Bargaining theory

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Classical two-person bargaining problem

A *bargaining problem* is a tuple $\langle X, D, \prec_1, \prec_2 \rangle$

- X is the set of possible *agreements*
- D is the *disagreement* outcome
- $\prec_i$ is player i's preference over $\Delta(X)$

Classical two-person bargaining problem (cont.)

Assumptions

- $D \prec_i x$ for both i and all $x \in X$, and, for some $x \in X$, $D \prec_i x$ for both i
- for any $x, y \in X$ and any $p \in [0, 1]$, there exists $z \in X$ such that $p\delta_x + (1 - p)\delta_y \sim_i z$ for both i
- for each i there is a unique $B_i \in X$ such that $x \prec_i B_i$ for all $x \in X$
- for each i, $B_i \sim_j D$ for $j \neq i$

Utility space

Let u_1 and u_2 be the von Neumann-Morgenstern utility functions representing $\prec_1$ and $\prec_2$, respectively

- let $U = \{(u_1(x), u_2(x)) : x \in X\}$ and $d = (u_1(D), u_2(D))$
- we can choose u_1 and u_2 such that $d = (0, 0)$
- then, the bargaining problem may be reduced to $\langle U, d\rangle$

The assumptions become

- for some $(v_1, v_2) \in U$, $v_1 > 0$ and $v_2 > 0$
- U is a compact and convex set

Given $\langle U, d\rangle$ and scales $a > 0$ and b,the *rescaled* problem $\langle U', d'\rangle$ w.r.t. (a, b) is

$$U' = \{au + b : u \in U\}, \text{ and } d' = ad + b. \tag{1}$$

Nash solution

The *Nash solution* assigns to the problem $\langle X, D, \prec_1, \prec_2 \rangle$ an agreement $x^* \in X$ for which

$$x^* \prec_i p\delta_x \Rightarrow x \precsim_j p\delta_{x^*} \text{ for all } x \in X \text{ and } p \in [0,1] \tag{2}$$

Theorem

The agreement $x^* \in X$ is a Nash solution of the problem $\langle X, D, \prec_1, \prec_2 \rangle$ if and only if

$$x^* \in \arg\max_{x \in X} [u_1(x) - u_1(D)][u_2(x) - u_2(D)]. \tag{3}$$

Moreover, such agreement is unique.

General solution

A *solution*, denoted by F, is a function that maps a bargaining problem, U, d, into a unique agreement point, $F(U,d) = (F_1(U,d), F_2(U,d)) \in U$

- here we consider the utility space
- the Nash solution is a particular solution

A bargaining problem, $\langle U, d \rangle$, is *symmetric* if there is a function $\phi : U \to U$ such that

- $\phi(d) = d$;
- $\phi(u) = v$ if and only if $\phi(v) = u$

The axiomatic approach

Four axioms on a solution:

- **PO** There is no other agreement $(u_1, u_2) \in U$ such that $F_i(U, d) \leq u_i$ for both $i = 1, 2$ with strict inequality for at least one i
- **SYM** If $\langle U, d \rangle$ is symmetric w.r.t. ϕ, then $\phi(F(U, d)) = F(U, d)$
- **SI** Given a bargaining problem $\langle U, d \rangle$ and its rescaled version $\langle U', d' \rangle$ w.r.t. (a, b), then $F(U', d') = aF(U, d) + b$
- **IIA** Consider two bargaining problems, $\langle U, d \rangle$ and $\langle U', d \rangle$ with $U \subset U'$; if $F(U', d) \in U$, then $F(U, d) = F(U', d)$

Characterization of Nash solution

Theorem

Nash solution is the unique solution that satisfies PO, SI, SYM, and IIA.

Bargaining in search-theoretic literature

Two players: *buyer* and *seller*

- seller produces y consumption good for buyer
- in exchange of some payment p from buyer, bounded by capacity z

Preferences over outcome $(y, p) \in \mathbb{R}_+ \times [0, z]$:

$$\begin{aligned} u^b &= u(y) - p, \\ u^s &= -v(y) + p \end{aligned}$$

- $u'(y) > 0,\ u''(y) < 0,\ u'(0) = +\infty,$
- $v'(y) > 0,\ v''(y) < 0,\ v'(0) = 0,\ u(0) = v(0) = 0$
- for some $y^* > 0$, $u'(y^*) = v'(y^*)$

The Nash solution

$(y, p) \in \arg\max (u^b)(u^s)$ s.t. $p \leq z$

- solution: $p = p^N(y) = \min\left\{z, p^N(y^*)\right\}$ where

$$p^N(y) \equiv [1 - \Theta(y)]u(y) + \Theta(y)v(y),\ \Theta(y) = \frac{u'(y)}{u'(y) + v'(y)}$$

- $u^b = \Theta(y)[u(y) - v(y)]$ is not monotone in z

When $z \geq p^N(y^*) = [u(y^*) - v(y^*)]/2$, production level is y^*; otherwise, it is lower than y^*

Optimal payment capacity

The buyer chooses z before the negotiation, with cost ιz

- anticipating the bargaining outcome determined by Nash
- ι is the opportunity cost of obtaining payment z

Formally, the buyer's problem is

$$-\iota z + u^b(z), \tag{4}$$

where $u^b(z)$ is buyer payoff from bargaining with payment capacity z

Inefficiency of Nash bargaining

Theorem

For all $\iota \geq 0$, optimal $z < p^N(y^*)$.

- main result from Lagos and Wright (2005)

The result can be avoided by another bargaining solution, the proportional solution

The proportional solution

Kalai (1977): $(y, p) \in \arg\max u^b$ s.t. $u^b = u^s$, $p \leq z$

- solution: $p = p^K(y) = \min\left\{z, p^K(y^*)\right\}$, where

$$p^K(y) \equiv [u(y) + \upsilon(y)]/2$$

- $u^b = [u(y) - v(y)]/2$ is monotone in z

The Rubinstein game

Potentially infinitely many stages

- buyer first makes an offer (y, p), $y \leq y^*, p \leq z$
- if accepted, the game ends with agreement (y, p)
- otherwise,
  - with prob. ξ^s, seller makes an offer
  - with prob. $1 - \xi^s$, the game ends
- if seller offer rejected, buyer gets to make another offer with prob. ξ^b
- and so on....

SPE without liquidity constraint

Consider the case where $p \leq z$ does not bind

Equilibrium conditions for equilibrium offers, (y^b, p^b, y^s, p^s)

- $y^b = y^* = y^s$, for otherwise there will be unexploited gain from trade
- (p^b, p^s) solves

$$\begin{aligned} -\upsilon(y^*) + p^b &= \xi^s \left[-\upsilon(y^*) + p^s\right] \\ u(y^*) - p^s &= \xi^b \left[u(y^*) - p^b\right]. \end{aligned}$$

Calculate the equilibrium (p^b, p^s)