

## Microeconomic Theory I Midterm [11/12/2010]

Please Note:

1. Write your answers in English.
2. You have 3 hours (2:20-5:20pm); there are a total of 150 points (and would count for “15% + 6% (in lieu of computer assignment)” toward your final grade). Allocate your time wisely. You do not have to do the questions in order.
3. If you cannot find the appropriate function/game trees/matrices to answer question A1, B2, or B9, you may request them (from the TA) to solve subsequent questions. However, you will have to forfeit some points for each request.

### Part A (32%): The Undercover Signaling Game ( 黑社會如何識別臥底 )

Read the following Financial Times article regarding signaling in the underworld:

#### **You can't afford to get signals crossed in the underworld**

**By Tim Harford (2010/9/3 Financial Times)**

“A wiseguy sees things if there are wiseguy things to see,” wrote Joe Pistone, the FBI agent better known as Donnie Brasco – the name under which he managed to infiltrate the mob. But what are the wiseguy things to see? And how is a wiseguy to know he isn't dealing with the likes of Joe Pistone?

Such questions are among those that fascinate Diego Gambetta. Professor Gambetta, an Italian sociologist based at Oxford University, has managed to wrap himself in the language of economics as capably as Pistone wrapped himself in the language of organised crime. Gambetta is an authority on the Sicilian mafia, but deploys the tools of an economist to understand them and other criminals.

A key concept in modern economics is the “signal”, an idea developed by the Nobel laureate Michael Spence. A signal is an action that distinguishes one type of person from a would-be mimic because it would be too costly for the mimic to carry out. Spence suggested that the decision to acquire a degree might be a signal. The degree may be of no practical value but employers may still value it and quite rationally pay higher salaries to graduates. Why? Because a degree will distinguish good applicants from bad – if bright, energetic candidates are willing to go to the trouble of acquiring one, while dim, lazy candidates are not. The degree serves no educational purpose but the employer uses it to separate the wheat from the chaff.

For a criminal, the stakes are higher and the dividing lines sharper. However similar the boiled-down textbook model might seem, employing a graduate who turns out to disappoint is not the same as plotting an offence with a colleague who turns out to be an undercover cop. But while it is no easy matter to study criminal signals, the danger and purity of the signalling problem that criminals face makes them a tempting group for Gambetta to study in his new book, *Codes of the Underworld*.  
(...omitted...)

An interesting implication of Michael Spence's model is that if degrees really were largely signals, the world would be a better place if universities were closed down. There is an interesting parallel in Gambetta's work: he points out that prison time provides a wonderfully credible signal. Few undercover police are likely to sign up for four or five years in jail, so an extended prison sentence can be an asset to any criminal trying to establish his credentials. Prisons are sometimes called universities of crime, but surely this is a parallel nobody expected.

**Answer the following questions:**

1. (4%) Consider the following criminal signaling game: A newbie is about to join the gang, and the boss has to determine whether to accept him into the circle or not. The boss knows that the newbie is either a true ally or an undercover cop with probability  $(p, 1-p)$ , and wants to take the appropriate action. In particular, the boss would like to accept ( $A$ ) the newbie if he is a true ally (payoff =  $b_2$ ), and kill him ( $K$ ) if he is a cop (payoff =  $b_1$ ). Taking the wrong action is fatal (killing an ally or allowing a cop to join) and yields a payoff of 0. The newbie can first send a signal ( $S$ ) by staying in jail for five years (which costs  $c$  for both types), and if successfully join the circle, receives a payoff of  $a_1$  if he is a cop and  $a_2$  if he is a true ally. Draw the game tree of this undercover signaling game. (If you are not sure, you may request it from the TA and forfeit 4 points.)
2. (8%) Suppose  $\bar{\beta} = \frac{pb_2}{(1-p)b_1} < 1, 0 < c < a_1 < a_2, b_1, b_2 > 0$ . Show that the strategy profile [(Not|cop, Not|ally), (Kill|Send, Kill|Not)] is a perfect Bayesian Equilibrium, or Bayesian Nash equilibrium (with rationalizable out-of-equilibrium beliefs) as discussed in BGT A1.2. What are the supporting beliefs?
3. (2%) What is the economic interpretation of this pooling equilibrium?
4. (8%) For the same set of parameters, show that  $[(\beta S + (1 - \beta)N|cop, S|ally), ((1 - \alpha)K + \alpha A|Send, K|Not)]$  is a perfect Bayesian Equilibrium, or Bayesian Nash equilibrium (with rationalizable out-of-equilibrium beliefs) as discussed in BGT A1.2. What is  $\alpha, \beta$ ? What are the supporting beliefs?
5. (2%) What is the economic interpretation of this semi-pooling equilibrium?
6. (4%) Consider the experimental data reported in Potters and van Winden (IJGT 1996):

