and 21. In test 20, the hypothesis is that output is explained by its own lags and lags of expected inflation innovations. Relative to the hypothesis in test 2, this is the additional restriction that the coefficients on lagged real rates are zero. Using Akaike's criterion, this additional restriction is rejected on the full sample, though not on either partial sample. In test 21, the hypothesis is that output is explained by its own lags, lags of the real rate, and lags of the level of expected inflation. The fit is nearly as good as that of the restricted system of test 2 for both partial samples; however, for the full sample, the fit is much worse.

TABLE VII (CONT.)

HYPOTHESIS TEST RESULTS

Partial Data Set I: 49:2–83:2 with 50:2–51:2 removed

Null Hypothesis	Alternative	Log Determinant	Degrees of Freedom	χ^2 Statistic	Marginal Significance Level	Akaike's Criterion
1. r exogenous	A	−16.9366	15	21.596	.1188	2.61
2. Y explained by r, Q	B	−17.1340	20	27.497	.1218	4.14
3. r constant	A	−16.6379	16	55.946	.0000	−16.11
4. r a random walk	A	−16.8525	16	37.262	.0125	−1.95
5. Y exogenous	A	−16.8140	12	35.693	.0004	−8.49
6. Y, R block exogenous	A	−16.8645	16	29.888	.0186	−1.15
7. Y explained by R	A	−16.9627	8	18.589	.0172	−2.67
8. Y explained by M	A	−16.9368	8	21.573	.0058	−4.38
9. Y explained by Π	A	−16.8875	8	27.239	.0006	−7.64
10. Y exogenous	B	−16.8301	28	61.238	.0003	−7.31
11. Y, R block exogenous	B	−16.8545	32	58.526	.0029	−1.75
12. Y explained by R	B	−16.9900	24	43.488	.0087	−1.08
13. Y explained by M	B	−16.9591	24	46.920	.0034	−3.05
14. Y explained by Π	B	−16.9083	24	52.553	.0007	−6.30
15. Y explained by $\tilde{R}$	B	−16.9546	24	47.414	.0030	−3.34
16. Y explained by $\tilde{M}$	B	−16.9166	24	51.634	.0009	−5.77
17. Y explained by $\tilde{\Pi}$	B	−16.9633	24	46.449	.0039	−2.78
18. Y explained by r	B	−16.9899	24	43.497	.0087	−1.08
19. Y explained by $\hat{\Pi}$	B	−16.9990	24	42.483	.0114	−0.50
20. Y explained by Q	B	−17.0804	24	33.453	.0949	4.71
21. Y explained by r, $\hat{\Pi}$	B	−17.1246	20	28.543	.0972	3.54

Alternative Vector Autoregressions

Alternative	Lag in Equation R	Y	M	Π	Log Determinant	Period	Correction Factor	Effective Number of Observations[a]
A	4	4	4	4	−17.1244	49:2–50:1, 51:3–83:2	17	115
B	4	8	4	4	−17.3818	51:3–83:2	17	111

[a] Sims' [17] adjustment for degrees of freedom is incorporated.

Thus, we see from Table VII that not only is our procedure of testing restrictions relative to an unrestricted vector autoregression quite capable of rejecting hypotheses similar to those tested in Sections 3 and 4, but when corrected for degrees of freedom, those earlier hypotheses fit the data better than any of the alternative hypotheses we tried.

6. SUMMARY

This paper has examined the empirical support for a number of hypotheses about the link between money, interest, and output. Because the relevant real rate is unobservable, an appropriate empirical counterpart suggested by a particular class of structural models was formulated. This class of model might be considered "dynamic IS-LM" with rational expectations. Although this class

TABLE VII (CONT.)

HYPOTHESIS TEST RESULTS

Partial Data Set II: 49:2–83:2 with 50:2–51:2 and 79:4–82:1 removed

Null Hypothesis	Alternative	Log Determinant	Degrees of Freedom	χ^2 Statistic	Marginal Significance Level	Akaike's Criterion
1. r exogenous	A	−18.5822	15	17.648	.2816	4.75
2. Y explained by r, Q	B	−18.7737	20	27.768	.1150	3.78
3. r constant	A	−18.3535	16	41.653	.0004	−8.20
4. r a random walk	A	−18.3245	16	44.707	.0002	−9.97
5. Y exogenous	A	−18.4288	12	33.749	.0007	−7.61
6. Y, R block exogenous	A	−18.4041	16	36.342	.0026	−5.11
7. Y explained by R	A	−18.5636	8	19.593	.0120	−3.38
8. Y explained by M	A	−18.5653	8	19.414	.0128	−3.28
9. Y explained by Π	A	−18.4990	8	26.385	.0009	−7.32
10. Y exogenous	B	−18.4912	28	56.294	.0012	−4.89
11. Y, R block exogenous	B	−18.4095	32	64.546	.0006	−5.71
12. Y explained by R	B	−18.6422	24	41.050	.0164	.02
13. Y explained by M	B	−18.6342	24	41.852	.0134	−.45
14. Y explained by Π	B	−18.5698	24	48.355	.0023	−4.25
15. Y explained by $\tilde{R}$	B	−18.5753	24	47.799	.0027	−3.92
16. Y explained by $\tilde{M}$	B	−18.6272	24	42.563	.0111	−.86
17. Y explained by $\tilde{\Pi}$	B	−18.5931	24	45.999	.0044	−2.87
18. Y explained by r	B	−18.5826	24	47.060	.0033	−3.49
19. Y explained by $\hat{\Pi}$	B	−18.6462	24	40.642	.0182	.26
20. Y explained by $\tilde{Q}$	B	−18.7085	24	34.346	.0787	3.93
21. Y explained by r, $\hat{\Pi}$	B	−18.7725	20	27.885	.1122	3.71

Alternative Vector Autoregressions

Alternative	Lag in Equation R	Y	M	Π	Log Determinant	Period	Correction Factor	Effective Number of Observations[a]
A	4	4	4	4	−18.7502	49:2–50:1, 51:3–79:3, 82:2–83:2	17	105
B	4	8	4	4	−19.0486	51:3–79:3, 82:2–83:2	17	101

[a] Sims' [17] adjustment for degrees of freedom is incorporated.

does not include those models which explicitly posit barriers to information flows, some of our results bear on their empirical validity.

The first test sought to identify the determinants of the real interest rate. Specifically, we could not reject the hypothesis that the real rate is governed only by its own past history, with no separate influence coming from money, output, nominal rates, or prices. Although this hypothesis is not an implication of any particular alternative to the Keynesian theory, it is incompatible with Keynesian models, except for some very restrictive and economically uninteresting special cases. Taken literally, our results imply that monetary policy has not discernibly affected the real rate, although it has causally influenced nominal interest rates. Our results also show a strongly negative correlation between expected real rates and inflation innovations. Since both inflation and expected real rates have some persistent component, this can explain the well-documented negative correlation between the level of current period inflation and real rates, even in the absence of any structural link between past inflation and future real rates.

Our second test showed that expected inflation innovations are a sufficient statistic for predicting real variables, given current and past real variables. The effect of an inflation innovation on future output is unambiguously negative, a result which seems incompatible with most demand driven models of output. We interpret this result as being consistent with a "classical" model in which output is structurally exogenous to money and prices, but that new information is first reflected in expected inflation and nominal interest rates. Several other hypotheses were tested which, although not derived from any completely articulated theory, are of independent interest and show that our test procedure has power to discriminate among alternatives.

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