History and background

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The "Crisis" in Economics–a Century Ago

There is, in my on country, a general impression among educated men and even among professed economists themselves that our science has no methods or results which are universally accepted or worthy of being universally accepted, that it consists of an indefinite number of conflicting systems between which there is little to choose, that it is at best a sort of philosophy of economic life without much bearing upon practice, in short its state is simply chaotic and it is immersed in permanent crisis.

The "Crisis" in Economics–a Century Ago

there are many reasons for this [...] one of which consists in the inability, both of our public and of ourselves, to draw the right line between theorems and their applications, and between theorems and philosophical or quasi-philosophical doctrines.

Schumpeter, Journal of Economic Literature, 1982

Economics and game theory

Economics was under crisis in late 19th century

- one central question is explanation for relative prices

- another one is theory of capital and interest

Mathematics was also under crisis in the same period

- success in set theory leads to fruitful real analysis

- consistency of set theory questioned by Russell's paradox

Game theory emerges as mathematician searching for a resolution

The marginal revolution of economics

In late 19th century
- Menger (1871), Jevons (1871), Walras (1874)

Marginal value as the central concept
- following the tradition in calculus

Central question: determination of relative price
- the water/diamond paradox

The proof-theory evolution in mathematics

Informal mathematics presumes correspondence between
- symbolic expressions used by mathematicians

- objects in the outer world

Proof theory aims at an explicit formulation that distinguish the two and then connect to two
- this leads to mathematical logic, study of syntax (forms) and semantics (meaning)

- and to reformulation of set theory, and

- to computer science and game theory

Monopoly pricing

A monopoly facing market demand $P = a - bQ$

- P is market price

- Q is the quantity produced

Assuming that marginal cost is c, what is optimal quantity and price?

Model of Oligopoly

The Cournot (1838) model of duopoly

- two firms, $i = 1, 2$

- competing along quantities

- market demand depends on the quantities produced

Formalism

Firm i's production is Q_i, $i = 1, 2$
- market price is $P = a - b(Q_1 + Q_2)$

The two firms simultaneously determine productions
- but the optimal quantity depends on the other's decision

Given the firm 2's quantity Q_2, firm 1's profit is

$$P \times Q_1 - c \times Q_1 = \{[a - b(Q_1 + Q_2)] - c\} \times Q_1$$

Solve the optimal Q_1, as a function of Q_2, denoted by $f_1(Q_2)$

Equilibrium

A pair of productions, (Q_1^*, Q_2^*), is an equilibrium if
- $Q_1^* = f_1(Q_2^*)$, that is, Q_1^* is optimal given Q_2^*
- $Q_2^* = f_2(Q_1^*)$, that is, Q_2^* is optimal given Q_1^*

Prove that there is an unique equilibrium

Stackelberg competition

Now assume that firm 1's production decision is visible to firm 2 before firm 2's decision

- equivalently, we can assume that firm 1 can *commit* to its decision and make it known

Oligopoly

Firm i's production is Q_i, $i = 1, 2, ..., N$
- market price is $P = a - b \sum_{i=1}^{N} Q_i$

The N firms simultaneously determine productions
- but the optimal quantity depends on the others' decision

Given the other firms' *aggregate* quantity Q_{-i}, firm i's profit is

$$P \times Q_i - c \times Q_i = \{[a - b(Q_i + Q_{-i})] - c\} \times Q_i$$

Solve the optimal Q_1, as a function of Q_{-i}, denoted by $f(Q_{-i})$

N-firm equilibrium

A pair of productions, $(Q_1^*, Q_2^*, ..., Q_N^*)$, is an equilibrium if for all i,

- $Q_i^* = f\left(Q_{-i}^*\right)$, that is, Q_i^* is optimal given Q_{-i}^*

Prove that there is an unique equilibrium

Perfect competition

When there are many firms, competition becomes stronger

- equilibrium price converges to c

This is efficient

- marginal benefit to the society equal to marginal cost