Session	$\bar{\beta}$	Cop send	Ally send	$c/a_1$	Accept   Not	Accept   Send
3	25%	16%	85%	75%	0%	53%

Which of the above equilibrium predicts the data better? Why?

7. (4%) Do the above equilibrium you find match the situation described in Tim Harford's article, namely, "prison time provides a wonderfully credible signal"? Why or why not?

**Part B (68%): Being Rich is Good (有錢真好) vs. Being Rich is Better (有錢卡司好)**

Consider the following three events:

Event 1: 作者: xxxxxx (xxx) 看板: NTU

標題: [問題] 後門 118 巷內"有錢真好"的炒飯  
時間: Wed Nov 18 08:50:31 2009

經過該店門口，發現它的炒飯價格比較便宜。但也要 40 元。  
有人吃過嗎？

炒飯的內容會有肉塊嗎？還是就是一般的炒飯內容，幾片碎葉、幾片肉片？  
向大家打聽一下，來做為我是否前去光顧炒飯的參考，謝謝說明囉～

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※ 發信站: 批踢踢實業坊(ptt.cc)

Event 2: Coupon for NT\$10 off each meal →

**吃飯的學問**  
不過是找對小吃店而已

**有錢。卡司好。現炒飯麵**

**折價券 10元**

【每客僅限用一張】

歡迎外帶

台北市辛亥路2段209號  
TEL: 23772289      002218

Event 3: 作者: xxxxxx (aaaa aaaa) 看板: NTU

標題: [問題] 後門的有錢系列要開戰了?  
時間: Fri Oct 29 19:32:39 2010

以前去"卡司好"都點牛腩燴飯，覺得 80 塊才 4 條牛腩，  
和"真好"一比顯得有點虛，不過還好有無限飯的優點，打平差距。

今天去吃"卡司好"，發現不只招牌換了，  
大多數餐點包括牛腩燴飯的定價都變成 50(港式 60)，剩下 6 樣 80 的餐，  
其他附餐類品質沒變，一樣有無限飯，  
豆腐羹裡豆腐塊膨脹成之前的 8 倍左右，  
這樣"卡司好"在後門有錢系列的競爭力  
一夕之間從最昂貴翻盤成最便宜，  
如此打破行情的舉動其他競爭者是否會跟進？  
還是會放棄競爭被淘汰，值得觀察。

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(簽名檔略)

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※ 發信站: 批踢踢實業坊(ptt.cc)

Answer the following questions:

1. (4%) Consider the following game played between the two restaurants, “Being Rich is Better” (BRB) and “Being Rich is Good” (BRG) choosing prices  $p_1, p_2$ , respectively. Each restaurant has the same fixed cost  $C = 3,000$  (per day) and a constant marginal cost of  $c_1$  for BRG and  $c_2$  for BRB (per meal). The (daily) demand function for each restaurant is **hedonic**, or linear in product characteristics (such as own price and other’s price):

$$q_i(p_i, p_{-i}) = D_i - \varepsilon_i p_i + \eta_i p_{-i}$$

What is the economic interpretation of  $\varepsilon_i$  and  $\eta_i$ ?

