Microeconomic Theory I Midterm [11/13/2009]

Please Note:

1. You have 3 hours (2:20-5:20pm); there are a total of 90 points plus bonus 12 points (and would count for "30% + bonus 4%" toward your final grade). Allocate your time wisely.
2. If you cannot find the appropriate functions to answer question B1, B2, B8a, D6a or D7a, you may request suggested functions (from the TA) to solve subsequent questions. However, you will have to forfeit 2 point for each function you request.

Part A (18%): Predicting Your Own Classroom Experiment Results

Suppose there are two groups of consumers. Members of the first group each have the same endowment (44, 9), and utility function $U_i(x, y) = a\log x + (1-a)\log y$. Members of the second group each have the same endowment (6, 41), and utility function $U_i(x, y) = x^b y^{1-b}$.

1. (2%) Why can the two groups of consumers each be represented by one representative agent? Explain.
2. (4%) Find all Pareto efficient allocations (PEA).
3. (2%) Suppose a=0.6, b=0.8. Draw the Edgeworth box and depict all Pareto efficient allocations the two parties could agree upon (the contract curve).
4. (4%) What is the Walrasian equilibrium if consumers are all price takers?
5. (2%) Suppose a=0.6, b=0.8. Draw the Walraisian equilibrium allocation on your Edgeworth box and the budget line that connects it with the endowment allocation.
6. (2%) How could you transform the above two utility function so that the new utility functions satisfies *u*(x,0) = *u*(0,y)=0 and *u*(50,50)=1000? (You need not worry about uniqueness, just give one transformation that does the job.)? Do your results in the previous questions change? Why or why not?
7. (**bonus** 2%) Can your answers predict the behavior of you and your colleagues in the classroom experiment we conducted last week? Explain.

Part B (44%): Edgeworth Box Bargaining and the Competitive Equilibrium

Consider two parties, Daiwan and Dailiok, bargain over two goods, Gingtse (G) and Tzukuan (T). Total endowment is G=100, T=100.

1. (2%) Suppose Dailiok only cares about Tzukuan (T). Write down a utility function that represents Dailiok's preferences assuming that it is homothetic.
2. (2%) Suppose Daiwan view the two goods as perfect substitutes (units defined so the rate of substitution is 1). Write down a utility function to represent the preferences of Daiwan assuming that it is also homothetic.
3. (2%) Are these the only utility functions that can represent such preferences? Explain.
4. (4%) Find all Pareto efficient allocations the two parties could agree upon.

5. (2%) Draw the Edgeworth box and depict all Pareto efficient allocations (contract curve).
6. (4%) Initially, Daiwan has G=20, T=80, and Dailiok has G=80, T=20. What is the Walrasian equilibrium if both parties are price takers?
7. (4%) Verify that the Kuhn-Tucker conditions are satisfied for both parties in equilibrium.
8. Next, suppose Daiwan view the two goods as substitutes: If Daiwan owns more G, the rate of substitution is X units of G for one unit of T. If Daiwan owns more T, the rate of substitution is X unit of T for one unit of G. Assume X=10. Redo questions 1, 4-7 as follows: (Hint: You may apply what you learned from the Sample Question!)
    a. (2%) Write down new a utility function to represent the preferences of Daiwan assuming that it is also homothetic.
    b. (4%) Find all Pareto efficient allocations the two parties could agree upon.
    c. (2%) Draw the Edgeworth box and depict the contract curve.
    d. (4%) Initial endowment is G=20 and T=80 for Daiwan, and G=80, T=20 for Dailiok. What is the Walrasian equilibrium (for each state) if both parties are price takers?
    e. (4%) Verify that the Kuhn-Tucker conditions are satisfied for both parties in equilibrium.
9. (2%) Are the above Walrasian equilibrium allocations (Pareto) efficient? Explain.
10. (**bonus** 2%) Now suppose there are two groups of people in Daiwan: Half of the people view the two goods as perfect substitutes (denote group PS) and the other half view the two goods as perfect complements (denote group PC). Moreover, Daiwan can select a delegate from one group to bargain with Dailiok according to the delegate's own preferences. Suppose the Walrasian equilibrium accurately predicts the bargaining outcome. Which group would the people in group PS and group PC each pick from?
11. (**bonus** 2%) How could your answers above apply to the on-going ECFA bargaining between Taiwan and China? (Hint: What kind of negotiator should Taiwan send? Why?)
12. (**bonus** 2%) How would your answers to the above questions change if the Walrasian equilibrium may not be a good predictor of the bargaining outcome?

Part C (40%): The Sleeping Game

Read the (abridged) article below and answer the following questions:

1. (2%) Consider the following game played between the two sleepy pilots: Each pilot chooses to either sleep or stay awake. Falling asleep gives the sleepy pilot some rest, which is worth NT$2,000 to each pilot. The plane flies safely if at least one pilot to stay awake, which is worth NT$10,000 to each pilot. If both pilots fall asleep, the plane would be in danger, which would cost the pilot NT$100,000 each. Draw the game matrix (assuming each pilot only cares about the sum of their own monetary payoffs).
2. (1%) Is it consistent with equilibrium for both pilots to stay awake? Why or why not?
3. (6%) Solve for all of the pure and mixed Nash equilibrium of this game.
4. (3%) Which equilibrium could result in the case described in the news above where both pilots fall asleep despite FAA forbidding pilots sleeping? Which equilibrium

corresponds to the case where one pilot tells the other s/he is going to rest for a while? Which one has a higher monetary payoff for the pilots?