2. (4%) Write down the profit maximization problem of restaurant  $i$  given rival price  $p_{-i}$ . (If you are not sure, you may request it from the TA and forfeit 4 points.)
3. (8%) Suppose firms know their opponent’s price when choosing their actions. Show that the best response functions are linear for  $p_1 \in [c_1, 125], p_2 \in [c_2, 130]$ . I.e. Find the parameters  $(k_i, m_i)$  for the best response function  $p_i^{BR}(p_{-i}) = k_i + m_i p_{-i}$
4. (8%) Suppose both firms move simultaneously. What is the Nash equilibrium of this simultaneous pricing game? What is the equilibrium profit for each firm? How many meals does each firm serve?
5. (8%) Suppose you estimated from empirical data  $D_1 = 450, D_2 = 240, \varepsilon_1 = 12, \varepsilon_2 = 16$ , and  $\eta_1 = 15, \eta_2 = 11$ . Moreover, you found that BRB initially entered the market and set  $p_1 = 80$  against  $p_2 = 70$ , but then (Event 1) BRG countered with  $p_2 = 40$ . If you assume that BRB’s first move ( $p_1 = 80$ ) and BRG’s second move ( $p_2 = 40$ ) are both best responses to the then opponent price, what are the marginal costs of the two restaurants?
6. (4%) Consider the case of  $(p_1, p_2) = (80, 70)$  and that of  $(p_1, p_2) = (80, 40)$ , calculate the (daily) profit for each firm in both cases? How many meals (per day) does each firm serve in each case?
7. (8%) Suppose now BRB counters BRG by offering coupons of \$10 off per meal (Event 2). What is the profit and meals sold for each restaurant now? Is this a best response? Why or why not?
8. (4%) Knowing that BRG is still doing pretty well, Mr. Rich, the owner of BRB, and decides to lowers his price to  $p_1 = 50$  (Event 3). Is this a best response? Why or why not?
9. (12%) Can you come up with a utility function that “rationalizes” the behavior of Mr. Rich? Find the appropriate parameters to make  $p_1 = 50$  a best response. (Hint: You may request the Guilt-Envy model and forfeit 4 points.)
10. (8%) What do you think BRG would do in response? Can you predict the profit and meals sold for each restaurant if this happens? How close is this to the equilibrium prediction for the simultaneous game that you solved in Question 4?

### Part C (20%): Two-Person Guessing Games

- (4%) Consider the following 2-person guessing games: Each player chooses a number  $x_i \in [a_i, b_i]$ . Payoffs are higher if  $x_i$  is closer to the target  $g_i = m_i x_{-i}$  (depends on opponent's number):  $u_i(x_i, x_{-i}) = |x_i - g_i|$ . What are the Nash equilibrium of the following games?
  - $[a_1, b_1] = [300, 900], m_1 = 0.7; [a_2, b_2] = [100, 900], m_2 = 1.3.$
  - $[a_1, b_1] = [100, 900], m_1 = 0.5; [a_2, b_2] = [100, 500], m_2 = 1.5.$
- (6%) Assume L0 players play randomly, and Lk players best response to L(k-1) players. What is the L1, L2, L3 prediction for each player in these games?
- (2%) Does the Lk prediction coincide with Nash equilibrium for some k? Why or why not?
- (2%) According to the experimental data reported in Costa-Gomes and Crawford (AER 2006), Subject#101 played  $x_1 = 350, x_2 = 780$  in game (a) and  $x_1 = 150, x_2 = 500$  in game (b). Which type is this subject? Is it a best response to play the Nash equilibrium against this person? Why or why not?
- (2%) According to the experimental data reported in Costa-Gomes and Crawford (AER 2006), Subject#108 played  $x_1 = 546, x_2 = 455$  in game (a) and  $x_1 = 250, x_2 = 225$  in game (b). Which type is this subject? Is it a best response to play the Nash equilibrium against this person? Why or why not?
- (4%) How does your answer to this part provide insight to the pricing game considered in Part B?

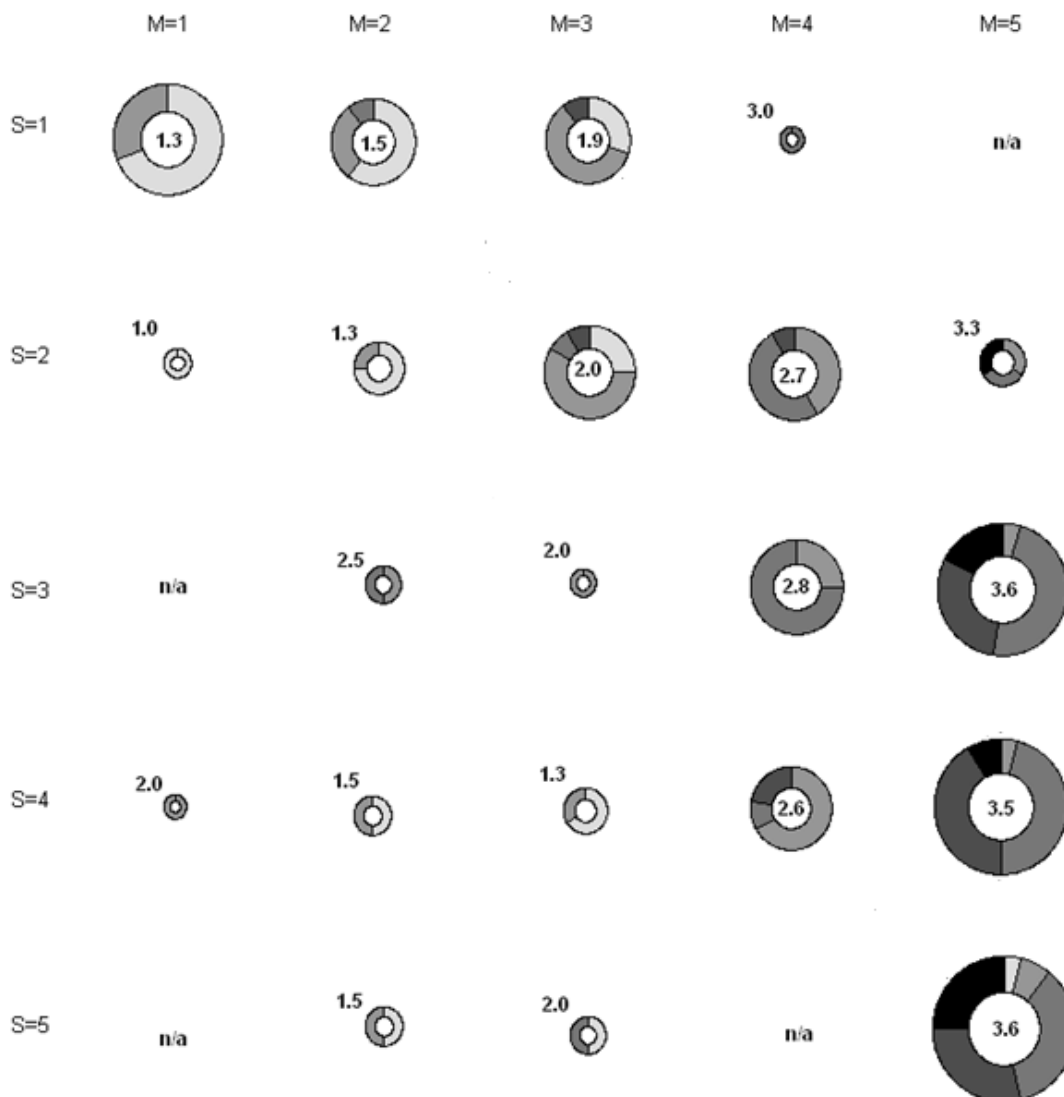
## Part D (30%): Stock Analyst Cheap-talk

The following story was reported by Wang, Spezio and Camerer (AER 2010):