5. (**bonus** 2%) How did different rules (across countries) select different equilibrium? Do you think the experts rightly blame the FAA for this “accident”? Why or why not?
6. Now consider the case where the two pilots are altruistic and have the same utility function. Assume the pilots discount the payoffs of others by a factor of α.
    a. (2%) Write down a utility function to represent the pilots’ altruistic preferences and draw the new payoff matrix.
    b. (6%) Solve for all of the pure and mixed Nash equilibrium of this game.
    c. (4%) Can this explain both the outcome in the US (where both pilots occasionally fell asleep) and that in Taiwan (where one pilot asks the other to cover him when he is taking a nap)? Why or why not?
7. Now consider the case where the two pilots are inequality averse in the sense of Fehr and Schmidt (1999) and have the same utility function. Assume the pilots dislike earning less than the other player by a factor of α, but feel guilty about earn more by a factor of β.
    a. (2%) Write down a utility function to represent the pilots’ inequality-averse preferences and draw the new payoff matrix.
    b. (6%) Solve for all of the pure and mixed Nash equilibrium of this game.
    c. (4%) For what parameter values can this explain the intended outcome of the FAA (where both pilots always stay awake) and the outcome in Taiwan (where one pilot asks the other to cover him when he is taking a nap)? Why or why not?
8. (**bonus** 2%) Which model of pilot’s preferences do you think is more realistic and explains more empirical facts? Justify your answer.

美客機「睡」過頭 怪ＦＡＡ不人道　　2009/10/26 中國時報【劉屏／華盛頓廿五日電】

兩位客機駕駛員都睡著了。誰的錯？美國聯邦民航總署（ＦＡＡ）成為眾矢之的。專家說，ＦＡＡ不把駕駛員當人，不准他們輪流小睡，才導致這種緊急事件。

美國西北航空班機日前在明尼亞波利斯市上空過門不入，多飛了二百四十餘公里才降落。其間地面航管呼叫，但得不到回應，以為發生劫機，遂請軍方派遣戰鬥機升空。最後是空服員從客艙內打機內電話給駕駛員，才結束這場烏龍。兩位駕駛說是因為討論公司的政策太專心，以致忘了降落。…(中略)…專家普遍認為，最可能的解釋是「兩位駕駛員都睡著了」。…(中略)…迄今為止，沒有任何專家指責兩位駕駛，反倒頗多同情之詞，把矛頭指向ＦＡＡ。飛安專家、退休客機駕駛約翰.南斯接受ＡＢＣ主播吉布遜訪問時說，發生這種事，是因為「多年來，ＦＡＡ不承認駕駛員是人，不承認他們會睏，不准許駕駛在駕駛艙裡睡覺」。

由於ＦＡＡ不准睡，有的駕駛心想，「既然同伴不會睡，我小睡一下應無妨」，說不定兩人都這麼想，就都睡著了。專家說，很多國家准許駕駛員在嚴格的前提小睡一會兒，這些前提包括啟動自動駕駛儀、不能離開駕駛座、告知空服員等。這種作法的立論基礎是：一位擺明了要睡，另一位就絕不敢睡，反而比較安全。就像據傳台灣曾有客機駕駛告訴同僚，「我要在座位上打坐一會兒」，另一位於是格外專心，倒也平安。

Sample Question for the Midterm of Microeconomic Theory I (Fall 2009)

NOTE: This is a sample question that was initially in the midterm, but was subsequently dropped due to space limit (and time limit) of the exam.

Part Z (18%): Air Defense by Aircrafts or Missiles

Consider Daiwan's Ministry of National Defense (MND) who plans to defend Daiwan's airspace using either aircrafts (A) or anti-air missiles (M). The super computers of the MND's "office of war games has converted the effect of different weapons (say, F-16 vs. IDF or PAC-3 vs. Tien Kung-II) into standardized (continuous) units. Suppose Daiwan view the aircrafts and missiles as substitutes: If Daiwan owns a stronger fleet of aircrafts, the rate of substitution is X units of aircrafts for one unit of missiles. If Daiwan owns a stronger array of anti-air missiles, then the rate of substitution is X units of missiles for one unit of aircrafts. Assume initially that X=3.

1. (2%) Write down a utility function to describe the preferences of MND assuming that it is homothetic.
2. (4%) Solve for and draw the income expansion path for MND given the price for aircrafts is US$18.8 million per unit[1] and that for missiles is US$9 million per unit.[2]
3. (2%) Derive the indirect utility function of MND.
4. (4%) Can you use the Roy's Identity to derive MND's demand? Why or why not?
5. (4%) Hence, or otherwise, derive MND's demand functions for both aircrafts and missiles.
6. (**bonus** 2%) What kind of preferences does Daiwan have if X goes to infinity?

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[1] Wiki (http://en.wikipedia.org/wiki/F-16_Fighting_Falcon) states each F-16C/D costs US$18.8 million.
[2] Wiki (http://en.wikipedia.org/wiki/MIM-104_Patriot) states that each PATRIOT unit costs US$ 1 to 3 million. Here we are (rather arbitrarily) assuming that the most advanced PAC-3 system costs US$3 million, and three sets of PAC-3 is equivalent to one standardized unit (that would match one F-16C/D).

List of suggested functions:

B1: $U_i(x, y) = \min\{x + Xy, Xx + y\}$

C1: $U_i(G,T) = T$

C2: $U_i(G,T) = G + T$

D6: $U_i(X) = x_i + \alpha x_{-i}$

D7: $U_i(X) = x_i - \frac{\alpha}{n-1}\sum_{k\neq i}\max(x_k - x_i, 0) - \frac{\beta}{n-1}\sum_{k\neq i}\max(x_i - x_k, 0)$

Game matrix of part D:

	Sleep	Awake
Sleep	-98, -98	12, 10
Awake	10, 12	10, 10