“During the tech-stock bubble, Wall Street security analysts were alleged to inflate recommendations about the future earnings prospects of firms, in order to win investment banking relationships with those firms. Specifically, analysts in Merrill Lynch used a five-point rating system (1=Buy to 5=Sell) to predict how the stock would perform. They usually gave two 1-5 ratings for short run (0-12 months) and long run (more than 12 months) performance separately. Henry Blodget, Merrill Lynch’s famously optimistic analyst, “did not rate any Internet stock a 4 or 5” during the bubble period (1999 to 2001). In one case, the online direct marketing firm LifeMinders, Inc. (LFMN), Blodget first reported a rating of 2-1 (short run “accumulate”—long run “buy”) when Merrill Lynch was pursuing an investment banking relationship with LFMN. Then, the stock price gradually fell from \$22.69 to the \$3-\$5 range. While publicly maintaining his initial 2-1 rating, Blodget privately emailed fellow analysts that “LFMN is at \$4. I can’t believe what a POS [piece of shit] that thing is.” He was later banned from the security industry for life and fined millions of dollars.”

Consider the following sender-receiver game: One player is the sender, and the other is the receiver. The sender is informed about the true state of the world,  $S$ , uniformly drawn from the state space  $\mathbf{S} = \{1, 2, 3, 4, 5\}$ , and the bias  $b=1$ . The receiver knows the bias  $b$ , but not the realization of the state  $S$ . The sender then sends a message to the receiver, from the set of messages  $\mathbf{M} = \{1, 2, 3, 4, 5\}$ . After receiving a message from the sender, the receiver chooses an action from the action space  $\mathbf{A} = \{1, 2, 3, 4, 5\}$ . The true state and the receiver’s action determine payoffs. In particular, the receiver’s payoff is  $u_R = 110 - 20 \cdot |S - A|^{1.4}$ , and the sender’s payoff is  $u_S = 110 - 20 \cdot |S + b - A|^{1.4}$ . Note that the receiver earns the most if her action matches the true state (since her payoff falls with the absolute difference between  $A$  and  $S$ ). The sender prefers the receiver to choose an action equal to  $S+b$ . The basic structure of the game is commonly known to both players.

1. (2%) Suppose  $L_0$  senders would simply tell the truth and  $L_0$  receivers follow the message (either naively or best responding to  $L_0$  senders). In addition,  $L_k$  senders best respond to the  $L(k-1)$  receivers and  $L_k$  receivers best respond to  $L(k-1)$  senders. What would a  $L_1$  sender send when seeing  $S=1, 2, 3, 4, \text{ or } 5$ ? What about  $L_1$  receivers?
2. (4%) What is the strategy of a  $L_2$  sender? What about a  $L_2$  receiver? What are her beliefs when seeing each message?
3. (4%) What is the strategy of a  $L_3$  sender? What about a  $L_3$  receiver? What are her beliefs when seeing each message?
4. (4%) What is the prediction for a  $L_4$  sender? Are they distinguishable from  $L_3$  senders? Why or why not? What about a  $L_4$  receiver?
5. (2%) What is the perfect Bayesian equilibrium of this game? Does the  $L_k$  prediction coincide with Nash equilibrium for some  $k$ ? Why or why not?
6. (4%) Show that it is a perfect Bayesian equilibrium for senders to send  $M=5$  regardless of the true state  $S$ , and receivers to choose  $A=3$  regardless of the message.



Wang, Spezio and Camerer (2010) report experimental data with the above figure: “The true states 1-5 correspond to the five rows. The sender messages 1-5 correspond to the five columns. Within each stage-message cell, there is a pie chart. The area of the pie-chart in each cell is scaled by the number of occurrences for the corresponding state and message ; i.e., the most common state-message pairs have the largest pies. Hence, the rows indicate senders’ behavior with respect to different states and the columns represents the “informativeness” of each message, determined by the distribution of states conditional on each particular message. ...Each pie chart also shows the distribution of actions chosen by the receiver for that state and message, using a gray-scale ranging from white (action 1) to black (action 5). The average receiver action is the number inside the pie.”

7. (4%) Does the equilibrium prediction match this (aggregated) empirical data? Why or why not?
8. (6%) Does the level-k prediction match the aggregate data? Why or why not? How could you test the level-k model prediction if you had individual data?